Relativistic Twin Paradox from FOL point of view

Gergely Székely

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Coauthors:
Judit X. Madarász, István Németi

A fast review

$d \geq 2$ – the dimension of space-time

$$\mathfrak{M} = \langle U; \mathrm{B}, \mathrm{Ob}, \mathrm{IOb}, \mathrm{Ph}, \mathrm{Q}, +, \cdot, \leq, \mathrm{W} \rangle$$

The world-view relation:
$\mathrm{W}(m, b, p)$ – "observer m sees body b at coordinate point p."

AxFrame $\mathrm{Ob} \cup \mathrm{Ph} \subseteq \mathrm{B}$, $\mathrm{IOb} \subseteq \mathrm{Ob}$, $U = \mathrm{B} \cup \mathrm{Q}$, $\mathrm{W} \subseteq \mathrm{Ob} \times \mathrm{B} \times \mathrm{Q}^d$ and the **field reduct** $\mathfrak{F} = \langle \mathrm{Q}; +, \cdot, \leq \rangle$ is an **Euclidean ordered field**, i.e. a linearly ordered field in which positive elements have square roots.

The **event** (the set of bodies) observed by observer m at coordinate point p is:

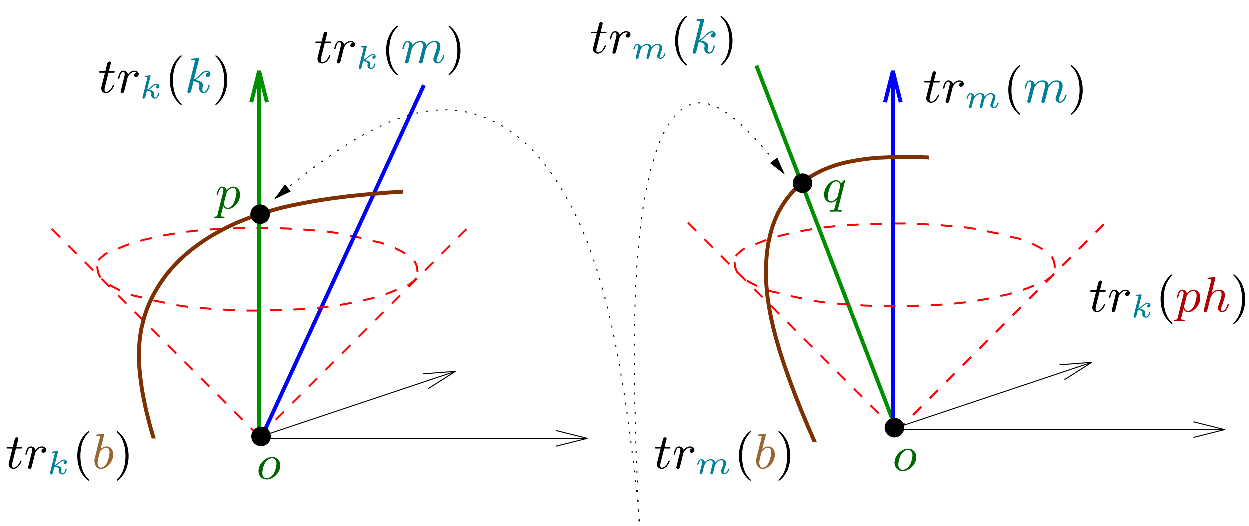
$$ev_m(p) := \{b \in \mathrm{B} : \mathrm{W}(m, b, p)\}$$

The **life-line** (or **trace**) of body b as seen by observer m is defined as the set of coordinate points where b was observed by m:

$$tr_m(b) := \{p \in \mathrm{Q}^d : \mathrm{W}(m, b, p)\}$$

The **world-view transformation** between the world-views of observers k and m is the set of pairs of coordinate points $\langle p, q \rangle$ such that m and k observe the same nonempty event in p and q, respectively:

$$f_m^k := \{\langle p, q \rangle \in \mathrm{Q}^d \times \mathrm{Q}^d : ev_k(p) = ev_m(q) \neq \emptyset\}$$



$ev_k(p) = ev_m(q)$, that is $\langle p, q \rangle \in f_m^k$

AxSelf Each observer sees himself resting at the origin.

AxPh For every inertial observer, the lines of slope 1 are exactly the traces of the photons.

AxEv All inertial observers observe the same events.

AxSymm If events e_1 and e_2 are simultaneous for both inertial observers m and k, then m and k agree on the spatial distance between e_1 and e_2.

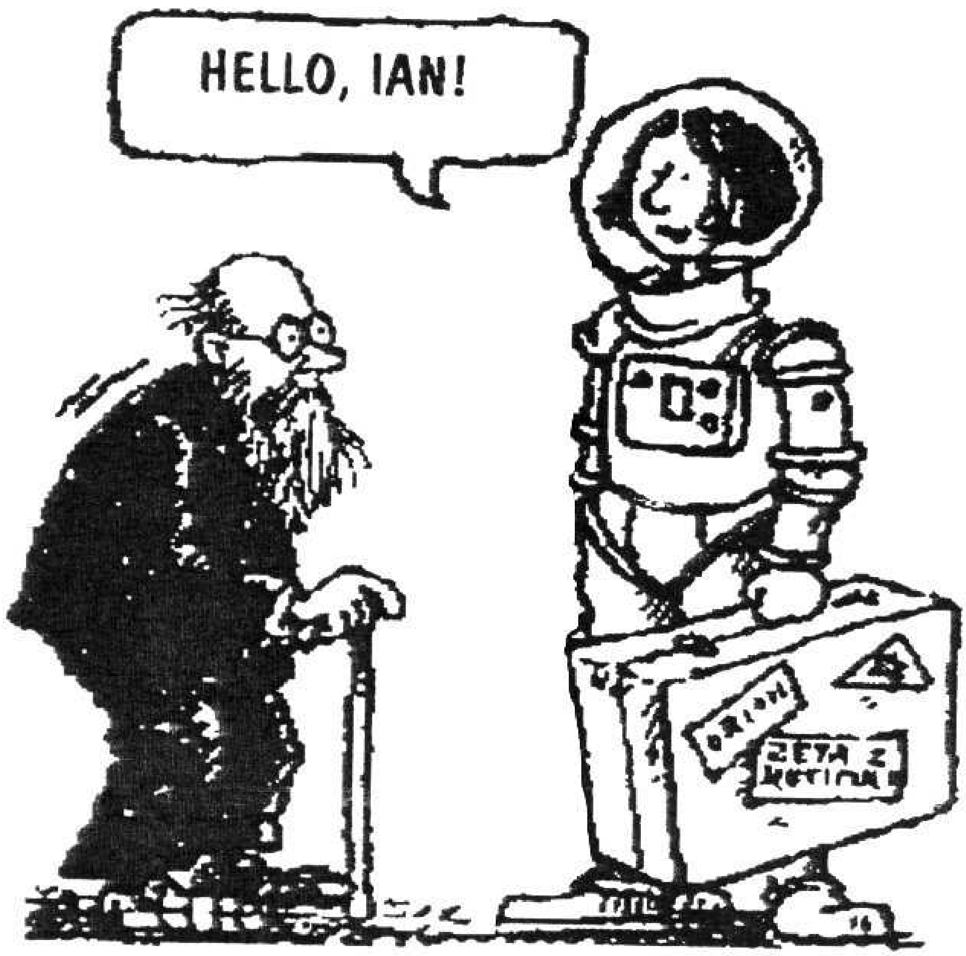
$$\boxed{\mathsf{Specrel} := \{\mathsf{AxFrame}, \mathsf{AxSelf}, \mathsf{AxPh}, \mathsf{AxEv}, \mathsf{AxSym}\}}$$

AxAcc At each moment of his life-line, each observer sees the nearby world for a short while as an inertial observer does.

$$\boxed{\mathsf{AccRel} := \mathsf{Specrel} \cup \{\mathsf{AxAcc}\}}$$

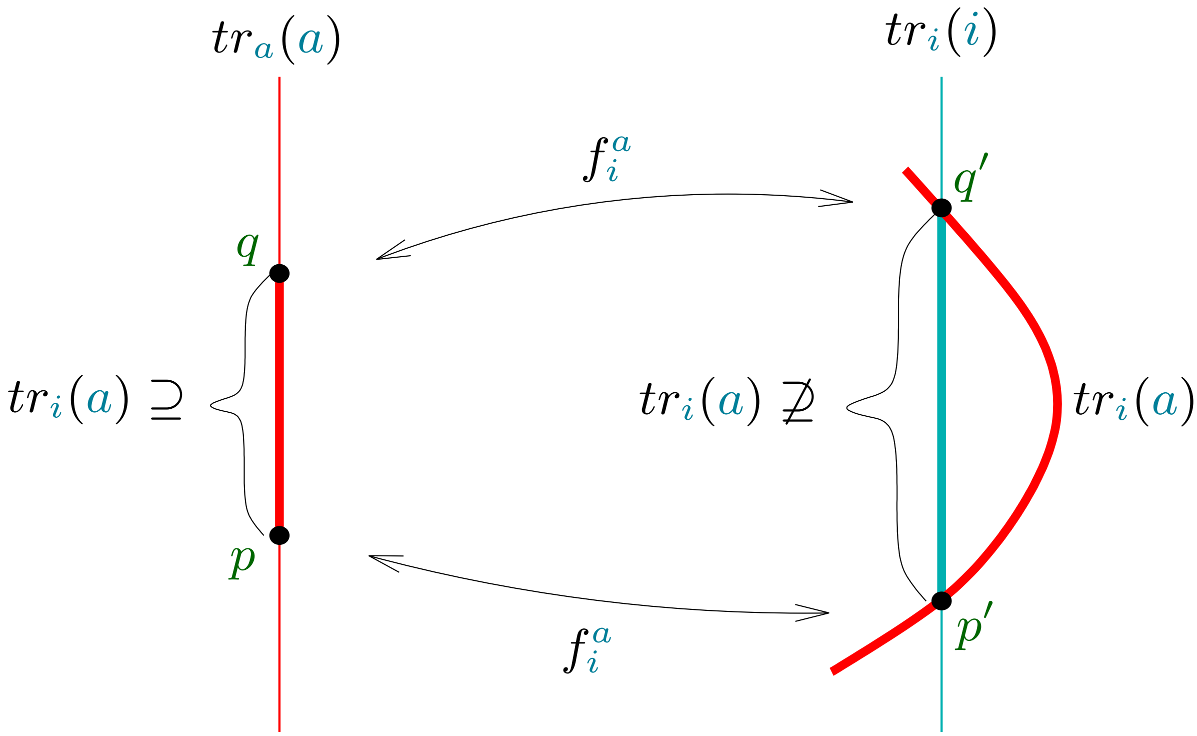
The Twin Paradox

Twin Paradox (TP) concerns two twin siblings whom we shall call Ann and Ian. ("A" and "I" stand for accelerated and for inertial, respectively). Ann travels in a spaceship to some distant star while Ian remains at home. TP states that when Ann returns home she will be *younger* than her *twin brother* Ian.



We say that observer a is in **twin-paradox relation** with observer i iff whenever a leaves i between two meetings, a measures less time between the two meetings than i:

In notation: $\mathsf{Tp}(a < i)$

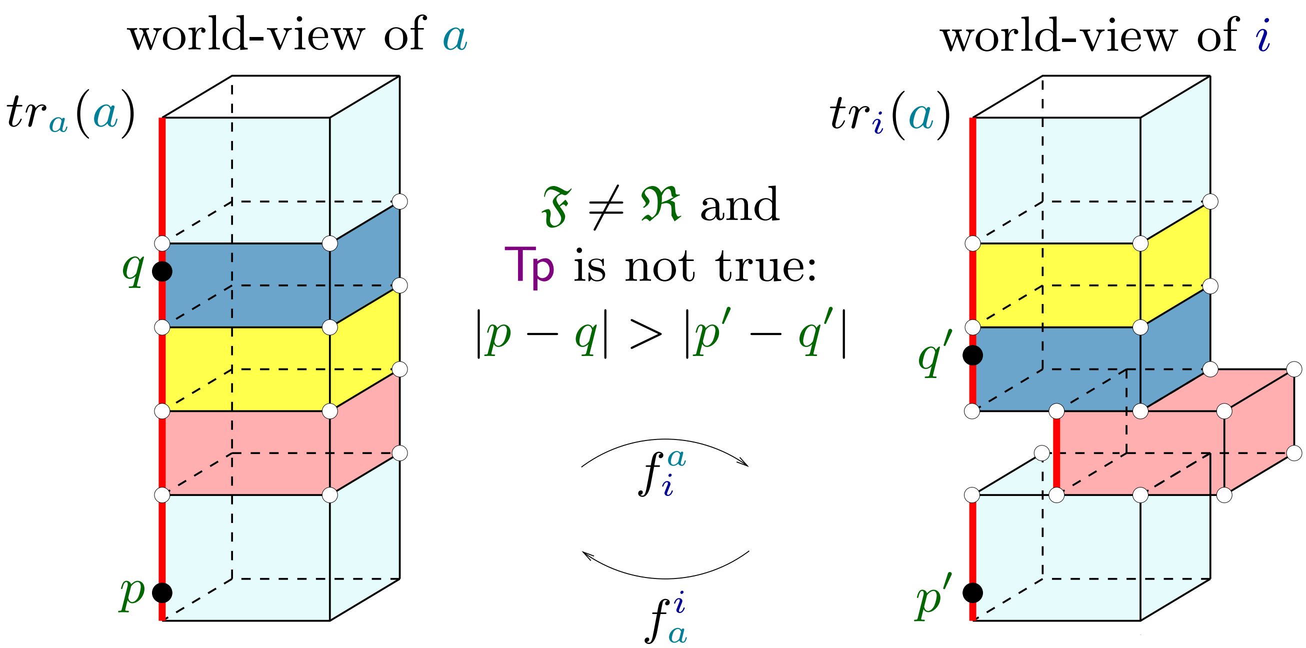


$$\Longrightarrow \; |q-p| < |q'-p'|$$

Tp Every observer is in twin-paradox relation with every inertial observer:

$$\forall a \in \mathrm{Ob} \;\; \forall i \in \mathrm{IOb} \quad \mathsf{Tp}(a < i)$$

Theorem Let $\mathfrak{F}$ be an Euclidean ordered field. There is a model $\mathfrak{M}$ of AccRel such that Tp is not true in $\mathfrak{M}$ with field reduct $\mathfrak{F}$ if and only if $\mathfrak{F}$ not isomorphic to $\mathfrak{R}$.



This theorem has strong consequences, it implies that to prove the Twin Paradox, it does not suffice to add all the FOL-formulas valid in $\mathfrak{R}$ (to AccRel). Let $Th(\mathfrak{R})$ denote the set of all FOL-formulas valid in $\mathfrak{R}$.

Corollary Even assuming AccRel $\cup\, Th(\mathfrak{R})$ is not enough to prove Tp.

The IND scheme

IND If a nonempty and bounded subset of $\mathbb{Q}$ is parametrically definable by a first-order formula of *our language*, then it has a supremum.

$$\boxed{\mathsf{AccRel}^{+} := \mathsf{AccRel} \cup \mathsf{IND}}$$

Theorem

Tp follows from AccRel^{+} if $d \geq 3$.

The strength of IND comes from the fact that the formulas in IND can "talk" about more "things" than just those in the language of $\mathfrak{R}$, they can talk about the world-view relation W, too.