A CONCEPTUAL ANALYSIS OF THE RELATIVISTIC CLOCK PARADOX

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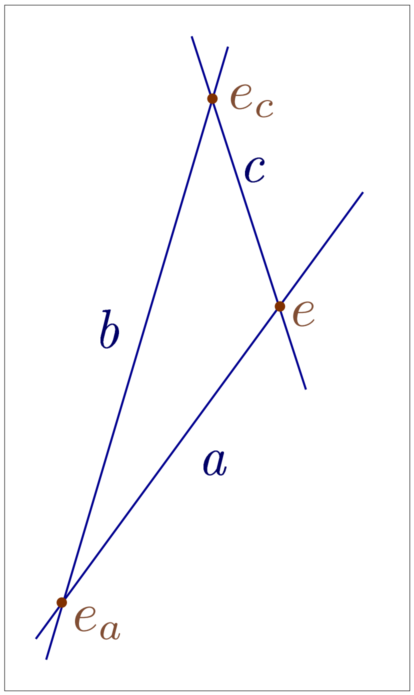
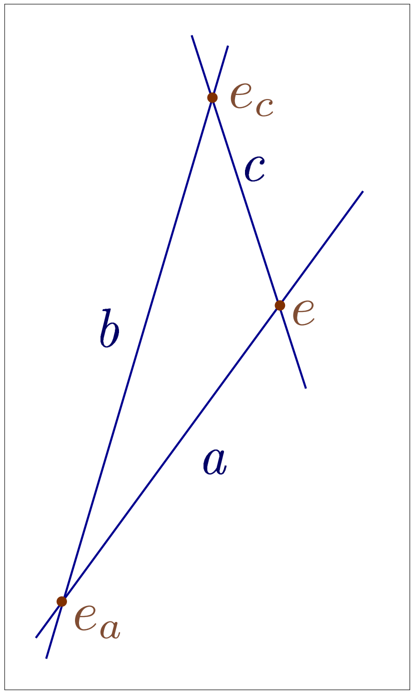
CLOCK PARADOX



ClkP: $\mathsf{time}_b(e_a, e_c) > \mathsf{time}_a(e_a, e) + \mathsf{time}_c(e, e_c)$

VARIANTS OF CLOCK PARADOX

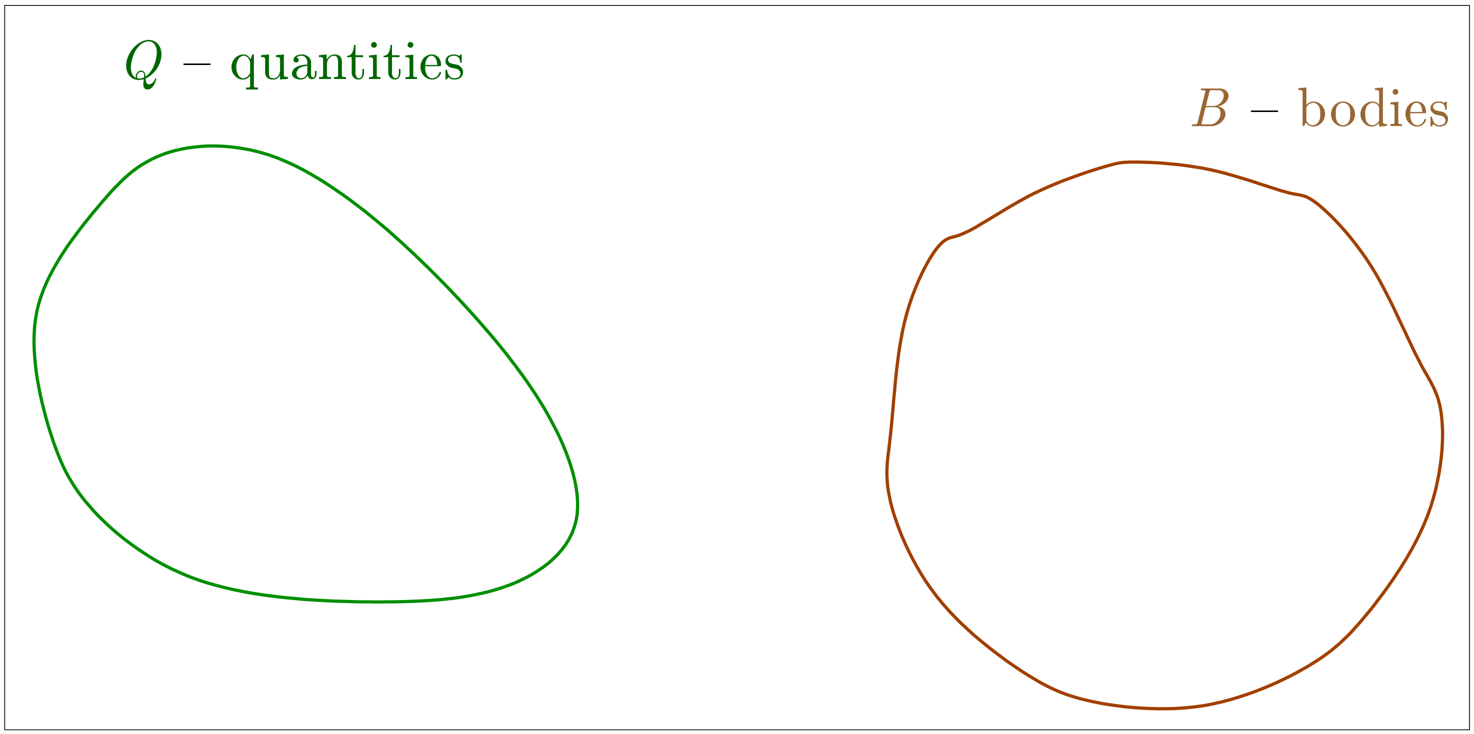


NoClkP: $\text{time}_b(e_a, e_c) = \text{time}_a(e_a, e) + \text{time}_c(e, e_c)$

AntiClkP: $\text{time}_b(e_a, e_c) < \text{time}_a(e_a, e) + \text{time}_c(e, e_c)$

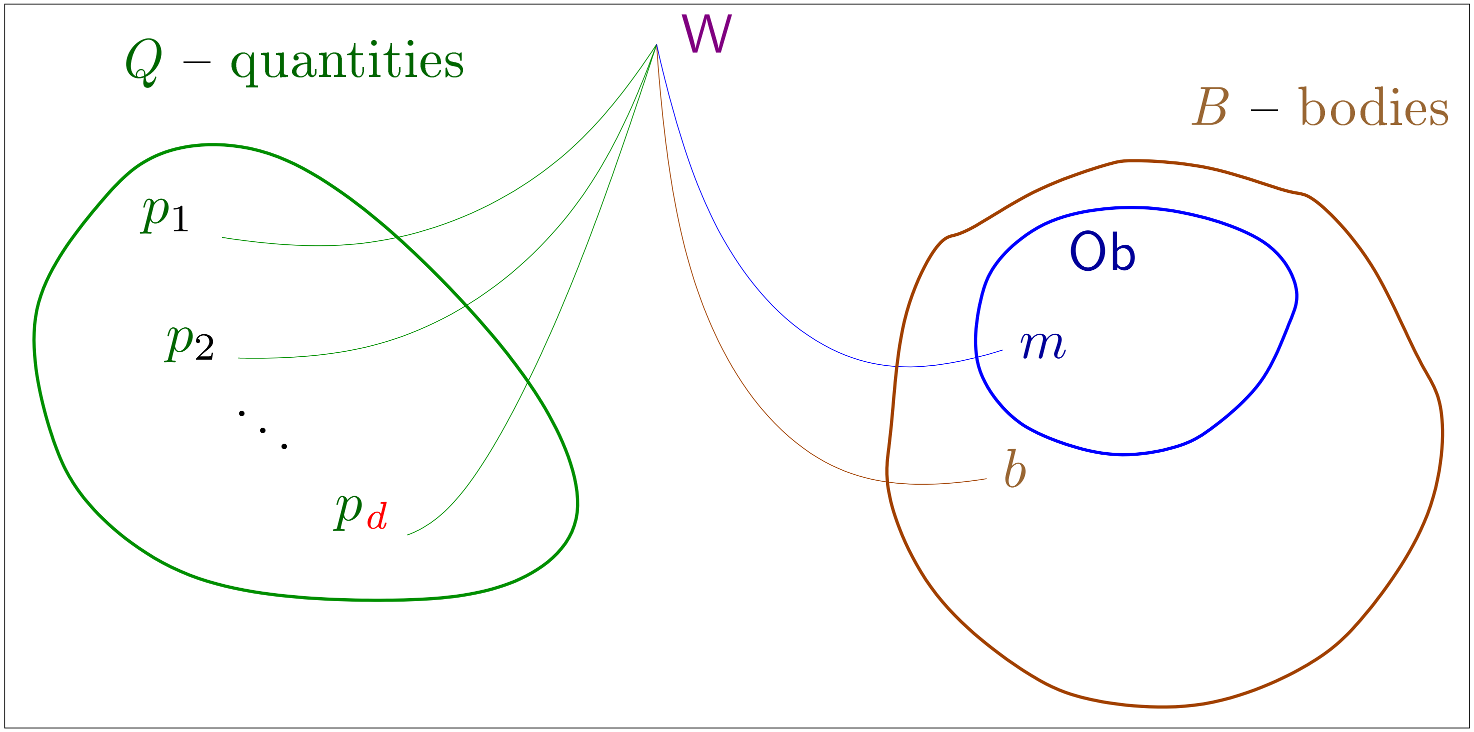
FIRST-ORDER LOGIC FRAMEWORK FOR SPACE-TIMES

Language: $\langle Q : \qquad ; B : \qquad ; \quad \rangle$



Dimension of space-time: $d \geq 2$.

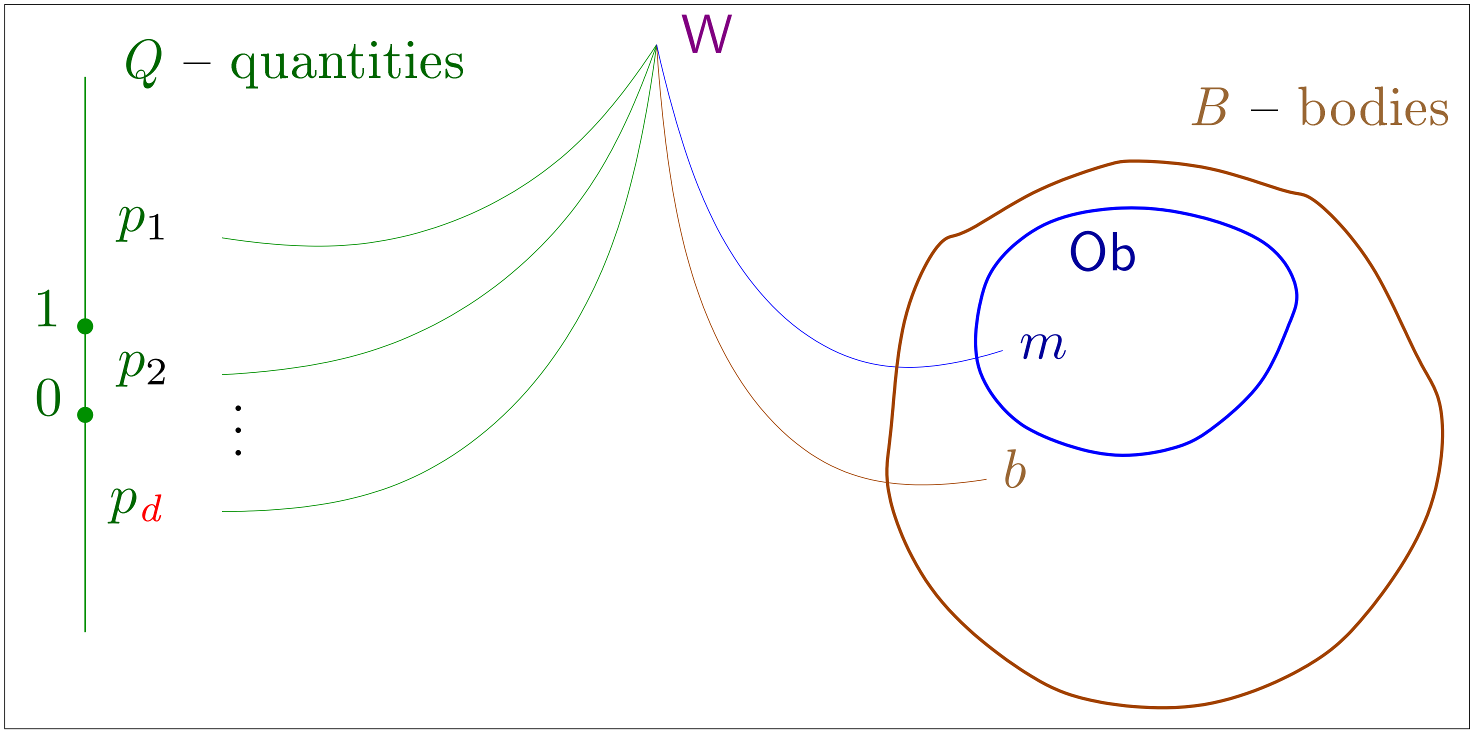
Language: $\langle Q : \qquad ; B : \mathsf{Ob} \qquad ; \mathsf{W} \rangle$



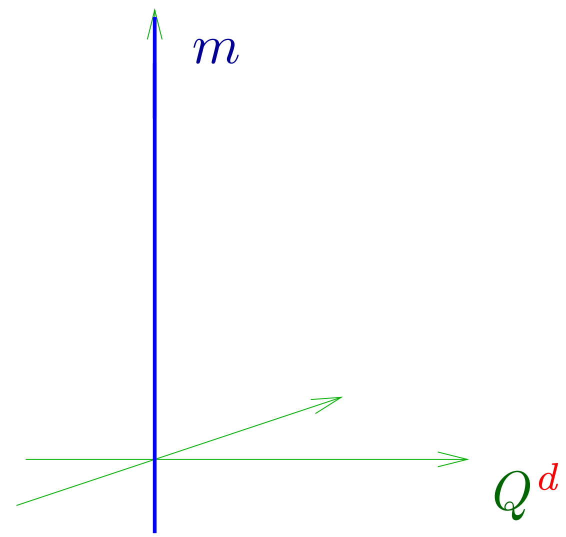
World-view relation: $\mathsf{W}(m, b, \vec{p})$ – "Observer m coordinatizes body b at space-time location $\vec{p}$" (at time p_1 and space $\langle p_2 \dots p_d \rangle$)

STRUCTURE OF QUANTITIES

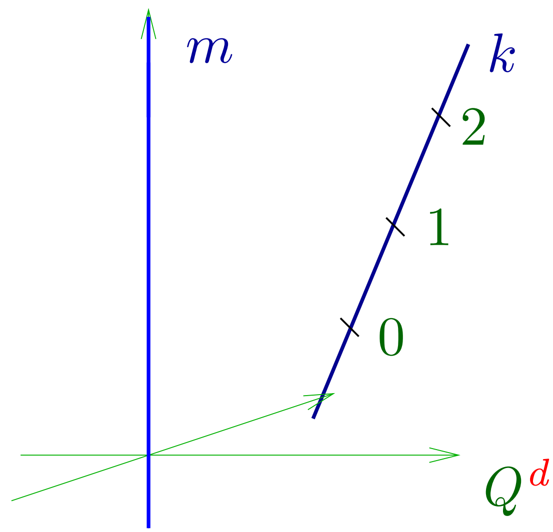
Language: $\langle Q : <, +, \cdot, 0, 1; B : \mathsf{Ob} \quad ; \mathsf{W}\rangle$



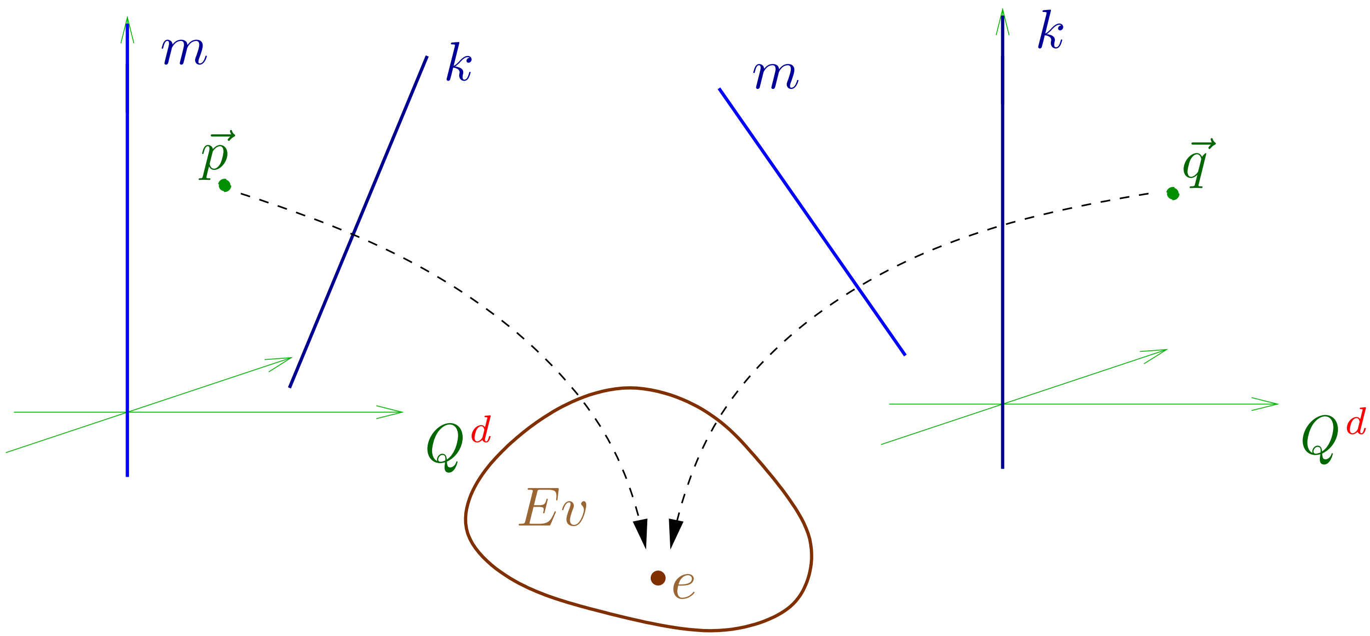
AxEOF: The quantity part $\langle Q; +, \cdot, <, 0, 1\rangle$ is a Euclidean ordered field. (Positive elements have square roots.)



AxSelf: The observer are in rest according to themselves.



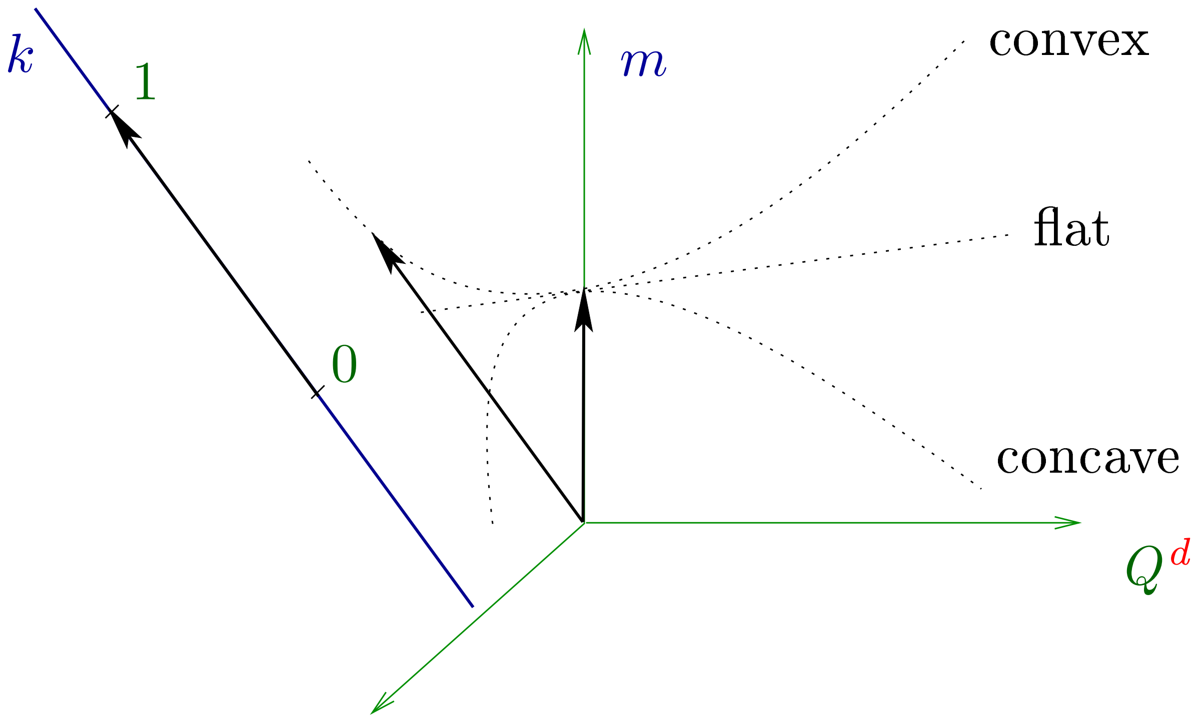
AxLinTime: The life-lines of observers are lines and time is passing uniformly on the life-lines.



AxEv: Every observer coordinatize the same events.

$$\boxed{\mathsf{Kinem}_0 := \{\mathsf{AxEOF}, \mathsf{AxSelf}, \mathsf{AxLinTime}, \mathsf{AxEv}\}}$$

MINKOWSKI SPHERE



MS_m is the set of time-unit vectors.

Theorem: Assume Kinem_0. Then

$$\forall m \in \mathsf{Ob}\ \ MS_m \text{ is convex} \quad \Longrightarrow \quad \mathsf{ClkP},$$

$$\forall m \in \mathsf{Ob}\ \ MS_m \text{ is flat} \quad \Longrightarrow \quad \mathsf{NoClkP},$$

$$\forall m \in \mathsf{Ob}\ \ MS_m \text{ is concave} \quad \Longrightarrow \quad \mathsf{AntiClkP}.$$

Theorem: Assume $\mathsf{Kinem}_0+\mathsf{AxDispl}$. Then

$$\forall m \in \mathsf{Ob}\ \ MS_m \text{ is convex} \iff \mathsf{ClkP},$$
$$\forall m \in \mathsf{Ob}\ \ MS_m \text{ is flat} \iff \mathsf{NoClkP},$$
$$\forall m \in \mathsf{Ob}\ \ MS_m \text{ is concave} \iff \mathsf{AntiClkP}.$$

AxDispl is technical axiom. It is used to displace observers in order to create twin paradox situations.

AxUnivTime: The observers measure the same time between events.

Theorem: AxEOF + AxUnivTime $\models$ NoClkP

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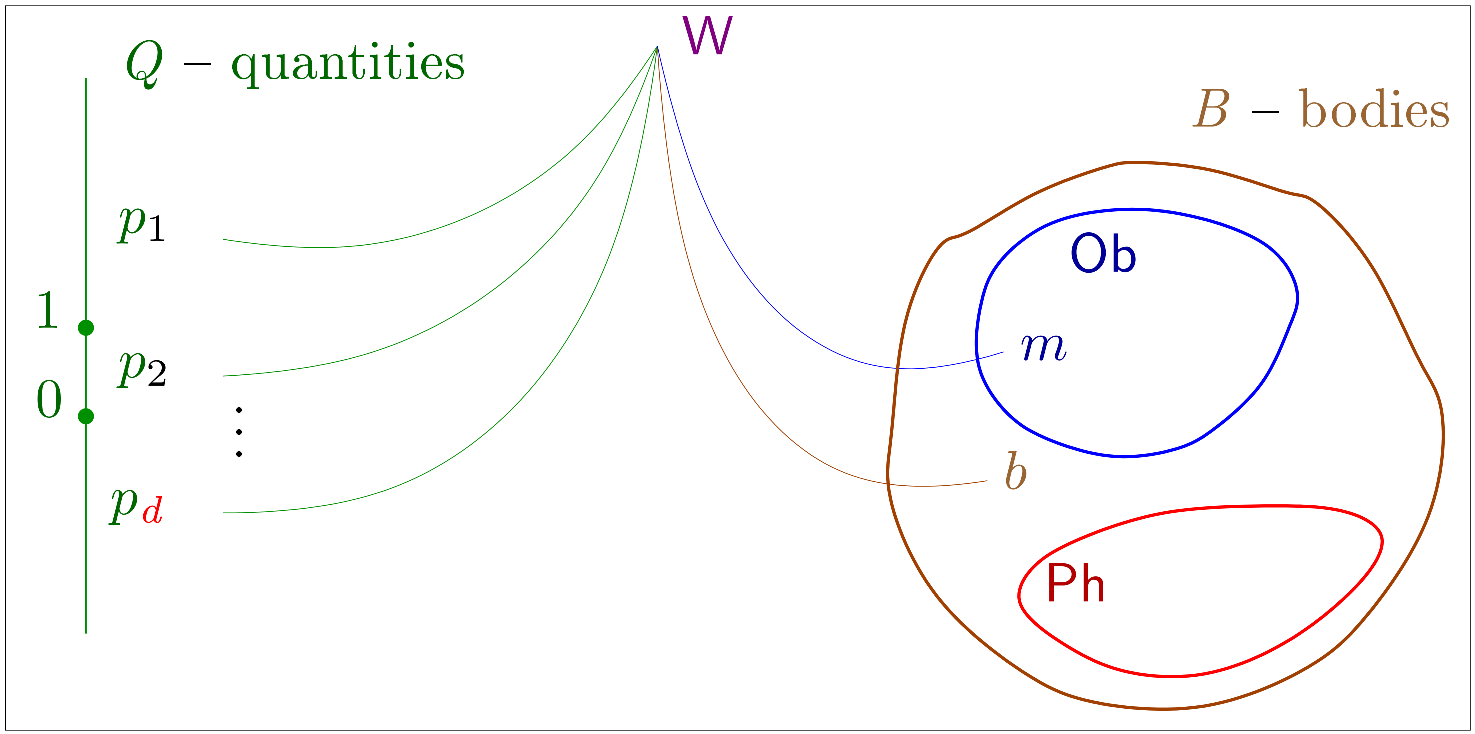
Theorem: AxEOF + AxUnivTime $\models$ NoClkP

AxOb$^+$: Observers can move in any direction with any finite speed.

Theorem: Kinem_0 + AxOb$^+$ + NoClkP $\not\models$ AxUnivTime

Language: $\langle Q : <, +, \cdot, 0, 1; B : \mathsf{Ob}, \mathsf{Ph}; \mathsf{W}\rangle$



AxPh: For every observer, the speed of light is 1.

$$\boxed{\mathsf{SpecRel}_0^d := \{\mathsf{AxEOF}, \mathsf{AxSelf}, \mathsf{AxPh}, \mathsf{AxEv}\}}$$

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We have to weaken AxOb^+ since $\mathsf{SpecRel}_0^d$ implies the impossibility of faster than light motions for observers (if $d \geq 3$).

AxOb: Observers can move in any direction with any speed less than 1 (less that the speed of light).

SlowTime: Relatively moving observers' clocks slow down.

Thm($d \geq 3$): $\mathsf{SpecRel}_0^d + \mathsf{AxLinTime} + \mathsf{SlowTime} \models \mathsf{ClkP}$

$\mathsf{SlowTime}$: Relatively moving observers' clocks slow down.

Thm($d \geq 3$**):** $\mathsf{SpecRel}_0^d + \mathsf{AxLinTime} + \mathsf{SlowTime} \models \mathsf{ClkP}$

Thm($d \geq 3$**):** $\mathsf{SpecRel}_0^d + \mathsf{AxLinTime} + \mathsf{AxOb} + \mathsf{ClkP} \not\models \mathsf{SlowTime}$

AxSimDist: If events e_1 and e_2 are simultaneous for both observers m and k, then m and k agree on the spatial distance between e_1 and e_2.

Thm($d \geq 3$): $\mathsf{SpecRel}_0^d + \mathsf{AxSimDist} \models \mathsf{ClkP}$

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Thm($d \geq 3$): $\mathsf{SpecRel}_0^d + \mathsf{AxSimDist} \models \mathsf{ClkP}$

Thm($d \geq 3$): $\mathsf{SpecRel}_0^d + \mathsf{AxLinTime} + \mathsf{AxOb} + \mathsf{ClkP} \not\models \mathsf{AxSimDist}$

A QUESTION FOR FURTHER RESEARCH

Question: What is the logical connection between AxSimDist and SlowTime?

Remark: If $Q = \mathbb{R}$, then AxSimDist and SlowTime are equivalent in the models of $\mathsf{SpecRel}_0^d + \mathsf{AxLinTime} + \mathsf{AxDispl} + \mathsf{AxOb}$.