INTERSECTION THEOREMS FOR SYSTEMS OF SETS

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A version of Dirichlet's box argument asserts that given a positive integer a and any a^2+1 objects $x_0, x_1, \ldots, x_{a^2}$, there are always $a+1$ distinct indices ν $(0 \leqslant \nu \leqslant a^2)$ such that the corresponding $a+1$ objects x_ν are either all equal to each other or mutually different from each other. This proposition can be restated as follows. *Let N be an index set of more than a^2 elements and let, for each element ν of N, X_ν be a one-element set. Then there is a subset N' of N having more than a elements, such that all intersections $X_\mu X_\nu$ corresponding to distinct elements μ, ν of N' have the same value.* In this note we investigate extensions of this principle to cases when the sets X_ν are of any prescribed cardinal b. Both a and b are given cardinals, finite or infinite. In the case of finite a and b we obtain estimates for the number which corresponds to a^2 in Dirichlet's case, and we show that when at least one of a and b is infinite then a^{b+1} is the best possible value of that number.

We introduce some definitions‡. A system $\Sigma_1 : Y_\nu \, (\nu \in N)$ of sets Y_ν, where ν ranges over the index set N, is said to *contain* the system $\Sigma_0 : X_\mu \, (\mu \in M)$ if, for every μ_0 of M, the set X_{μ_0} occurs in Σ_1 at least as often as in Σ_0, i.e. if

$$|\{\nu : \nu \in N;\ Y_\nu = X_{\mu_0}\}| \geqslant |\{\mu : \mu \in M;\ X_\mu = X_{\mu_0}\}|.$$

If Σ_1 contains Σ_0 and, at the same time, Σ_0 contains Σ_1, then we do not distinguish between the systems Σ_0 and Σ_1.

The system Σ_0 is called a *(a, b)-system* if it consists of a (not necessarily distinct) sets of cardinal b, i.e., if $|M| = a$ and $|X_\mu| = b$ for $\mu \in M$. The system Σ_0 is called a *Δ-system* if it has the property that the intersections of any two of its sets§ have the same value, i.e. if for

$$\mu_0, \mu_1, \mu_2, \mu_3 \in M;\ \mu_0 \neq \mu_1;\ \mu_2 \neq \mu_3$$

we always have $X_{\mu_0} X_{\mu_1} = X_{\mu_2} X_{\mu_3}$. More specifically, Σ_0 is a *$\Delta(a)$-system with kernel* K if $|M| = a$ and $X_{\mu_0} X_{\mu_1} = K$ whenever $\mu_0, \mu_1 \in M;\ \mu_0 \neq \mu_1$. In the special case when $|M| = 1$, say $M = \{\mu_0\}$, we stipulate that $K \subset X_{\mu_0}$, and the empty system Σ_0, for which $M = \emptyset$, is considered as a $\Delta(0)$-system with any arbitrary set K as kernel. Expressions such as

$$(> a,\ \leqslant b)\text{-system}, \quad \Delta(> a)\text{-system}$$

have their obvious meaning. Trivially, every $(> a, 0)$-system is a Δ-system, and the box principle stated above asserts that every

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‡ The cardinal of the set A is denoted by $|A|$, and set union by $A+B$ or $\Sigma(\nu \in N)\, A_\nu$ and set intersection by AB or $\Pi(\nu \in N)\, A_\nu$. $A \subset B$ denotes inclusion, in the wide sense. We use the obliteration operator ^ whose effect consists in removing from a well-ordered series the term above which it is placed. Unless the contrary is stated all sets are allowed to be empty.

§ Not necessarily distinct sets but sets having distinct indices μ.

$(> a^2, 1)$-system contains a $\Delta(> a)$-system. In what follows a and b denote arbitrary cardinals, and b^+ is the next larger cardinal to b.

THEOREM I.

(i) *If* $a, b \geqslant 1$ *then every* $(> b^+ b^b a^{b+1}, \leqslant b)$*-system contains a* $\Delta(> a)$*-system.*

(ii) *If* $a \geqslant 2$; $b \geqslant 1$; $a+b \geqslant \aleph_0$ *then every* $(\dot{>} a^b, \leqslant b)$*-system contains a* $\Delta(> a)$*-system.*

THEOREM II. *For every* a, b *such that* $a, b \geqslant 1$ *there exists a* (a^{b+1}, b)*-system which does not contain any* $\Delta(> a)$*-system.*

THEOREM III. *If* $1 \leqslant a, b < \aleph_0$ *and*

$$c = b!\,a^{b+1}\Big(1 - \frac{1}{2!a} - \frac{2}{3!a^2} - \dots - \frac{b-1}{b!\,a^{b-1}}\Big) \tag{1}$$

then every $(> c, \leqslant b)$*-system contains a* $\Delta(> a)$*-system.*

Remarks. 1. It follows from II that I(ii) is best possible, in the sense that, for $a \geqslant 2$; $b \geqslant 1$; $a+b \geqslant \aleph_0$ not every $(a^b, \leqslant b)$-system contains some $\Delta(> a)$-system.

2. The (a^{b+1}, b)-system of Theorem II will be constructed explicitly.

3. For $a = 2$; $b = 2$ the result III is best possible. For we have $c = 12$, and the following $(12, 2)$-system does not contain any $\Delta(3)$-system.

01, 01, 23, 23, 04, 04, 14, 14, 25, 25, 35, 35, where $xy = \{x, y\}$.

However, for $a = 3$; $b = 2$ Theorem III is not best possible. By II we see that III is best possible except for a factor between 1 and $b!$.

It is not improbable that in (1) the factor $b!$ can be replaced by $c_1{}^b$, for some absolute positive constant c_1. Such a sharpened version of III would have some applications in the theory of numbers, and in fact these applications originally gave rise to the present investigations.

Before proving Theorem I we establish a simple lemma which is at the root of a large number of combinatorial arguments.

RAMIFICATION LEMMA. *Let* α_0 *be an ordinal,* $c_0, c_1, \dots, \hat{c}_{\alpha_0}$ *be cardinals; let* S *be a set and* $M(s_0, s_1, \dots, \hat{s}_\alpha)$ *be a subset of* S *defined for* $\alpha < \alpha_0$ *and* $s_0, \dots, \hat{s}_\alpha \in S$, *such that* $|M(s_0, \dots, \hat{s}_\alpha)| \leqslant c_\alpha$. *Let* V *be a set of "vectors"* $(s_0, s_1, \dots, \hat{s}_{\alpha_0})$ *such that* $s_0, \dots, \hat{s}_{\alpha_0} \in S$ *and*

$$s_\alpha \in M(s_0, \dots, \hat{s}_\alpha) \text{ for } \alpha < \alpha_0. \tag{2}$$

Then
$$|V| \leqslant c_0 c_1 \dots \hat{c}_{\alpha_0}.$$

Proof. For every subset S' of S choose a representation of the form $S' = \{t_0, t_1, \dots, \hat{t}_k\}_{\neq}$, where k is the initial ordinal belonging to $|S'|$.

Define $\phi(S', t)$ for $t \varepsilon S'$ by putting $\phi(S', t_\kappa) = \kappa$ $(\kappa < k)$. Now let $s = (s_0, \ldots, \hat{s}_{\alpha_0}) \varepsilon V$. Then, by (2), we can define an ordinal $\kappa(\alpha, s)$ by putting

$$\kappa(\alpha, s) = \phi\Big(M(s_0, \ldots, \hat{s}_\alpha), s_\alpha\Big) \quad (\alpha < \alpha_0),$$

and a vector $\psi(s) = \Big(\kappa(0, s), \kappa(1, s), \ldots, \hat{\kappa}(\alpha_0, s)\Big)$.

Then $|\kappa(\alpha, s)| < c_\alpha$, and $s \neq s'$ implies $\psi(s) \neq \psi(s')$ as can be seen by considering the least α with $s_\alpha \neq s_\alpha'$. We conclude that

$$|V| = |\{s : s \varepsilon V\}| = |\{\psi(s) : s \varepsilon V\}| \leqslant c_0 c_1 \ldots \hat{c}_{\alpha_0}.$$

Proof of Theorem I. We suppose that the system

$$\Sigma : X_\nu \quad (\nu \varepsilon N)$$

is a $(|N|, \leqslant b)$-system which does not contain any $\Delta(> a)$-system, and our aim is to deduce that

$$|N| \leqslant b + b^b a^{b+1}. \tag{3}$$

Throughout the proof the letters α, β denote ordinals such that $|\alpha|, |\beta| \leqslant b$, and μ, ν denote elements of N. A subset N' of N is called a Δ-*set with kernel* K if the system $X_\nu (\nu \varepsilon N')$ is a Δ-system with kernel K. Put $X = \Sigma\,(\nu \varepsilon N) X_\nu$ and choose an object θ such that $\theta \notin X$. Well-order the sets X and N as well as the set of all subsets of N.

We define elements $f_\alpha(\nu)$ as follows. Let α_0 be fixed, and suppose that $f_\alpha(\nu)$ has already been defined for $\alpha < \alpha_0$ and for all ν, and that

$$f_\alpha(\nu) \varepsilon X_\nu + \{\theta\} \quad (\alpha < \alpha_0 ; \nu \varepsilon N).$$

Let $\nu_0 \varepsilon N$. We now proceed to define $f_{\alpha_0}(\nu_0)$. Put, for any functions $g_0(\nu), \ldots, \hat{g}_{\alpha_0}(\nu)$ defined for $\nu \varepsilon N$ and for any $x_0, \ldots, \hat{x}_{\alpha_0} \varepsilon X + \{\theta\}$

$$F(g_0, \ldots, \hat{g}_{\alpha_0}; x_0, ., \hat{x}_{\alpha_0}) = N\,\Pi\,(\alpha < \alpha)\{\nu : g_\alpha(\nu) = x_\alpha\}.$$

Let $N' \subset N$; $K \subset X + \{\theta\}$. Put

$$H(N') = \Sigma(\nu \varepsilon N') X_\nu$$

and define $\Gamma(N', K)$ to be the first subset N'' of N such that (i) $N'' \subset N'$, (ii) N'' is a Δ-set with kernel K, (iii) N'' is maximal such that (i), (ii) hold. Then, by hypothesis about Σ,

$$|\Gamma(N', K)| \leqslant a.$$

Put $$N(\alpha_0, \nu_0) = F\Big(f_0, \ldots, \hat{f}_{\alpha_0}; f_0(\nu_0), \ldots, \hat{f}_{\alpha_0}(\nu_0)\Big),$$

$$K(\alpha_0, \nu_0) = \{f_\alpha(\nu_0) : \alpha < \alpha_0\},$$

$$N^*(\alpha_0, \nu_0) = \Gamma\Big(N(\alpha_0, \nu_0), K(\alpha_0, \nu_0)\Big).$$

Then $\nu_0 \varepsilon N(\alpha_0, \nu_0)$; $N^*(\alpha_0, \nu_0) \subset N(\alpha_0, \nu_0)$; $|N^*(\alpha_0, \nu_0)| \leqslant a$, *Case* 1. $\theta \varepsilon K(\alpha_0, \nu_0)$. Then put $f_{\alpha_0}(\nu_0) = \theta$.

Case 2. $\theta \notin K(\alpha_0, \nu_0)$.

Case 2*a*. $\nu_0 \varepsilon N^*(\alpha_0, \nu.)$. Then put $f_{\alpha_0}(\nu_0) = \theta$.

Case 2*b*. $\nu_0 \notin N^*(\alpha_0, \nu_0)$. Then, by (iii) above, $N^*(\alpha_0, \nu_0) + \{\nu_0\}$ is not a Δ-set with kernel $K(\alpha, \nu_0)$. If we now assume that $N^*(\alpha_0, \nu_0) = \emptyset$ then this last fact implies that $K(\alpha_0, \nu_0) \not\subset X_{\nu_0}$ which, however, is false. Hence $N^*(\alpha_0, \nu_0) \neq \emptyset$, and there is a first element ν_1 of $N^*(\alpha_0, \nu_0)$ such that $X_{\nu_0} X_{\nu_1} \neq K(\alpha_0, \nu_0)$. Since we are in Case 2, we have $K(\alpha_0, \nu_0) \subset X_{\nu_0}$. Since $\nu_1 \varepsilon N^*(\alpha_0, \nu_0)$ we have $K(\alpha_0, \nu_0) \subset X_{\nu_1}$. Hence $K(\alpha_0, \nu_0) \subset\neq X_{\nu_0} X_{\nu_1}$, and we may define $f_{\alpha_0}(\nu_0)$ to be the first element of the set $X_{\nu_0} X_{\nu_1} - K(\alpha_0, \nu_0)$. This completes the definition of $f_\alpha(\nu)$. We have, in Case 2b,

$$f_{\alpha_0}(\nu_0) \varepsilon X_{\nu_1} \subset H\left(N^*(\alpha_0, \nu_0)\right). \tag{4}$$

Let $\nu \varepsilon N$. If for some α, we have $f_\alpha(\nu) \neq \theta$ then Case 2b applies to α, and hence also to each $\beta \leqslant \alpha$; the elements $f_\beta(\nu)$ $(\beta \leqslant \alpha)$ are therefore distinct elements of X_ν. Hence, in view of $|X_\nu| \leqslant b$, there is β_ν such that $f_\alpha(\nu) \varepsilon X_\nu$ for $\alpha < \beta_\nu$, and $f_{\beta_\nu}(\nu) = \theta$. Then Case 2a applies to β_ν, and we have $\nu \varepsilon N^*(\beta_\nu, \nu)$. This shows that†

$$N = \Sigma(\nu \varepsilon N) N^*(\beta_\nu, \nu), \quad |N| \leqslant a |\{N^*(\beta_\nu, \nu) : \nu \varepsilon N\}|.$$

We now prove that on the right hand side N^* may be replaced by N. Let $N(\beta_\mu, \mu) = N(\beta_\nu, \nu)$. Then $\mu \varepsilon N(\beta_\mu, \mu) = N(\beta_\nu, \nu)$; $f_\alpha(\mu) = f_\alpha(\nu) \varepsilon X_\nu$ for $\alpha < \beta_\nu$; $\beta_\mu \geqslant \beta_\nu$ and hence, by symmetry, $\beta_\nu \geqslant \beta_\mu$. Therefore $\beta_\mu = \beta_\nu$,

$$K(\beta_\mu, \mu) = K(\beta_\nu, \nu) \; ; \; N^*(\beta_\mu, \mu) = N^*(\beta_\nu, \nu).$$

Thus

$$|\{N^*(\beta_\nu, \nu) : \nu \varepsilon N\}| \leqslant |\{N(\beta_\nu, \nu) : \nu \varepsilon N\}|.$$

For any α and any $x_0, \ldots, \hat{x}_\alpha \varepsilon X$ put

$$G(x_0, \ldots, \hat{x}_\alpha) = F(f_0, \ldots, \hat{f}_\alpha \, ; x_0, \ldots, \hat{x}_\alpha),$$

$$M(x_0, \ldots, \hat{x}_\alpha) = H\left(\Gamma\left(G(x_0, \ldots, \hat{x}_\alpha), \{x_0, \ldots, \hat{x}_\alpha\}\right)\right).$$

Then

$$N(\alpha, \nu) = G\left(f_0(\nu), \ldots, \hat{f}_\alpha(\nu)\right).$$

Let α_0 be fixed such that $\alpha_0 \varepsilon \{\beta_\nu : \nu \varepsilon N\}$. Choose any ν with $\beta_\nu = \alpha_0$.

Then

$$N(\beta_\nu, \nu) = G\left(f_0(\nu), \ldots, \hat{f}_{\alpha_0}(\nu)\right).$$

Hence $N(\beta_\nu, \nu)$ is determined if the vector $s = \left(f_0(\nu), \ldots, \hat{f}_{\alpha_0}(\nu)\right)$ is known. Denote by S the set of all such vectors s, i.e. the set of all s which correspond to choices of ν such that $\beta_\nu = \alpha_0$. Let now $s = (s_0, \ldots, \hat{s}_{\alpha_0}) \varepsilon S$; $\alpha < \alpha_0$. We proceed to show that $s_\alpha \varepsilon M(s_0, \ldots, \hat{s}_\alpha)$.

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† We remind the reader that $|\{N^*(\beta_\nu, \nu) : \nu \varepsilon N\}|$ denotes the number of distinct sets $N^*(\beta_\nu, \nu)$.

We can choose $\nu \varepsilon N$ such that $\beta_\nu = \alpha_0$, and $s_\beta = f_\beta(\nu)$ for all $\beta < \alpha_0$. Then

$$\begin{aligned} s_\alpha = f_\alpha(\nu) \,\varepsilon\, H\Big(N^*(\alpha, \nu)\Big) &= H\Big(\Gamma\Big(N(\alpha, \nu), K(\alpha, \nu)\Big)\Big) \\ &= H\bigg(\Gamma\Big(G\big(f_0(\nu), \ldots, \hat{f}_\alpha(\nu)\big), \{f_0(\nu), \ldots, \hat{f}_\alpha(\nu)\}\Big)\bigg) \\ &= M(s_0, \ldots, \hat{s}_\alpha). \end{aligned}$$

In addition, we have

$$|M(s_0, \ldots, \hat{s}_\alpha)| = |H\Big(N^*(\alpha, \nu)\Big)| \leqslant b\,|N^*(\alpha, \nu)| \leqslant ba.$$

Hence, by the ramification lemma, when ν ranges through all values for which β_ν has the fixed value α_0, there arise at most $(ba)^{|\alpha_0|}$ distinct vectors $\Big(f_0(\nu), \ldots, \hat{f}_{\alpha_0}(\nu)\Big)$. We deduce that

$$\begin{aligned} |N| &\leqslant a\,|\{N^*(\beta_\nu, \nu) : \nu \,\varepsilon\, N\}| \leqslant a\,|\{N(\beta_\nu, \nu) : \nu \,\varepsilon\, N\}| \\ &= a\,\Sigma(|\alpha_0| \leqslant b)\,|\{N(\beta_\nu, \nu) : \nu \,\varepsilon\, N\,;\, \beta_\nu = \alpha_0\}| \\ &\leqslant a\,\Sigma(|\alpha_0| \leqslant b)(ba)^{|\alpha_0|} \leqslant a(ba)^b b^+, \end{aligned}$$

which proves (3) and so establishes I(i).

Part (ii) of Theorem I follows from (i). For if $a \geqslant 2$; $b \geqslant 1$; $a + b \geqslant \aleph_0$; then $b^+ b^b a^{b+1} = a^b$.

Proof of Theorem II. Choose sets A, B such that $|A| = a$; $|B| = b$, and let F be the set of all mappings of B into A. Consider the system

$$\Sigma : X(t, f) = \Big(x, f(x)\Big) : x \,\varepsilon\, B\Big\} \quad (t \,\varepsilon\, A\,;\, f \,\varepsilon\, F).$$

We consider the members of Σ as indexed by the pairs (t, f). In fact, they do not depend on t. Then Σ is a (a^{b+1}, b)-system. Let us assume that Σ contains a $\Delta(> a)$-system Σ' with kernel K, say the system

$$\Sigma' : X_\rho = X(t_\rho, f_\rho) \quad (\rho \,\varepsilon\, R).$$

Then $|R| > a$ and

$$(t_\rho, f_\rho) \neq (t_\sigma, f_\sigma) \text{ for } \{\rho, \sigma\}_{\neq} \subset R. \tag{5}$$

Let $x \,\varepsilon\, B$. Then $|\{f_\rho(x) : \rho \,\varepsilon\, R\}| \leqslant |A| = a < |R|$, and hence there is $\{\rho_x, \sigma_x\}_{\neq} \subset R$ with $f_{\rho_x}(x) = f_{\sigma_x}(x)$. Then, for any $\rho \,\varepsilon\, R$,

$$\Big(x, f_{\rho_x}(x)\Big) \,\varepsilon\, X_{\rho_x} X_{\sigma_x} = K \subset X_\rho = \Big\{\Big(y, f_\rho(y)\Big) : y \,\varepsilon\, B\Big\},$$

$f_{\rho_x}(x) = f_\rho(x)$, so that f_ρ is independent of ρ. Since $|\{t_\rho : \rho \,\varepsilon\, R\}| \leqslant |A| < |R|$ there is $\{\rho_1, \sigma_1\}_{\neq} \subset R$ with $t_{\rho_1} = t_{\sigma_1}$. But then $(t_{\rho_1}, f_{\rho_1}) = (t_{\sigma_1}, f_{\sigma_1})$ which contradicts (5). This proves Theorem II.

Proof of Theorem III. Let $1 \leqslant a$, $b < \aleph_0$. By Theorem I there exists a least number d, where $d < \aleph_0$, such that every $(> d, \leqslant b)$-system contains a $\Delta(>a)$-system. Denote this number by $f(a, b)$. We have to show that

$$f(a, b) \leqslant c,$$

where c is defined by (1).

There is a least number $\phi(a, b)$ such that every $(>\phi, \leqslant b)$-system

$$\Sigma : X_\phi \quad (\mu \varepsilon M)$$

which satisfies $X_\mu \neq X_\nu$ for $\{\mu, \nu\}_{\neq} \subset M$, contains a $\Delta(>a)$-system. Clearly, $\phi \leqslant f$. Also, $\phi(a, 1) = a$. We first show that

$$f(a, b) \leqslant a\phi(a, b). \tag{6}$$

Let

$$\Sigma' : X_\nu \quad (\nu \varepsilon N)$$

be a $\big(> a\phi(a, b), \leqslant b\big)$-system which does not contain any $\Delta(>a)$-system. We have to deduce a contradiction. Let

$$\nu_0 \varepsilon N;\ K(\nu_0) = \{\nu : \nu \varepsilon N;\ X_\nu = X_{\nu_0}\}.$$

Then $X_\nu \ \big(\nu \varepsilon K(\nu_0)\big)$ is a Δ-system, and therefore $|K(\nu_0)| \leqslant a$. Hence, if

$$\{X_\nu : \nu \varepsilon N\} = \{X_\mu : \mu \varepsilon M\}, \quad X_\mu \neq X_\nu \text{ for } \{\mu, \nu\}_{\neq} \subset M,$$

then $|M| > \phi(a, b)$, and it follows from the definition of ϕ that the system $X_\mu (\mu \varepsilon M)$ contains a $\Delta(>a)$-system. This is the required contradiction.

There is a $\big(\phi(a, b), \leqslant b\big)$-system

$$\Sigma : X_\nu \quad (\nu \varepsilon N),$$

where $X_\mu \neq X_\nu$ for $\mu \neq \nu$, which does not contain any $\Delta(>a)$-system. Let N_0 be a maximal subset of N such that $X_\mu X_\nu = \emptyset$ for $\{\mu, \nu\}_{\neq} \subset N_0$. Then $|N_0| \leqslant a$, since $X_\nu\ (\nu \varepsilon N_0)$ is a Δ-system. Put $X^* = \Sigma(\nu \varepsilon N_0) X_\nu$. Then we can choose elements

$$x_\mu \varepsilon X_\mu X^* \quad (\mu \varepsilon N - N_0).$$

Let $\xi \varepsilon X^*$. Then there is $\nu_0(\xi) \varepsilon N_0$ with $\xi \varepsilon X_{\nu_0(\xi)}$. Then the system (of sets of at most $b-1$ elements)

$$X_{\nu_0(\xi)} - \{\xi\};\ X_\mu - \{\xi\} \quad (\mu \varepsilon N - N_0;\ x_\mu = \xi)$$

does not contain any $\Lambda(>a)$-system since any such system Σ'' would yield a $\Delta(>a)$-system contained in Σ if we add to each member of Σ'' the element ξ. Hence $1 + |\{\mu : \mu \varepsilon N - N_0;\ x_\mu = \xi\}| \leqslant \phi(a, b-1)$,

$$\begin{aligned}\phi(a, b) = |N| = |N_0| + |N - N_0| &\leqslant a + \Sigma(\xi \varepsilon X^*)\,|\{\mu : \mu \varepsilon N - N_0;\ x_\mu = \xi\}| \\ &\leqslant a + \big(\phi(a, b-1) - 1\big)\, ba = -a(b-1) + ab\phi(a, b-1),\end{aligned}$$

$$\frac{\phi(a, b)}{b!\,a^b} \leqslant -\frac{b-1}{b!\,a^{b-1}} + \frac{\phi(a, b-1)}{(b-1)!\,a^{b-1}}.$$

By means of $b-1$ successive applications of this inequality we obtain

$$\frac{\phi(a, b)}{b!\,a^b} \leqslant -\frac{b-1}{b!\,a^{b-1}} - \frac{b-2}{(b-1)!\,a^{b-2}} - \dots - \frac{1}{2!\,a} + \frac{\phi(a, 1)}{1!\,a}.$$

In view of (6) and $\phi(a, 1) = a$ this is the desired result.

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