Fine tuning the axioms of relativity to specific subjects

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Joint work with:

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Standard answers:

1. "It is so because we live in a 4-dimensional Minkowski spacetime over $\mathbb{R}$ (the field of real numbers)."
2. "It is an axiom of Special Relativity."

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These answers are not satisfactory for a logician.

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A better answer:

$$\mathsf{SpecRel} \models \forall ob_1, ob_2 \in \mathrm{IOb}\ \forall ph \in \mathrm{Ph} \quad speed_{ob_1}(ob_2) < speed_{ob_1}(ph)$$

where $\mathsf{SpecRel} := \{\mathsf{AxField}, \mathsf{AxSelf}, \mathsf{AxPh}, \mathsf{AxEv}, \boxed{\mathsf{AxSymd}}\}$

(cf., talk of Andréka and Németi)

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An even better answer:

$$\mathsf{SpecRel}_0 \models \forall ob_1, ob_2 \in \mathrm{IOb}\ \forall ph \in \mathrm{Ph} \quad speed_{ob_1}(ob_2) < speed_{ob_1}(ph)$$

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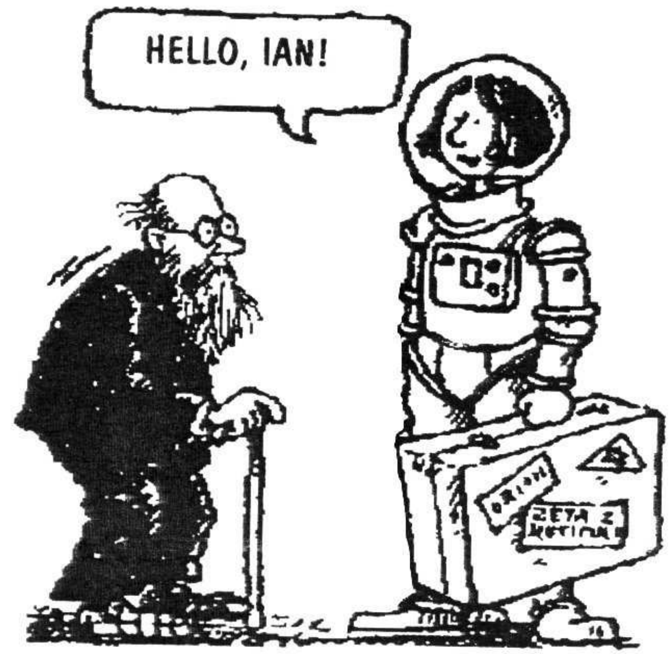
The Twin Paradox

Twin Paradox (TwP) concerns two twin siblings whom we shall call Ann and Ian. Ann travels in a spaceship to some distant star while Ian remains at home. TwP states that when Ann returns home she will be *younger* than her *twin brother* Ian.



Accelerated observers

AxCmv At each moment of his world-line, every observer coordinatizes the nearby world for a short while as an *inertial observer* does.

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Theorem: The world-view transformation between two observers is differentiable at the points where the two observers meet, and its derivative is a Lorentz transformation if AccRel_0 is assumed.

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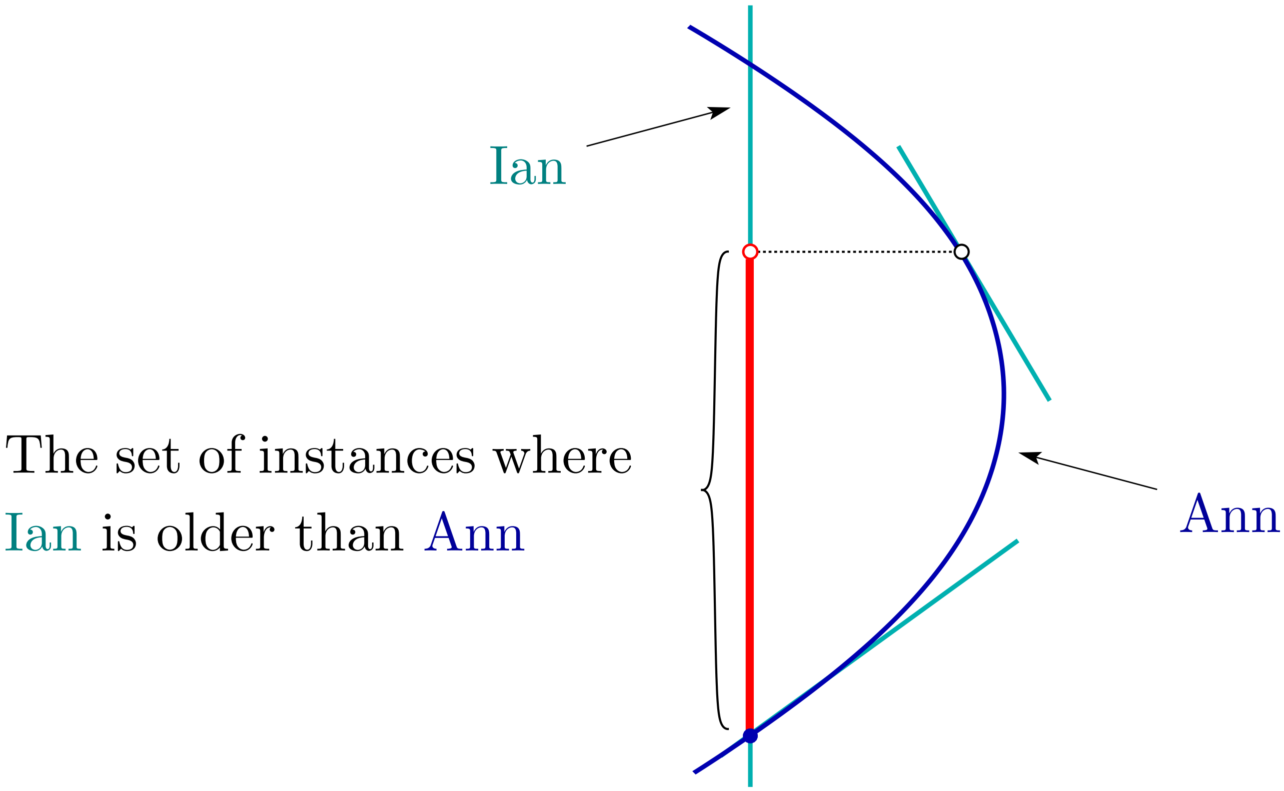

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This theorem has a strong consequence.

Corollary: Assuming even $Th(\mathbb{R})$ and AccRel_0 is not enough to prove the Twin Paradox.

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That is, even assuming AccRel_0 and every first-order formula which is true in $\mathbb{R}$ is not enough to prove the Twin Paradox.

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AxCont speaks not only about the number-line, but about its relation to the other parts of the models (e.g., to the observers).

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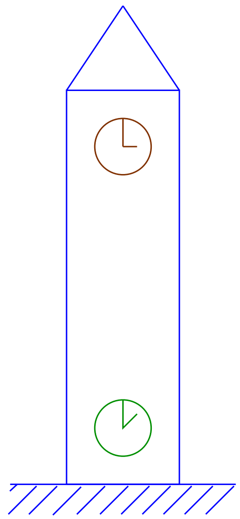
A question for further research is to find a better answer, that is, to prove Twin Paradox from fewer assumption.

Effect of gravitation on clocks within AccRel



Gravitational Time Dilation (GTD):

"The clocks in the bottom of a tower run slower than at its top."

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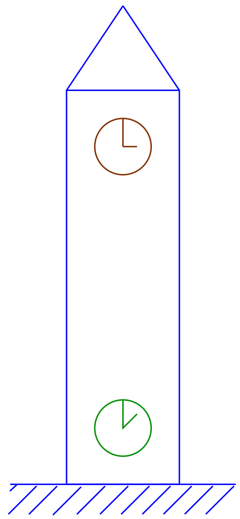

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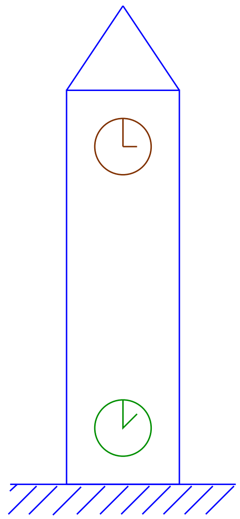

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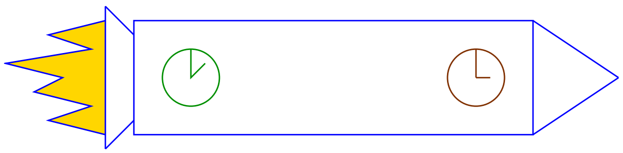
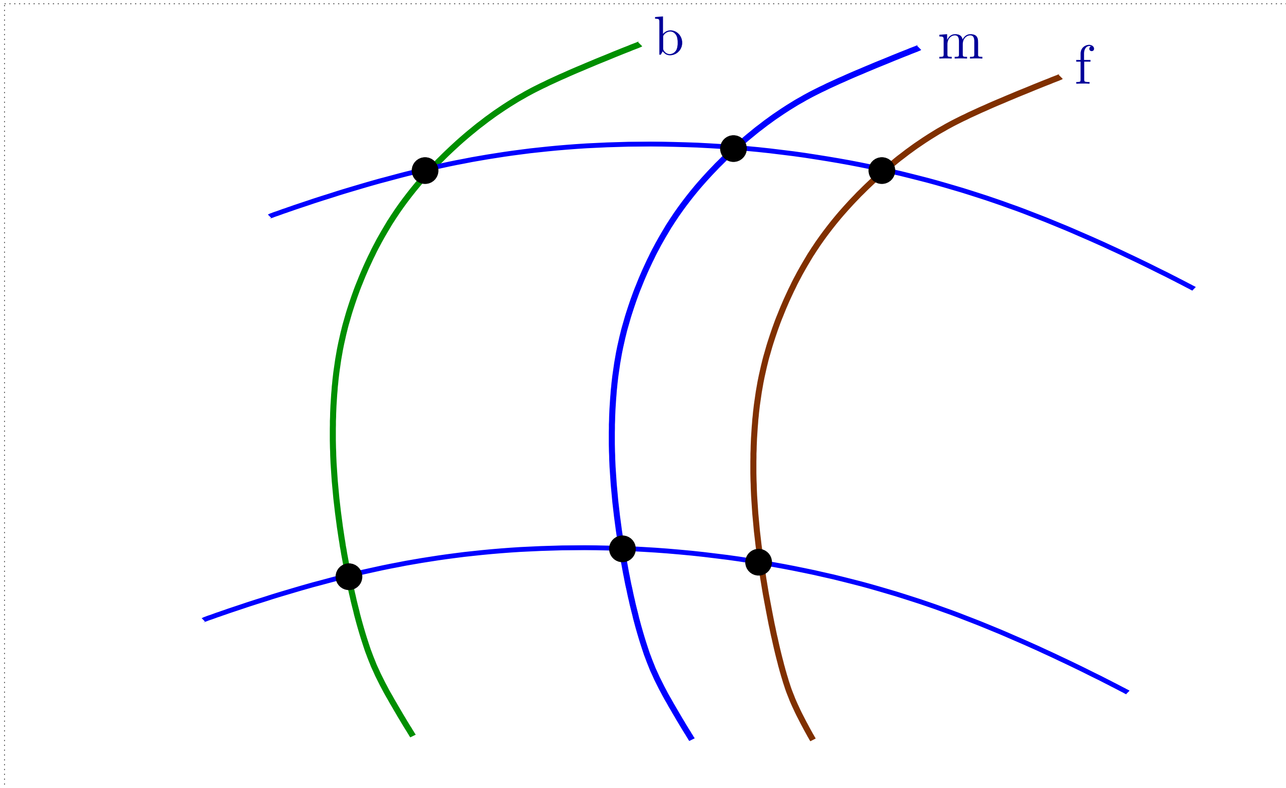
"The clocks in the back of an accelerated spaceship run slower than in its front."



How to formulate GTD within AccRel?



An accelerated spaceship $>\!\!\big|b, m, f\big\rangle$ is a triplet of observers with the following properties.

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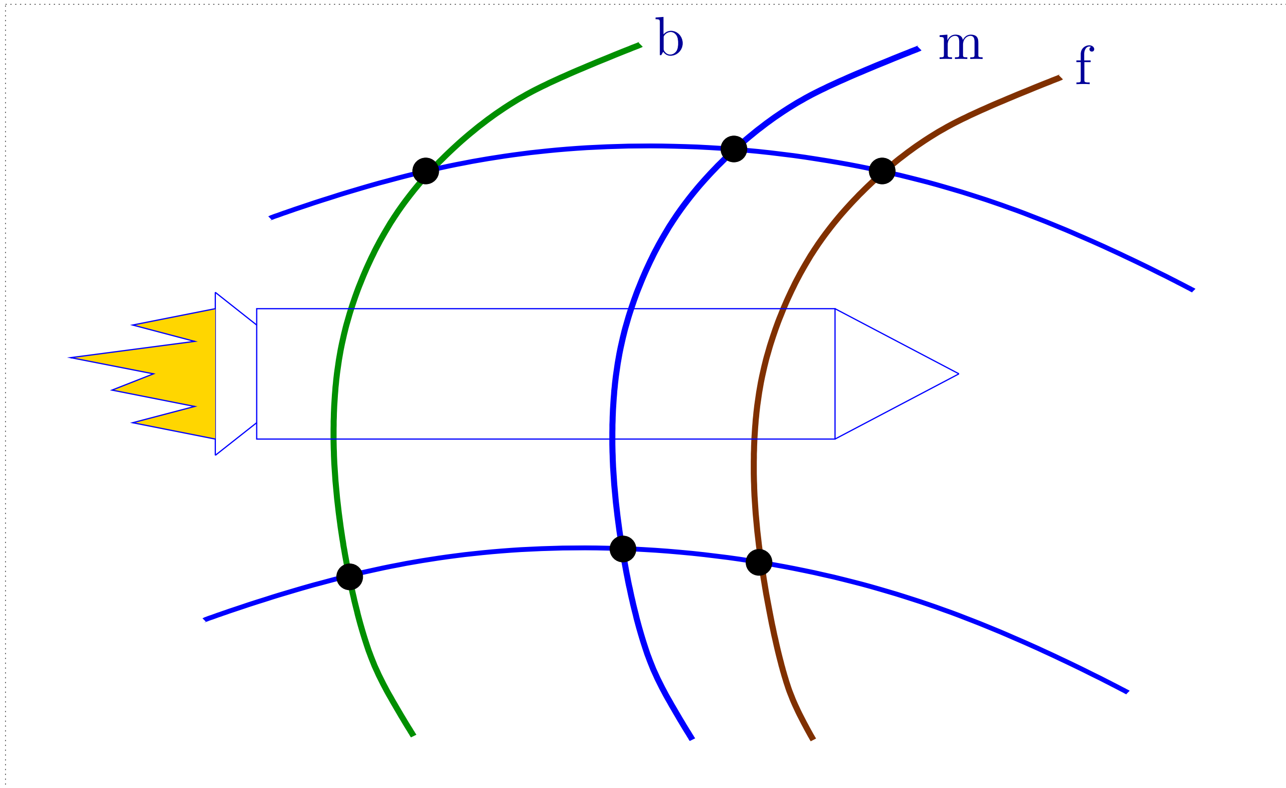

The "back" and the "front" of the spaceship are distinguished by the direction of the acceleration of the observer at the middle.

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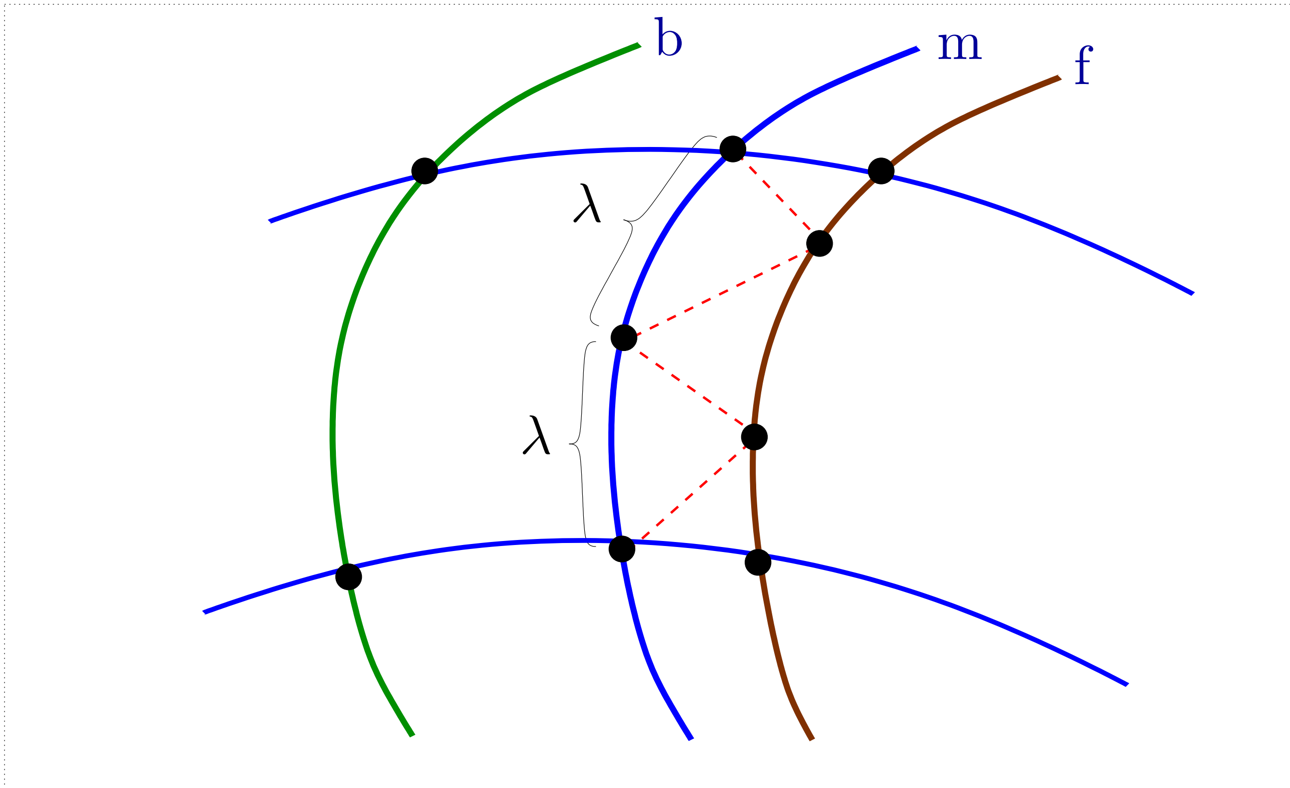

The observers at the front and at the back of the spaceship are of constant radar distance from the one at the middle.

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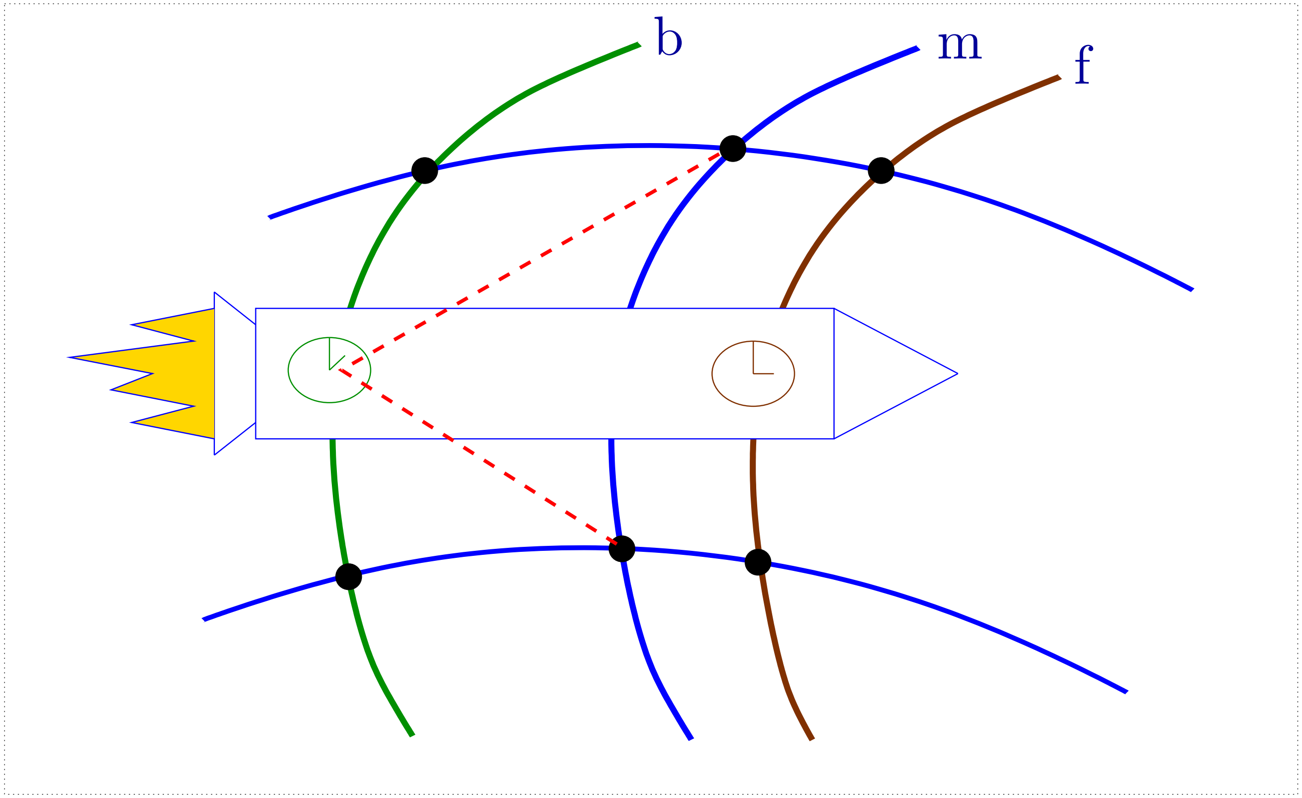
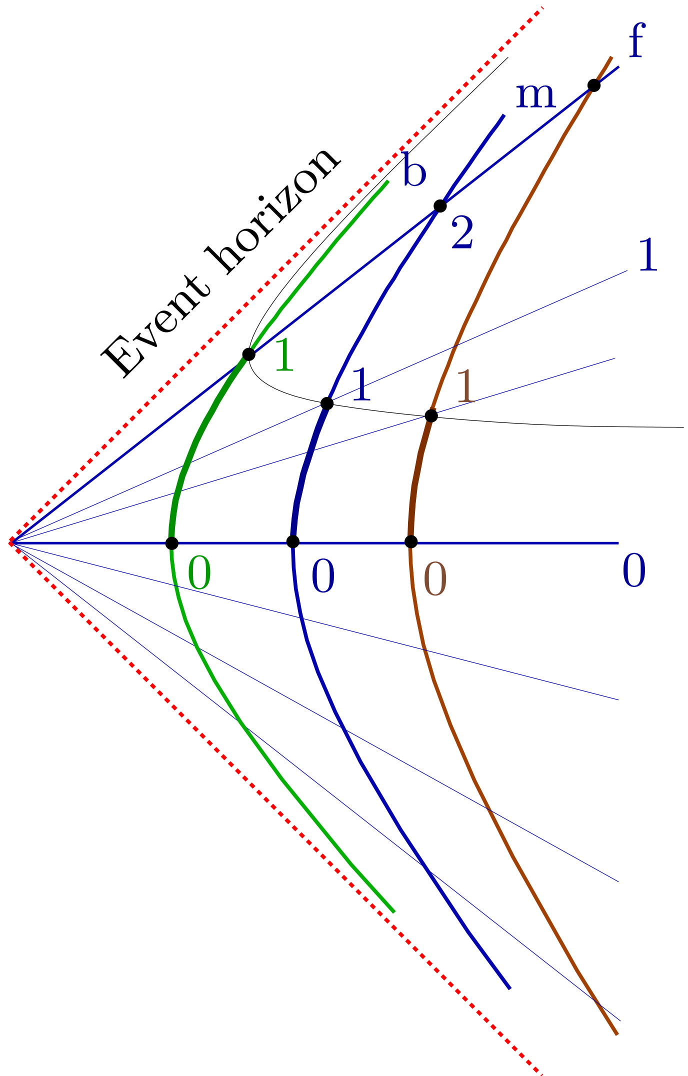


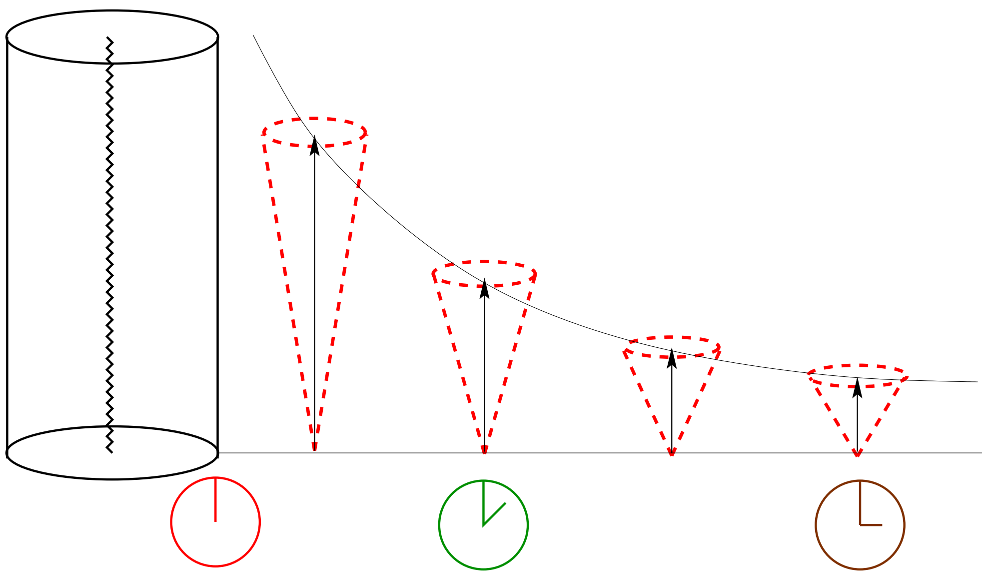
The observer at the middle reads off the clocks of the observers at the front and at the back by radar.

Theorem: The “gravitation causes slow time” follows from the theory $\mathsf{AccRel}_0 + \mathsf{AxCont}$.



Beyond the scope of AccRel

In the “black hole” models of our GenRel axiom system, the closer we are to the black hole, the slower time passes.



Moreover, the time stops at the event horizon.