Visualizing ideas about Gödel-type rotating universes

István Németi, Judit X. Madarász, Hajnal Andréka, and Attila Andai

Abstract. This paper consists mostly of pictures visualizing ideas leading to Gödel's rotating cosmological model. The pictures are constructed according to concrete metric tensor fields. The main aim is to visualize ideas.

Some kinds of physical theories describe what our universe looks like. Other kinds of physical theories describe instead what the universe could be like independently of the properties of the actual universe. This second kind aims for the "basic laws of physics" in some sense which we will not make precise here (but cf. e.g. Malament [**Mal84**, pp.98–99]). The present paper belongs to the second kind. Moreover, it is even more abstract than this, namely it aims for visualizing or grasping some mathematical or logical aspects of what the universe could be like.

The first few pages of this material are of a "science-popularizing" character in the sense that first we recall a space-time diagram from Hawking–Ellis [**HE73**] as "God-given truth", i.e. we do not explain why the reader should believe that diagram. Then we derive in an easily understandable visual manner an exciting, exotic consequence of that diagram: time-travel. This applies to the first few pages. The rest of this work is of a more ambitious character. *The reader does not have to believe anything* [1]. We do our best to make the paper self-contained and explain and visualize most of what we say.

In more detail, this work consists of Sections 1-8. Section 1 (p.2) is the just mentioned "popular" part. Section 2 (p.4) lays the foundation for discussing rotating universes. E.g. it shows how to visualize such space-times. The space-time built up in this section is called the "Naive Spiral world". The last part of section 2 (p.12) is about non-existence of a natural "now" in Gödel's universe GU. Section 3 (p.14) introduces co-rotating coordinates "transforming the rotation away". Section 4 (p.21) refines the Gödel-type universe (obtained in Section 2). Section

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[1]Not even the diagram recalled from Hawking–Ellis [**HE73**] in Figure 1 or any of the statements made in the first few pages.

5 (p.30) illustrates a fuller view of the refined version of GU. Section 6 (p.33) recoordinatizes the refined GU in order that the so-called gyroscopes do not rotate in this coordinatization. Section 7 (p.40) gives connections with the literature. E.g. it presents detailed computational comparison with the space-time metric in Gödel's papers. Section 8 (p.43) contains some technical data about how we constructed the figures illustrating Gödel's universe. More technical data can be found in [**NMAA**].

1. Prelude: Some facts from the literature and how they imply time-travel.

The following two figures represent Gödel's famous rotating universe. One of the many interesting features of Gödel's universe is that it contains closed timelike curves (CTC's for short), i.e. it permits "time-travel". In the following figures we use geodesics and light-cones in the spirit of e.g. [**AMN07**, sections 3.1–3.3] for visualizing Gödel's universe together with some of its main features. For these notions cf. p.5 herein.

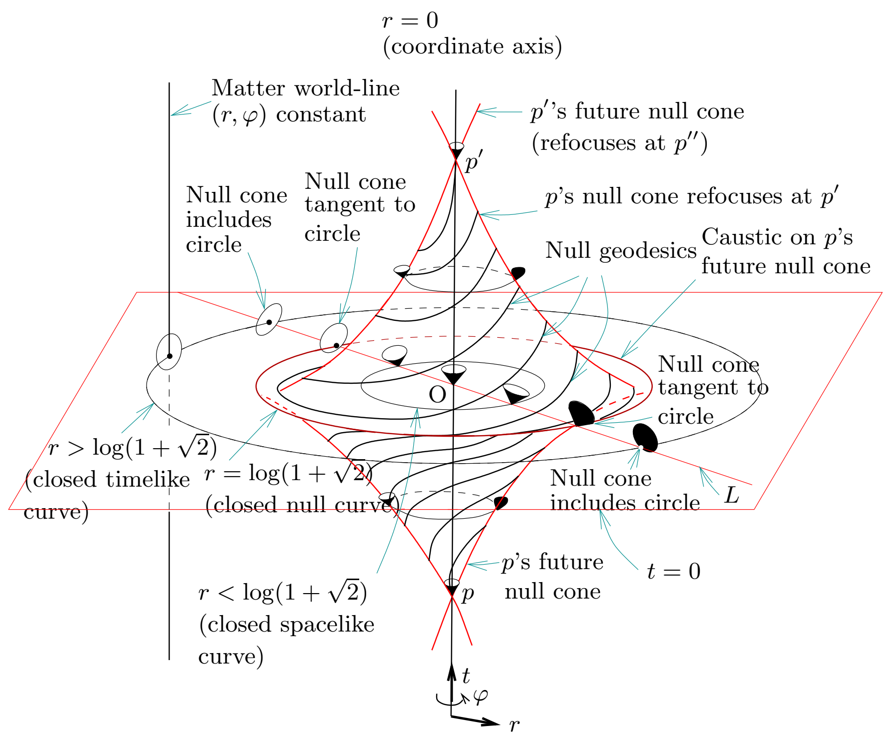

Figure 1: Gödel's universe in co-rotating cylindric-polar coordinates $\langle t, r, \varphi\rangle$. Irrelevant coordinate z suppressed. Light-cones (null-cones) and light-like geodesics (null geodesics) indicated. Light-cone *opens up* and tips over as r increases (see line L) resulting in closed time-like curves (CTC's). Drag effect (of rotation) illustrated. Photons emitted at p spiral out, reach CTC and reconverge at p'. This is a slightly corrected version of Hawking–Ellis [**HE73**, Figure 31,p.169] (cf. p.42 herein).

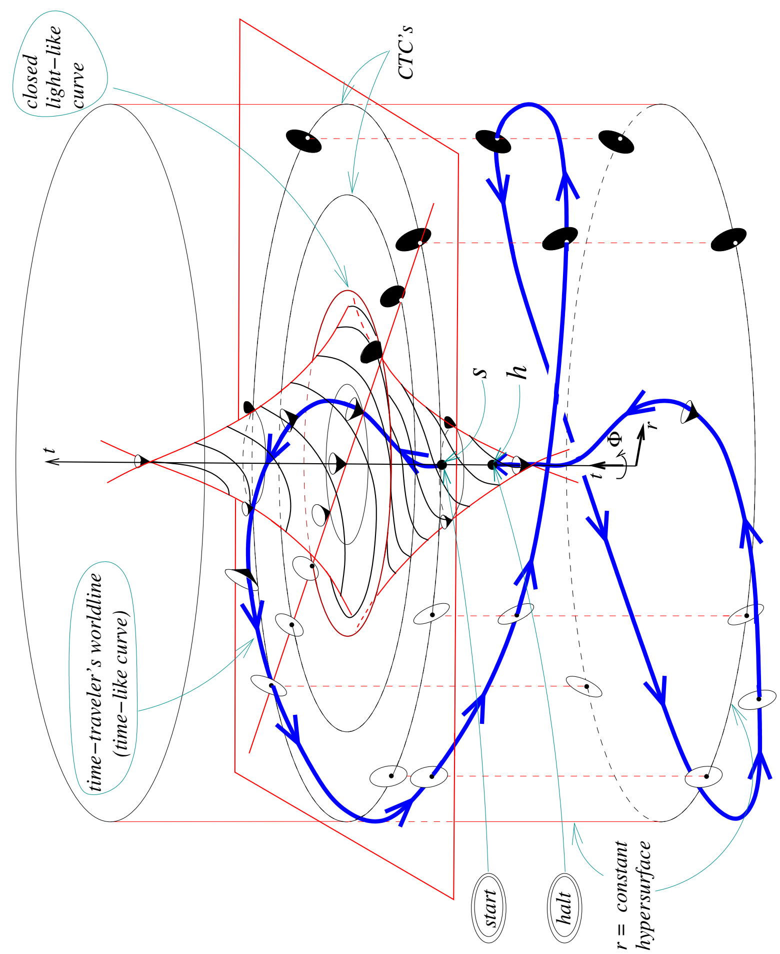

Figure 2: Gödel's universe with a time-traveler's (time-like) worldline indicated. The time-traveler's acceleration is bounded (but cannot be zero). The time-like curve C stays always inside the light-cones and spirals back to the past as m observes it. This is possible because the light-cones far away from the t-axis are so much tilted that they reach below the horizontal plane. See the explanation on p.3. Time-traveler starting at time s and arriving at time h, where h is earlier than s.

Explanation for Figure 2: Figure 2 illustrates the time-travel aspect in Gödel's universe. Assume observer m lives on the time axis $\bar{t}$. Assume p is a point far enough from $\bar{t}$. I.e. the radius r of p is large enough. Then at p the light-cones are so much tilted that a time-like curve C can spiral back into the past as observed by m. C involves only bounded acceleration. An observer, say k, can live on C. Then in m's view, k moves towards the past. Moreover, k can go back to the past as far as he wishes. It is an entertaining exercise to prolong curve C such that it starts at $s \in \bar{t}$ and ends at $h \in \bar{t}$ such that $h \prec s$, i.e. h is in the past of s, see Figure 2. Then our observer k can start its journey at s, spiral outwards to radius r, then spiral back along C and then spiral inwards to h. Then k can wait on the time axis $\bar{t}$ to meet itself at point s. The illustration of a time-traveler's worldline in Figure 2 is similar to the one in Horwich [**Hor87**, Figure 28, p.113].

2. Preparation for constructing Gödel style rotating universes. The Naive Spiral World.

In this part we populate Newtonian space with massive observers m_i for $i \in I$ which carry equal mass and are evenly distributed (where we understand "even" in the common sense). We will call these m_i's *distinguished observers* or *mass-carriers* or *galaxies* [2] . Then we rotate this inhabited space around the z axis. The galaxy in the origin is called m_0. We will make sure that nothing happens in the direction z, therefore we can suppress direction z in our pictures and discussion. So space-time becomes three-dimensional with axes t, x, y. We concentrate on the xy-plane inhabited by the galaxies (or distinguished observers) m_i. We rotate this plane of galaxies around the origin, i.e. around m_0. The rotation is rigid, i.e. the distances between the galaxies do not change. The *angular velocity* of this rotation is denoted by ω. We call the plane inhabited by the m_i's the *universe*. Hence ω is called the angular velocity of the universe. The rotation takes place in a Newtonian inertial frame of reference. [3] The angular velocity ω is chosen such that the resulting centrifugal force exactly balances the gravitational attraction between the m_i's. This is possible, cf. Gödel's paper [**Göd95b**, second half of p.270] for a proof. (Cf. [**Göd95b**, pp.261–289] for more detail.)

So our first pictures will show space-time diagrams in which the worldlines of the galaxies m_i appear as spirals around the t-axis (which happens to be the worldline of m_0). An extra feature is that, similarly to Gödel's papers, we assume the existence of certain kinds of *cosmic compasses*. Our cosmic compasses need not agree with what are called gyroscopes in physics. For the time being cosmic compasses constitute only certain conventions. Equivalently, they can be regarded as distinguished *local coordinate frames* or "local coordinate systems" for our distinguished observers or mass-carriers (the m_i's). These local frames need not be inertial. For the time being we do not associate any tangible or observational physical meaning to our compasses and local frames. [4] In Section 6 we will turn our attention to gyroscopes and local inertial frames, too.

We assume that all the m_i's agree with each other in that they have two cosmic compasses for carrying the original spatial directions x and y of our original Newtonian inertial reference frame with which we began our construction. This makes them equivalent (with each other) in the sense that any of them, say m, may think that he is at the center, he is not rotating and it is the rest of the observers who are rotating around m.

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[2]We use the world "galaxy" only in a metaphorical sense and it means nothing more than our distinguished observers carrying mass. Cf. Rindler [**Rin77**, p.203] for more on our usage for galaxies.

[3]Here we use the expression "inertial frame of reference" in the most classical (Newtonian) way, namely as it was given by L. Lange in 1885: "A reference frame in which a mass point thrown from the same point in three different (non co-planar) directions follows rectilinear paths each time it is thrown, is called an inertial frame."

[4]What they represent is mainly a logical "stage" in our construction of rotating universes. Though, in principle we could associate (a fairly complicated) observational meaning to them. We do not go into this here.

This paper is based on general relativity but we do not assume that the reader is familiar with the details of general relativity. What we do assume is familiarity with (i) the basics of special relativity and (ii) awareness of some of the basic principles of general relativity explained in items (1)-(2) below. All this can be found in [**AMN07**]. All what we need to know about special relativity in this paper can be found in [**AMN07**, sections 2.1–2.4]. What we need to know about general relativity theory in this paper can be found in [**AMN07**, sections 3.1–3.3] and is summarized in items (1)-(2) below.

(1) General relativity assumes that *special relativity holds locally.* This means, roughly, that in a general relativistic space-time, every point (event) is "surrounded" by a small, local coordinate frame (LF for short) and in each LF special relativity holds in some sense (cf. e.g. Rindler [**Rin77**] for a simple explanation of this). The LF's are local in the topological sense that space-time M comes together with a topology and then LF's are local in the sense that the "closer" we go to the point $p \in M$ the more accurately the local special relativity frame LF describes the behavior of light-signals and moving bodies. (For a precise formulation see [**AMN07**, sec.3.3, e.g., Def.3.3].)

In the case of Gödel's universe, M together with this topology is just the original (Newtonian) space-time $\mathbb{R}^4$. Thus, in the case of Gödel's universe $\langle M, \dots \rangle$ a single "global" coordinate system can cover the whole of M. This means that there exist coordinatizations $Co : \mathbb{R}^4 \longrightarrow M$ with Co a bijection which satisfy some natural requirements which we do not list here. E.g. Co involves one "time coordinate" and three "space coordinates", hence at first glance it looks similar to the familiar coordinatization of Newtonian space-time or special relativity. Further, one of the space coordinates turns out to be irrelevant, hence $Co : \mathbb{R}^4 \longrightarrow M$ will admit a 3-dimensional representation (via suppressing the irrelevant coordinate). So in our pictures there will be *one big coordinate system* Co covering the whole picture and there will be *many small coordinate systems* representing the LF's or other local coordinate systems. The big coordinate system represents the whole of our manifold M to be described.

When we describe a space-time M, the key ingredient is specifying how the little LF's are glued together to form the whole of M. We will do this by specifying a (fairly arbitrary) coordinatization C of M and then to each point $p \in M$ we describe how the LF at p is fitted into M at point p. [5] When specifying which LF is glued to what point, we use the coordinate system C as a tool for communication. Most of the time we will use geometric constructions for presenting the above data. In such a picture, the LF at p is represented by drawing the *light-cone* at p together with the *unit vectors* $\langle t_p, x_p, y_p \rangle$ of the LF at p. Sometimes we indicate only the future light-cones, sometimes we indicate both the future and the past light-cones. Most of the time we indicate the local simultaneity of the LF, too. [6] These pictures, beginning with Figure 7, represent *precise geometrical constructions*, hence they intend to specify the space-time in question completely (as opposed to being a mere "sketch" conveying only intuitive ideas). In Sections 7,8 we present constructions behind the

[5] The effect is somewhat similar to an Escher painting, e.g. he glues little birds together and there emerges an over-all pattern which has nothing to do with birds.

[6] To specify the LF, it is enough to specify the unit vectors $\langle t_p, x_p, y_p \rangle$. These determine the light-cones and the local simultaneity. However, the latter are very helpful in visualizing the space-time, that's why we indicate them in the pictures.

pictures together with the metric tensor field of the space-time in question. (To explain the latter, we note that a model of general relativity is usually given in the form $\langle M, \mathsf{g} \rangle$ where M is a manifold and g is a tensor field defined on M. We will not need these tensor-fields until Section 7.) We note that g can be reconstructed from the way the LF's are glued together in our pictures, hence if the reader understands the geometry of these pictures, he will automatically understand the space-time (or general relativity model) they represent.

(2) Occasionally we will mention so-called *geodesics*. Geodesics are the general relativistic counterparts of straight lines of special relativity, in particular, the worldlines of inertial bodies or freely falling bodies are called geodesics. The same applies to worldlines of photons. *Curves* are understood in the usual sense, e.g. geodesics are special curves. Properties of curves are generalized from special relativity to general relativity by saying that curve ℓ has property P if it has P locally (in the sense of special relativity). E.g. ℓ is *time-like* if for each $p \in \ell$ the LF surrounding p "thinks" that ℓ is time-like in the sense of special relativity. Similarly for *space-like*, *light-like* (and for other properties of geodesics).

We note that time-like curves are the possible worldlines of arbitrary bodies, i.e. of not necessarily inertial bodies. These may undergo acceleration. Both geodesics and time-like curves are curves in the usual sense. A curve is time-like if it always stays inside the light-cones. A curve ℓ is light-like if for any point $p \in \ell$, ℓ is tangent to the light-cone at p.

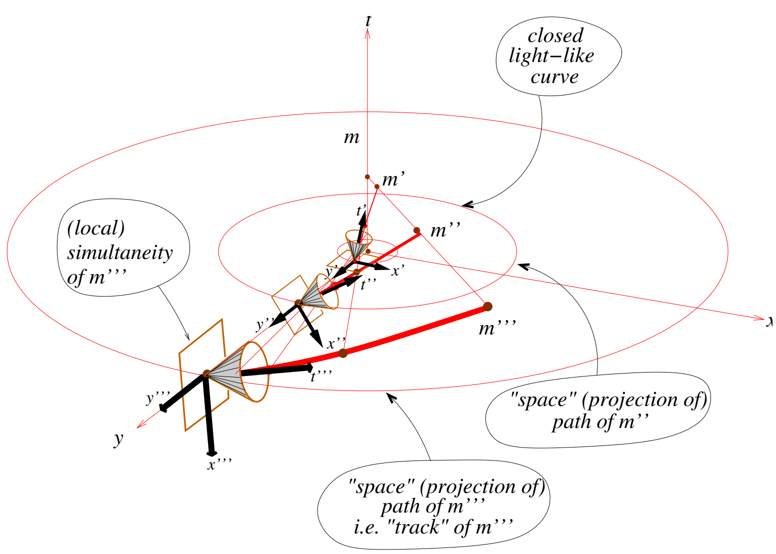

Figure 3: Observers m', m'', m''' perform a rigid rotation around observer m. Such observers are the only mass-carriers in this universe. Because of this rotation, m''' moves so fast that his light-cone tilts over so much that it is almost horizontal.

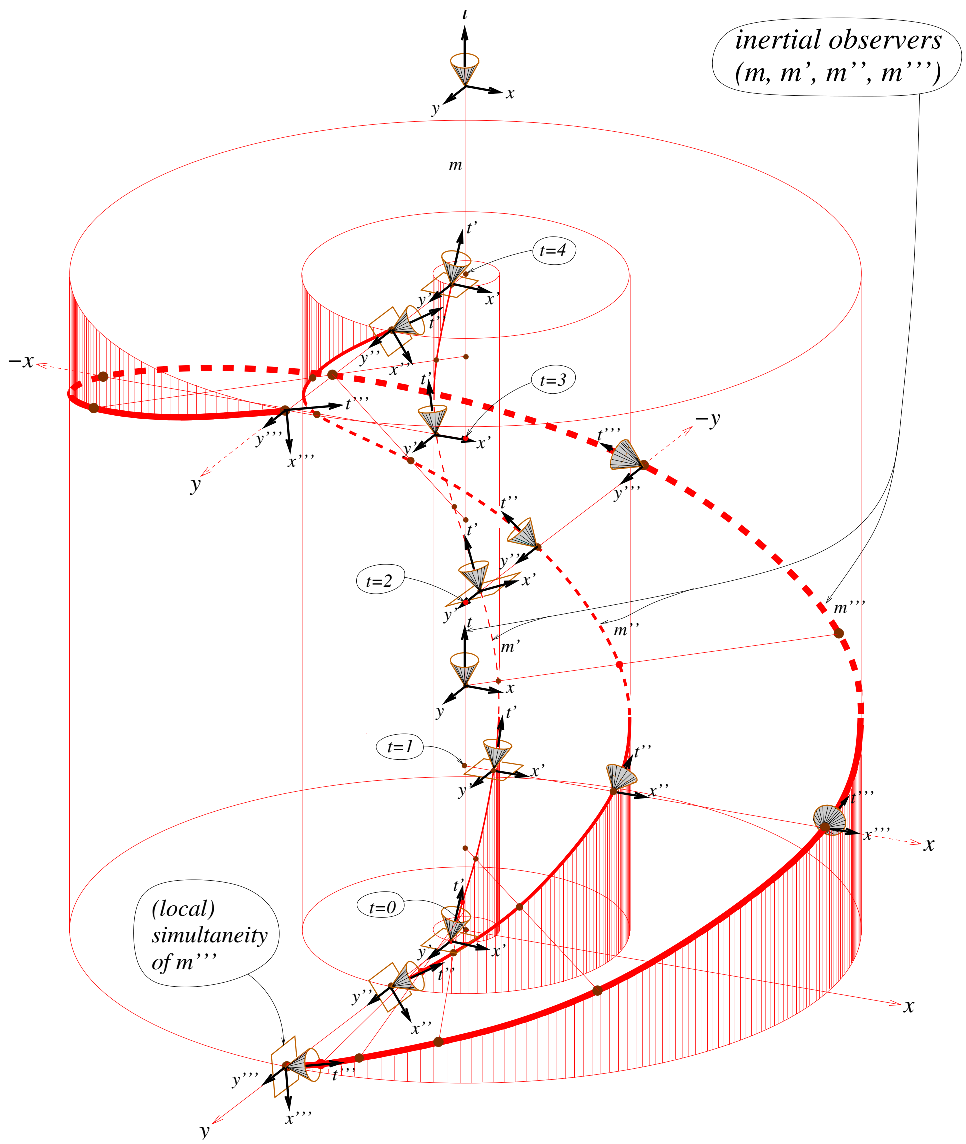

Figure 4: Gödel's Universe with emphasis on <u>inertial</u> observers instead of photons (the rotation is "rigid"). The coordinate system $\langle t', x', y' \rangle$ of say m' does <u>not</u> follow the rotation of the matter in this universe. The worldlines of $m, \ldots, m'''$ are (special) geodesics. $\langle t, x, y \rangle$, $\langle t', x', y' \rangle$ etc. are distinguished local coordinate systems. E.g. $\langle t'', x'', y'' \rangle$ is the local coordinate system of observer m''.

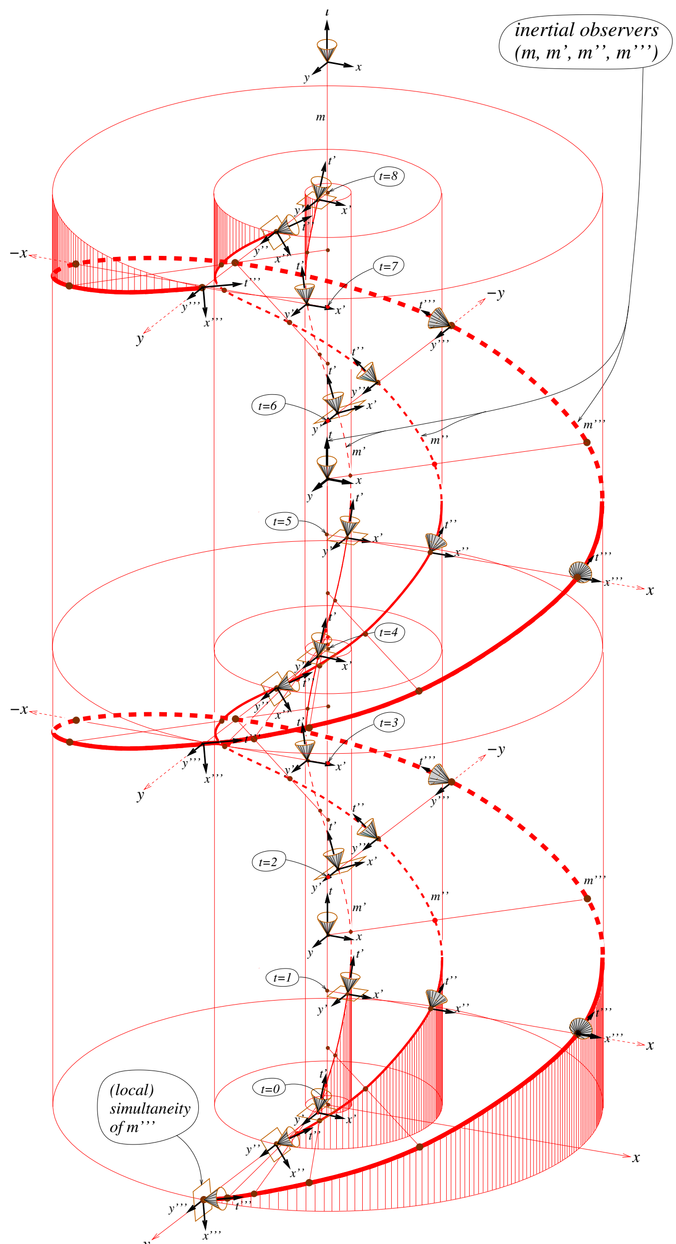

Figure 5: Previous figure copied on top of itself. It goes on like this in both directions forever. m', m'', m''' are (time-like worldlines of) observers “equivalent with” the observer m living on $\bar{t}$.

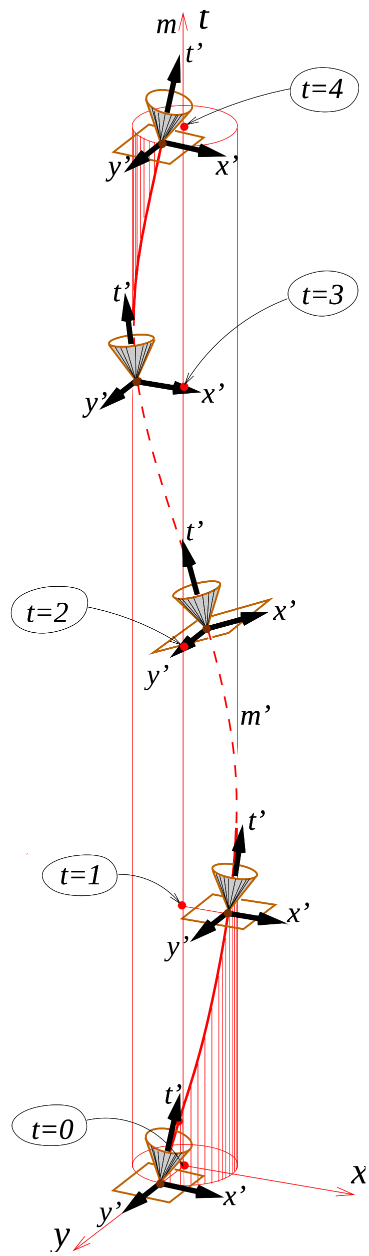

Figure 6: The coordinate system $\langle t', x', y' \rangle$ of say m' does <u>not</u> follow the rotation of the matter in this universe. The reader is asked to check that in a certain sense the direction x' remains parallel with the original direction x. This is why m' thinks that m is rotating around m'.

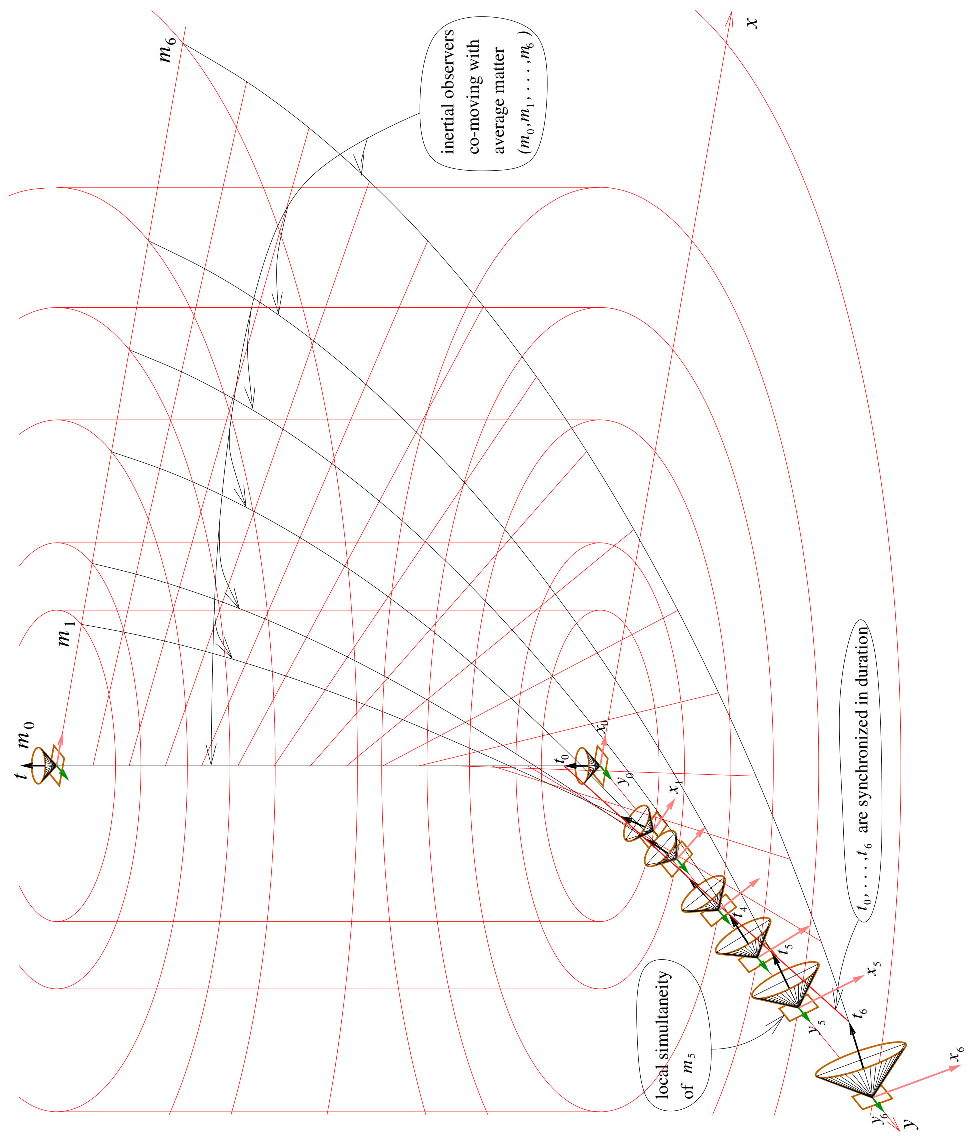

Figure 7: Each m_i can measure the time needed for a single turn of the universe. I.e. each m_i can measure the angular velocity ω of the universe. To ensure this we have to calibrate the t_i vectors of the m_i's such that in m_0's view the vertical components of all the t_i's are *equal* with that of t_0. See p.12 for more detail.

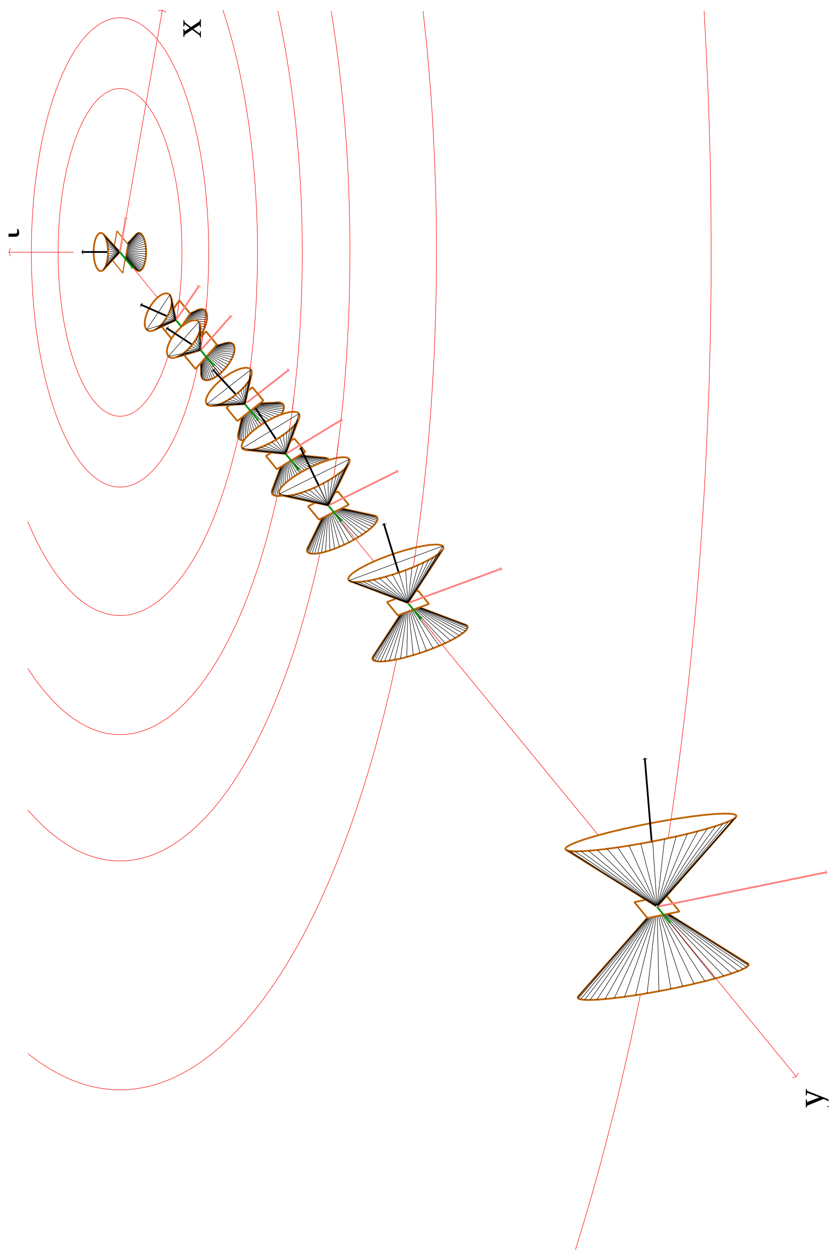

Figure 8: Arrangement of the light-cones in the Naive Spiral World (see p.12).

Gödel wanted the distinguished massive observers $m_0, \dots, m_i, \dots$ of his universe to be equivalent with each other. So far they are equivalent from the point of view that each of them thinks that the rest of the universe rotates around himself. This is so because the local coordinate systems (hence the cosmic compasses) of the distinguished observers m_i do not rotate, do not follow the rotation of the universe. At this point we can ensure one more symmetry property of the m_i's.

Each m_i can measure the time needed for a single turn of the universe, for example as follows: m_i picks a distinguished observer, say m_0, such that m_i's y-compass points in the direction of m_0 at an instant, and then measures the time passed until his y-compass again points in m_0's direction. [7] This is how m_i can measure the angular velocity ω of the universe. To ensure that all the distinguished observers get the same value for the angular velocity, we have to calibrate the t_i vectors of the m_i's such that in m_0's view the vertical components of all the t_i's are *equal* with that of t_0. This is ensured in Figure 7, and from now on we will always ensure this. [8] This choice of the local time-unit vectors ensures also that the local LF's measure a kind of "universal time", namely that of the big global reference frame. However, this "universal time" does not satisfy natural requirements about "time", see below (p.12).

Above we specified the time-unit-vectors of the local frames. Let us now specify three other vectors at each point p, these will specify the light-cone and the local frame at p. All what we say below in specifying the three unit vectors of the local frame is meant in the big global reference frame. The r-unit-vector at p points in the radial direction parallel to the xy-plane and has length 1. The (suppressed) z-unit-vector points in the direction of the (suppressed) z-axis and has length 1. Finally, the last local unit-vector is orthogonal to the three unit-vectors given so far and has the same length as the t-unit-vector. In the local frame at p, these 4 vectors constitute an orthonormal system. By this, we specified fully our general relativistic space-time. [9]

The *preliminary* version of Gödel's universe GU constructed above and depicted in Figures 3–8 will be referred to as "Naive GU" (NGU) or more specifically, "*Naive Spiral World*". The reason for this is that so far we have chosen the simplest possible arrangement of light-cones without checking whether they will satisfy certain properties we have in mind. Indeed, Section 4 will lead to some refinement/fine-tuning of the light-cone structure. However, the Naive GU has many of the desired properties already. Namely, the worldlines of the galaxies are geodesics, i.e., the distinguished observers m_i are really inertial observers. The radial straight lines parallel to the xy-plane are all geodesics, too.

In Gödel's Universe GU any two spacetime points (events) can be connected with a time-like curve. This is not hard to see by looking at Figure 2. Thus, the "future" of any event is the whole universe, and also the "past" of any event is the

[7] What does it mean that m_i's y-compass points in m_0's direction at some time t? We may use the following definition: there is a curve ℓ connecting m_i's worldline (starting with the event at t) with m_0's worldline such that at each point p of the curve ℓ the following holds: ℓ lies in the local simultaneity of the distinguished observer m passing through p and m's y-compass points in ℓ's direction in p.

[8] This will also ensure that each m_i will measure the same angular velocity for the universe, no matter which "partner" he chooses (in place of m_0) for the measurement.

[9] The corresponding metric tensor is given in section 7.

whole universe.[10] Figures 9–10 below illustrate that a natural "present" of an event consists of the whole universe, too.

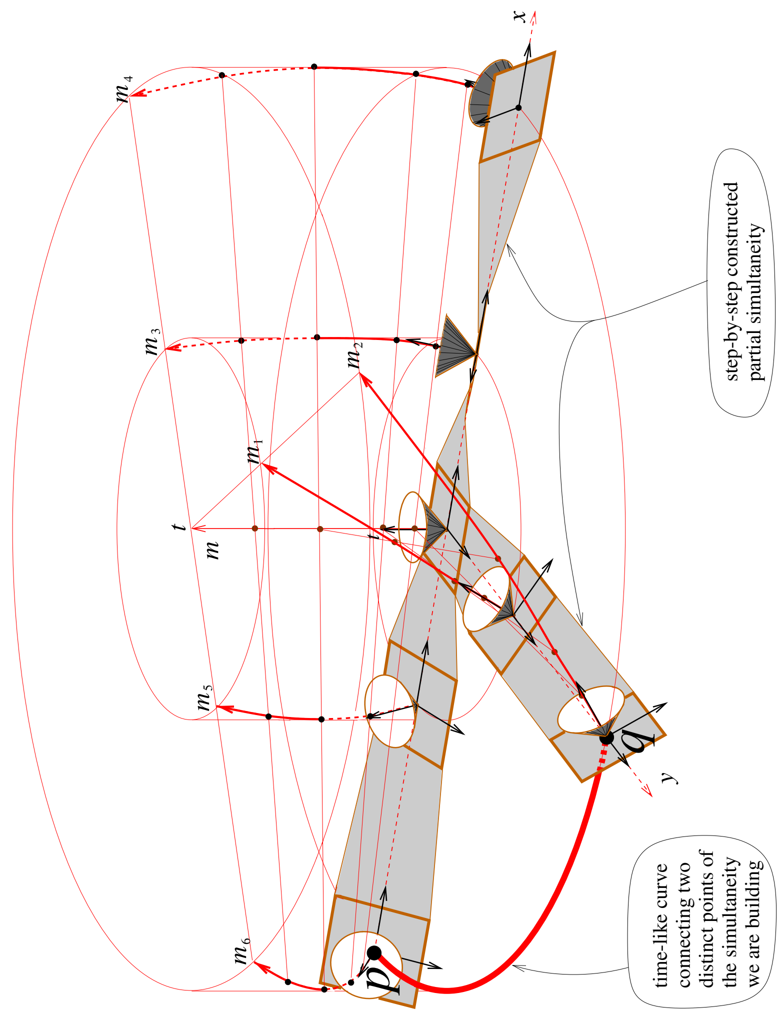

Figure 9: Idea of nonexistence of a global, natural simultaneity in Gödel's universe.

[10]This implies that GU enjoys the so-called Malament-Hogarth property, which requires the existence of an event which contains in its past a time-like curve of infinite length. Thus the general relativistic computer constructed in Etesi–Németi [**EN02**] (cf. also Hogarth [**Hog00**], Earman [**Ear95**], Németi–Dávid [**ND06**]) can be realized in the Gödel-type universes, too (in the literature general relativistic computers are usually constructed in Malament–Hogarth space-times).

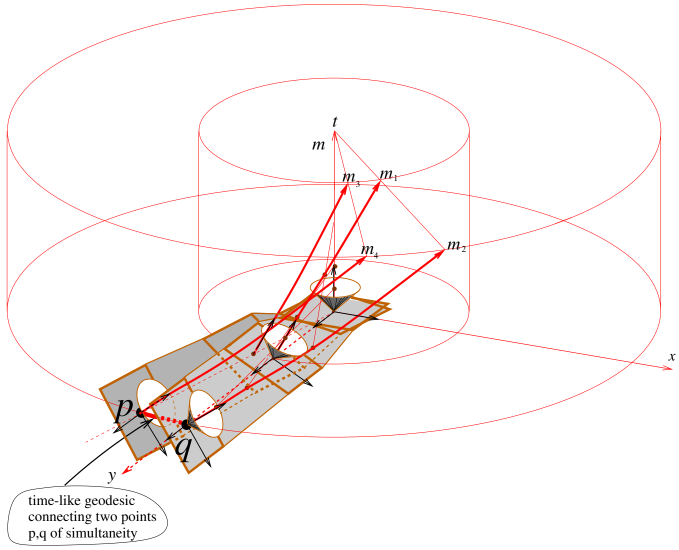

Figure 10: Previous figure but with the two strips of constructed simultaneity closer to each other, $p, \bar{0}$ and $q, \bar{0}$ are still simultaneous. The "informal logic" of these two figures generates a simultaneity connecting all points of space-time with each other.

3. Gödel's universe in co-rotating coordinates, "whirling dervishes". Transforming the rotation away.

Gott [**Got01**, p.91] writes "You could equally well view Gödel's universe as static[11] and non-rotating, as long as self-confessed 'nondizzy observers' would be spinning like whirling dervishes with respect to the universe as a whole." Gödel [**Göd95a**, p.271] writes: "Of course, it is also possible and even more suggestive to think of this world as a rigid body at rest and of the compass of inertia as rotating everywhere relative to this body." Below we introduce new coordinates $\langle T^r, X^r, Y^r, Z^r\rangle$ co-rotating with the matter content $m_0, \dots, m_i, \dots$ of the universe. In $\langle T^r, \dots\rangle$ the massive bodies m_i appear as being motionless with their worldlines vertical lines. We will call $\langle T^r, \dots\rangle$ "*Dervish World*" motivated by the above quotation from Gott. The transformation between the old spiral coordinates and the new rotating coordinates is elaborated later, on pp.43–46.

In the Spiral World, the "galaxies" $m_1, m_2, \dots, m_i$ appear as rotating around m_0 in direction φ with angular velocity ω while their cosmic compasses x_i, y_i appear fixed (non rotating). As a contrast, the Dervish World shows $m_1, \dots, m_i$ as motionless, while it shows their cosmic compasses as rotating in direction $-\varphi$ with angular velocity ω.

We will indicate on page 34 how this dervish world can be used to show that GU demonstrates that General Relativity (in its present form) does not imply the full version of Mach's principle.

[11]We note that Gott uses the world "static" here with its intuitive meaning, not with its technical meaning that has been adopted in the literature of general relativity.

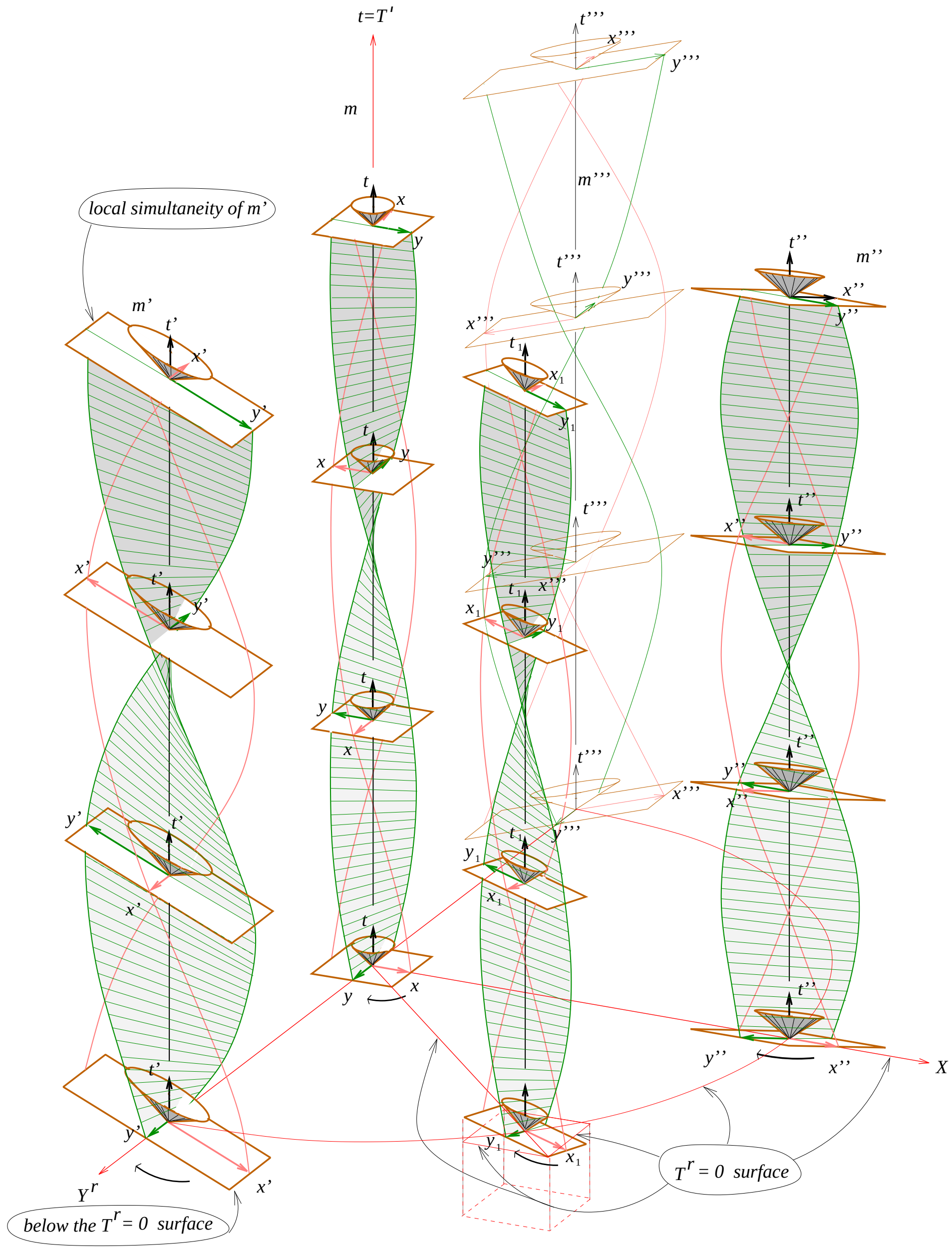

Figure 11: Gödel's universe GU in *rotating* coordinates $T^r = t,\ X^r,\ Y^r$. These coordinates co-rotate with GU, hence GU appears as *being at rest*. As a price, the local coordinate systems like $\langle t', x', y' \rangle$ appear as rotating backwards (in direction $-\varphi$) in the new coordinate system. The transformation between the old spiral coordinates and new rotating ones is elaborated on p.43.

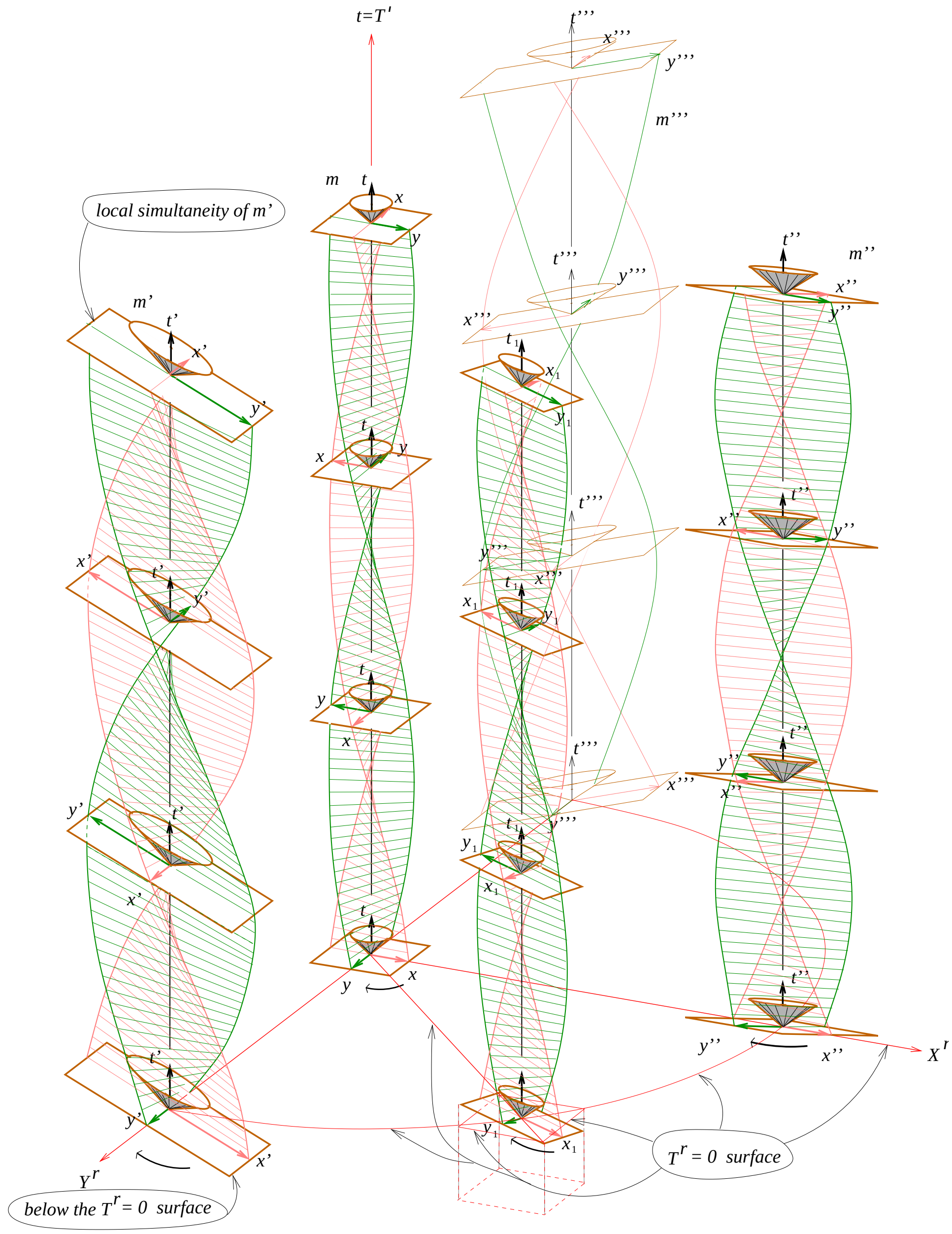

Figure 12: We have a system of non-moving massive observers m, m', m'' etc. (the same as in Figures 3–7) whose cosmic compasses i.e. whose local coordinate systems are spinning around creating a whirling effect. Gott [**Got01**, p.91] called these "whirling dervishes". This arrangement can be used to show that Mach's principle is violated, see p.34 for explanation.

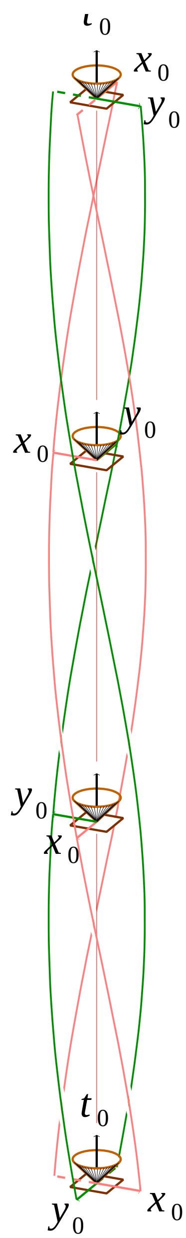

Figure 13: A typical dervish consisting of massive observer (or galaxy) m_0 and its cosmic compasses $\langle x_0, y_0, z_0 \rangle$. In other words, m_0's dervish is m_0's *local* coordinate system.

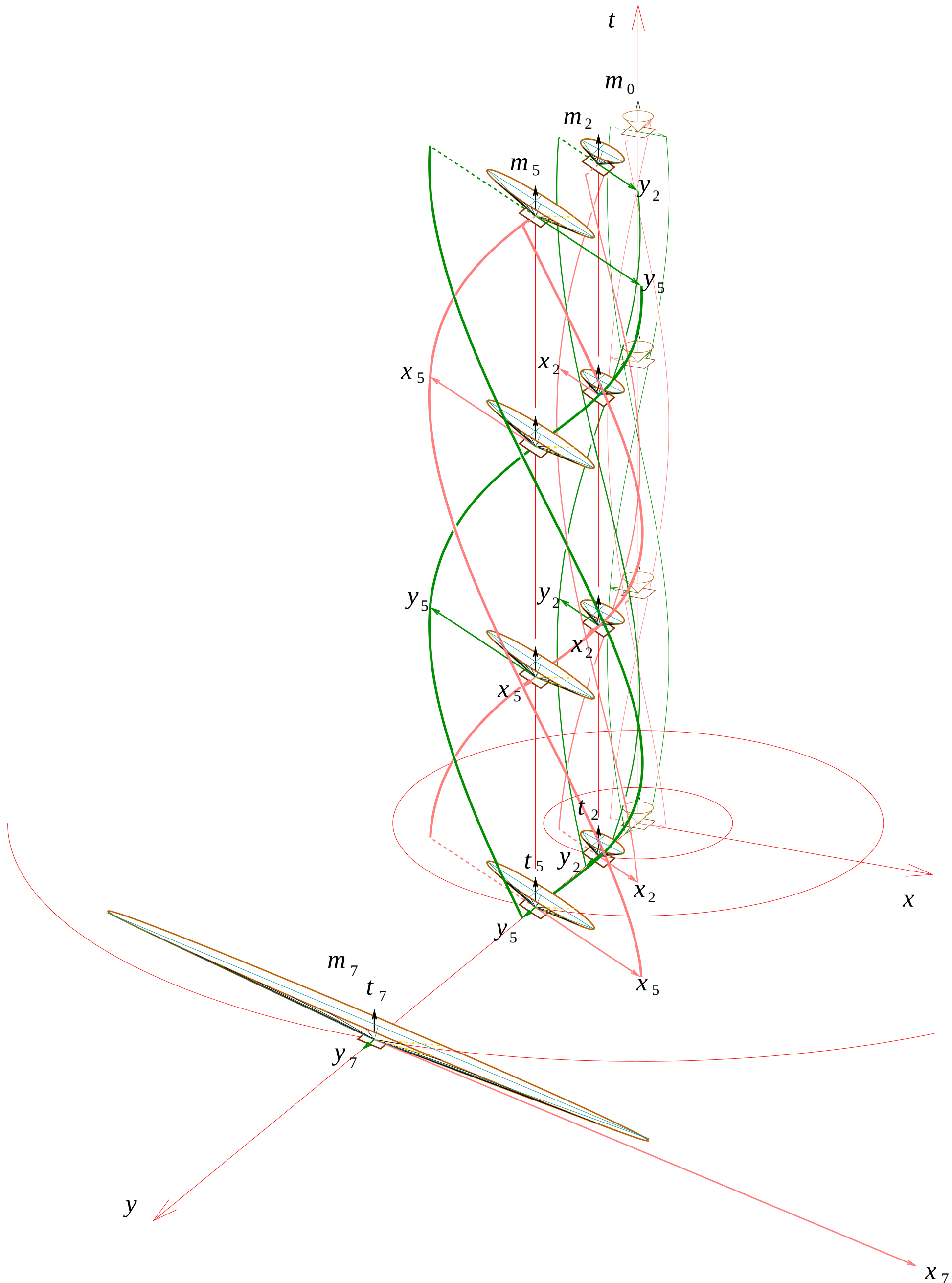

Figure 14: Dervishes $m_0, \ldots, m_7$ involving greater radiuses, hence more "violent" whirling effects.

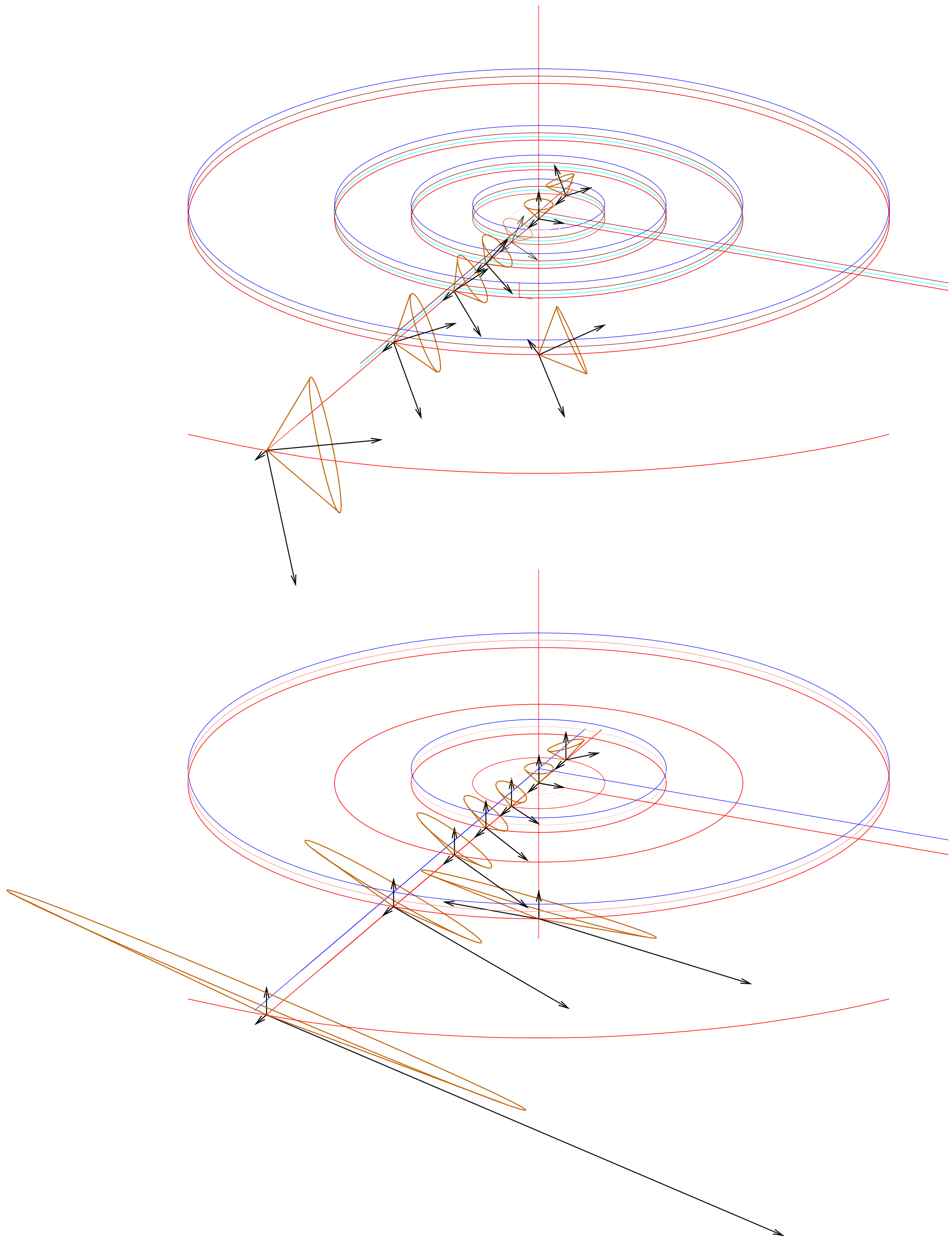

Figure 15: Light-cones and local unit vectors of spiral world above, and their counterparts in dervish world $\langle T^r, \ldots, Z^r \rangle$ below. Detailed representation of upper part is in Figures 7, 8 and that of lower part is in next Figure 16. See also Figures 11–14. The transformation between the two worlds is described on pp.43–46.

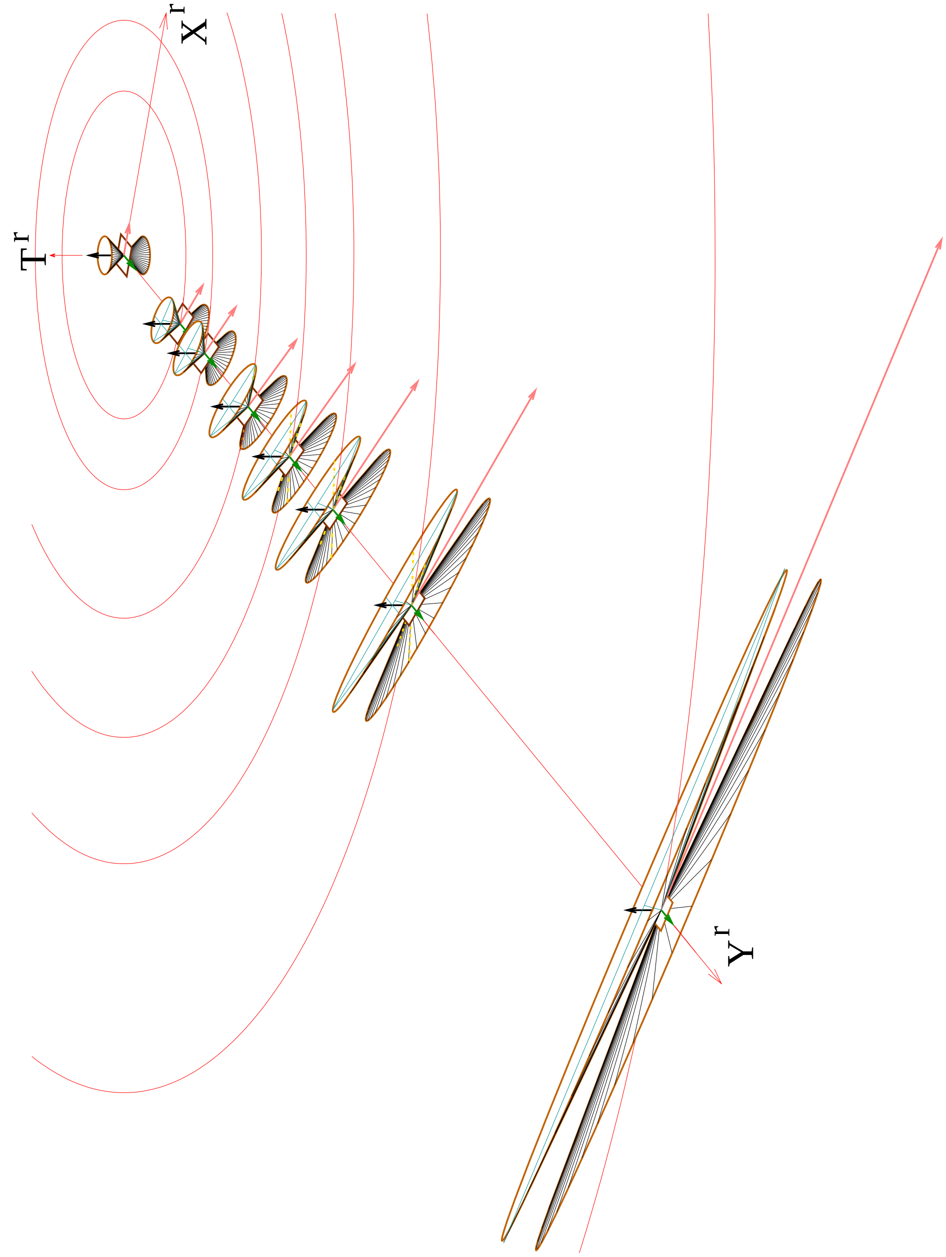

Figure 16: Light-cones with local unit vectors in dervish world $\langle T^r, \dots \rangle$. Compare with Figure 8 on p.11.

4. Fine-tuning the space-time structure of the Naive GU obtained so far. Tilting the light-cones.

First we show two pictures hinting at the fact that the lengths of unit-vectors of the local frames in our Naive Dervish World might be of inconvenient proportions.

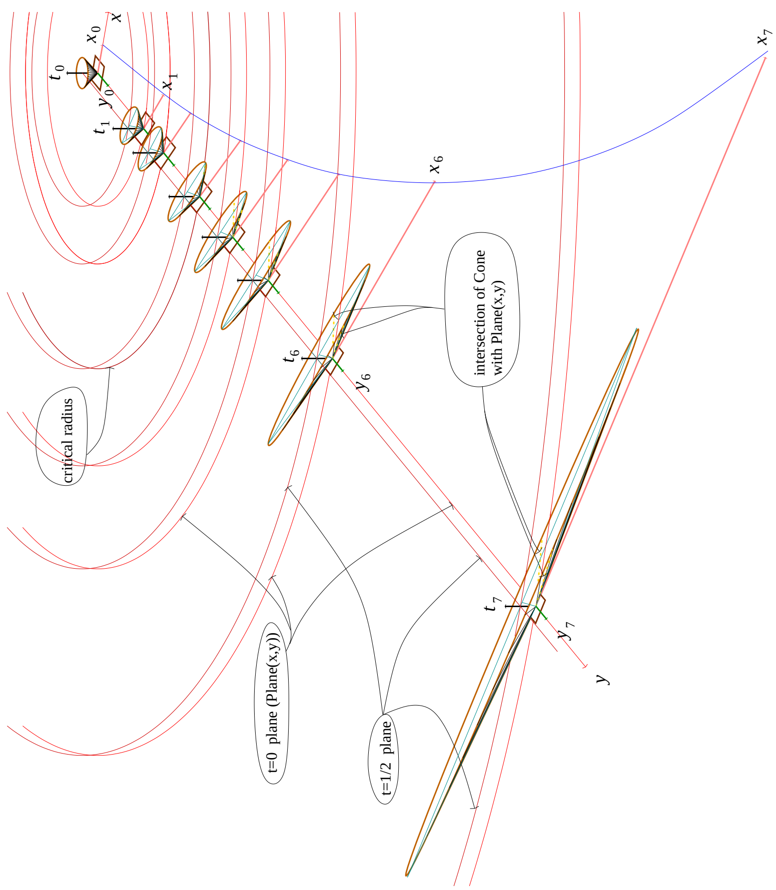

Figure 17: The x-unit-vectors of the local frames grow very fast in the Dervish World as we move outwards in the radial direction. (Critical radius means the radius where the light-cone touches the xy-plane.)

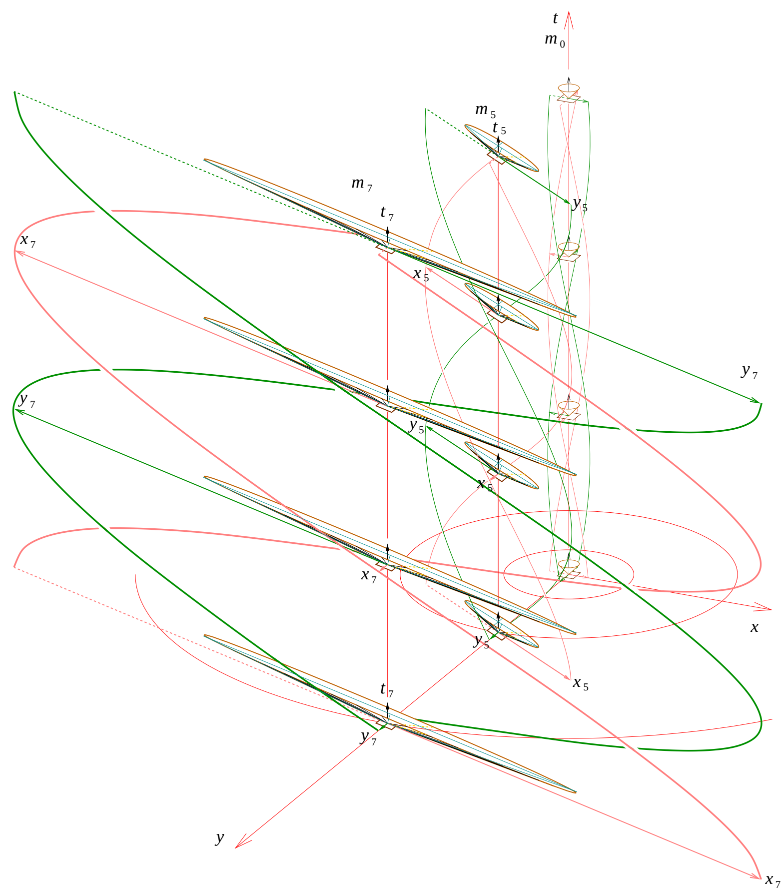

Figure 18: Whirling dervishes on larger radiuses.

The fact that the x_i vector of m_i has a much longer component parallel with coordinate X^r than x_0 (illustrated in the previous two figures) is the visual manifestation of the following fact, seen better in the spiral world. In the spiral world, m_i can send a photon ph upward almost parallel with the t axis such that ph reaches m_i again in a "rigidly bounded" time (an upper bound is $4\pi/\omega$) where the bound is independent of the choice of i. We choose the path of ph such that its distance from m_0 remains constant(ly the m_0–m_i distance). This path need not be geodesic but as Gödel wrote, we can use mirrors to force ph to follow this path. See Figure 19.

In Gödel's Universe the return-time of the photons sent around m_0 in a circle of radius r tends to infinity as r tends to infinity.

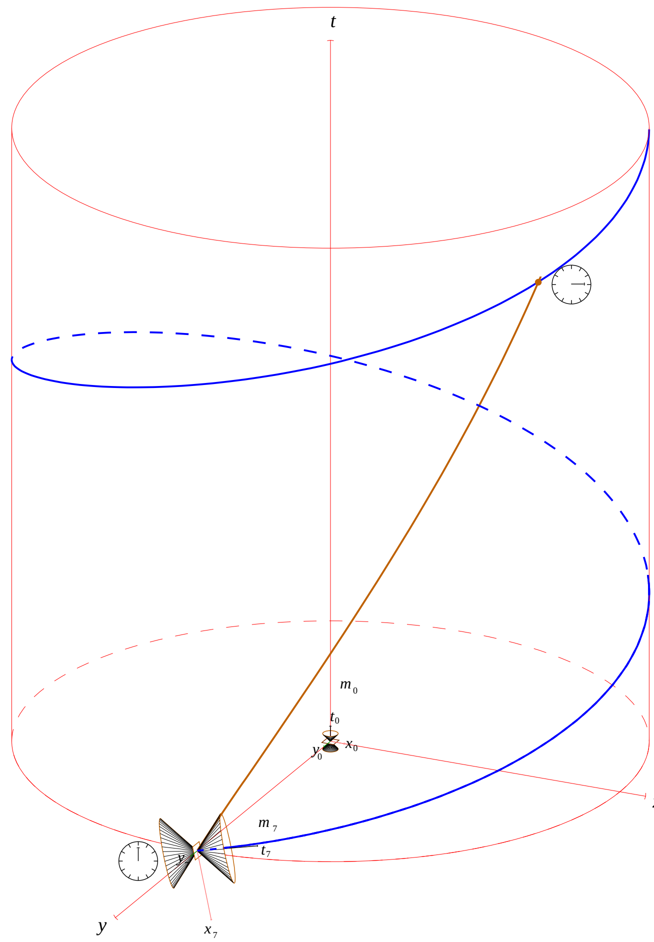

Figure 19: The time needed for a photon sent out by m_7 and kept with mirrors on a circle around m_0 to come back is a little more than the time needed for the universe to make a turn. This time is measured by m_7, cf. p.12.

Let us see how we can remove this difference with Gödel's universe without destroying the logic of our construction. How can we fine-tune our construction? We are aiming at the "smallest" and simplest change so that the logic of our construction would remain intact. Changing the length's of the x_i vectors and keeping the other unit-vectors as they were results in making the light-cones narrower. Since this will not lead to CTC's, we will "tilt" the light-cones, instead. So, in fine-tuning the Naive GU we will speak about tilting the light-cones, and we will call the new space-time Tilted GU.

Let us work in the dervish world.

Choice 1 We can tilt the light-cones forwards (in the positive φ direction) such that with increasing r (radius) we also increase the tilting. This can be done in such a manner that the difference we talked about disappears. The result of such tilting is represented in Sections 4–5 (Figures 21–28). The so obtained tilted universe resembles very closely the universes presented in Gödel's papers. (E.g. they agree in many structural properties [in Gödel's sense].)

Choice 2 We can also tilt the light-cones (in dervish world) backwards, opposite to the φ direction, carefully enough such that the difference goes away and we do not induce other undesirable effects. See Figure 20. This Choice 2 tilting is just Choice 1 tilting seen from another coordinate system (namely by using the coordinate transformation $\varphi \to -\varphi$). Below we will explore Choice 1, and in Section 6 (p.33) we explore Choice 2. We will see that both Choice 1 and Choice 2 have their advantages.

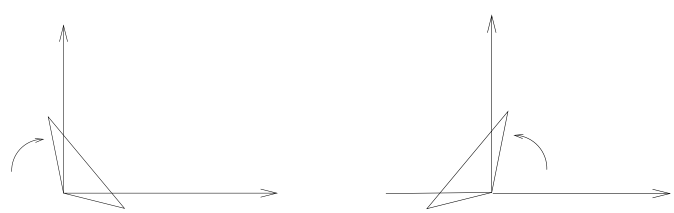

Figure 20: Choice 1 is that we tilt the light-cones forwards. Choice 2 is that we tilt the light-cones backwards.

From now on we concentrate on Choice 1 (till Section 6).

We will call the tilting in Choice 1 "*forward-tilting*", the so obtained dervishes *tilted dervishes*, and the so obtained (tilted) dervish world *Tilted Dervish World* or *Choice 1 Dervish World*. Recall that we describe a simple transformation between the spiral world $\langle t^s, \dots \rangle$ and the dervish world $\langle t^d, \dots \rangle$ in Section 8 (p.43). We use this transformation for transforming the new, tilted universe from the dervish world to the spiral world. We call the result *Tilted Spiral World* or use simply the adjective "new spiral" or "refined-spiral" for referring to the so obtained light-cones as new spiral cones or back rotated ones. The expressions "rotating back" and "back-rotating" intend to refer to application of the inverse transformation $\langle t^d, \dots \rangle \longrightarrow \langle t^s, \dots \rangle$ described in Section 8. In such contexts the inverse transformation is applied to the result of forward-tilting.

The result of the above outlined forward-tilting is the Gödel-type universe which we will describe in more detail in the coming parts. We will call this space-time Tilted GU (or sometimes new GU). In this paper, we do not specify the exact tilting of the cones according to which the figures were constructed. More detail is given in [**NMAA**].

In this section we describe "Tilted Dervish World", and in the next section, Section 5, we describe "Tilted Spiral World".

Tilted dervishes (fuller description of new GU in dervish world).

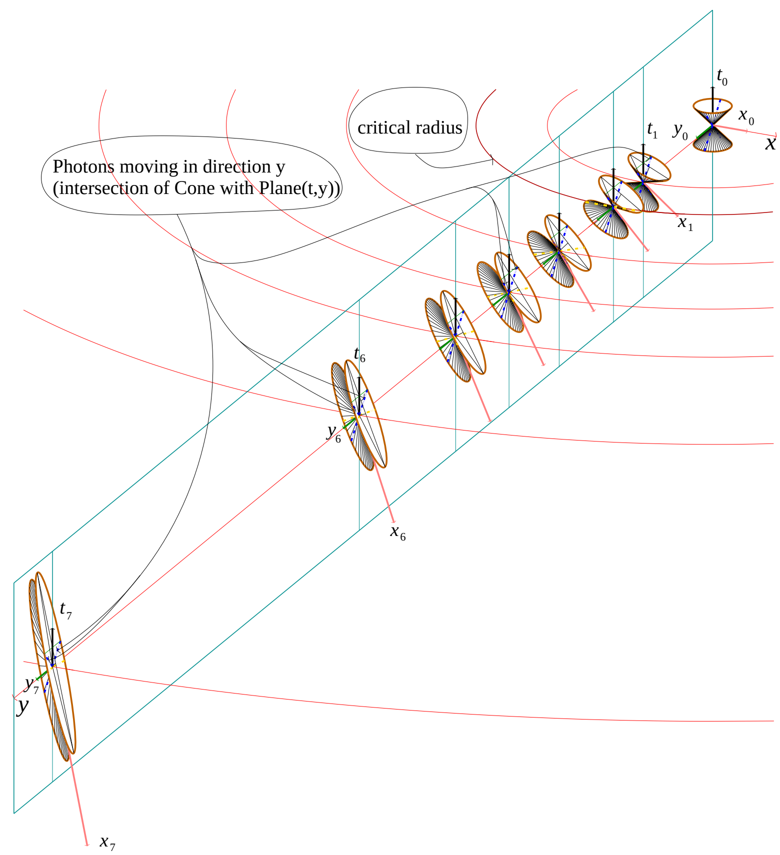

Figure 21: Tilted Dervish World (Choice 1 Dervish World).

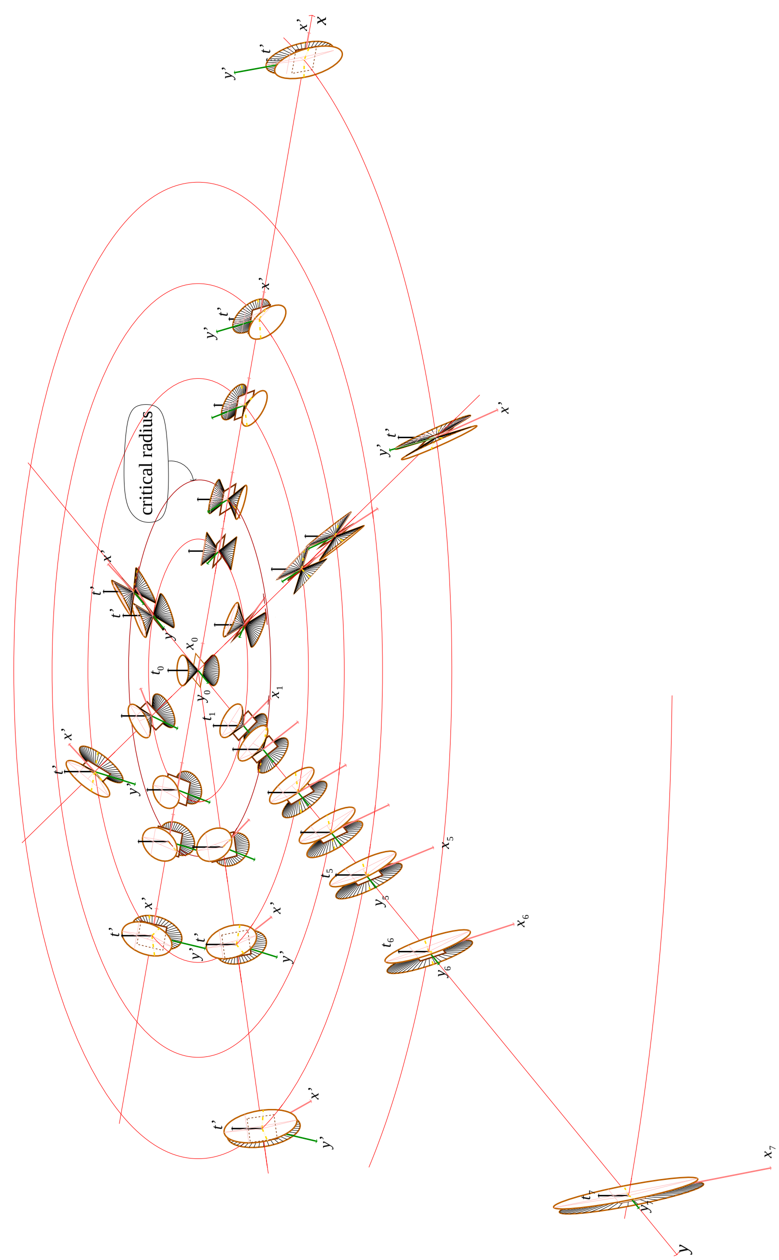

Figure 22: Tilted Dervish World.

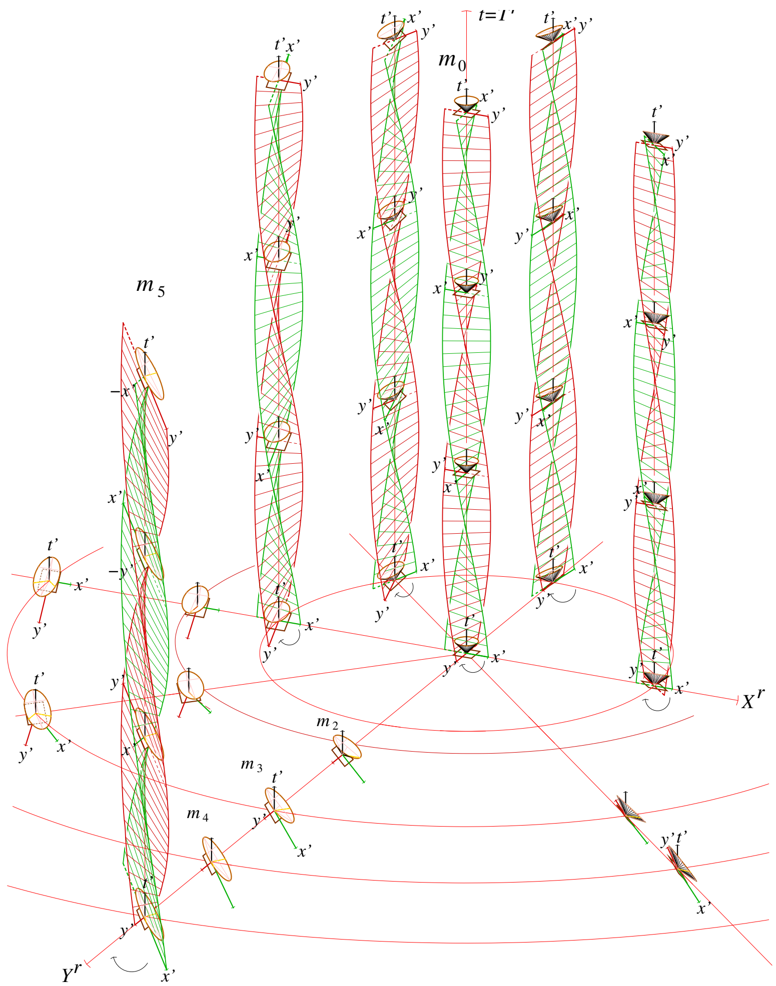

Figure 23: Tilted Dervish World. Spinning dervishes are artificially sped up (“artificial ω of dervishes” = $\pi/15$).

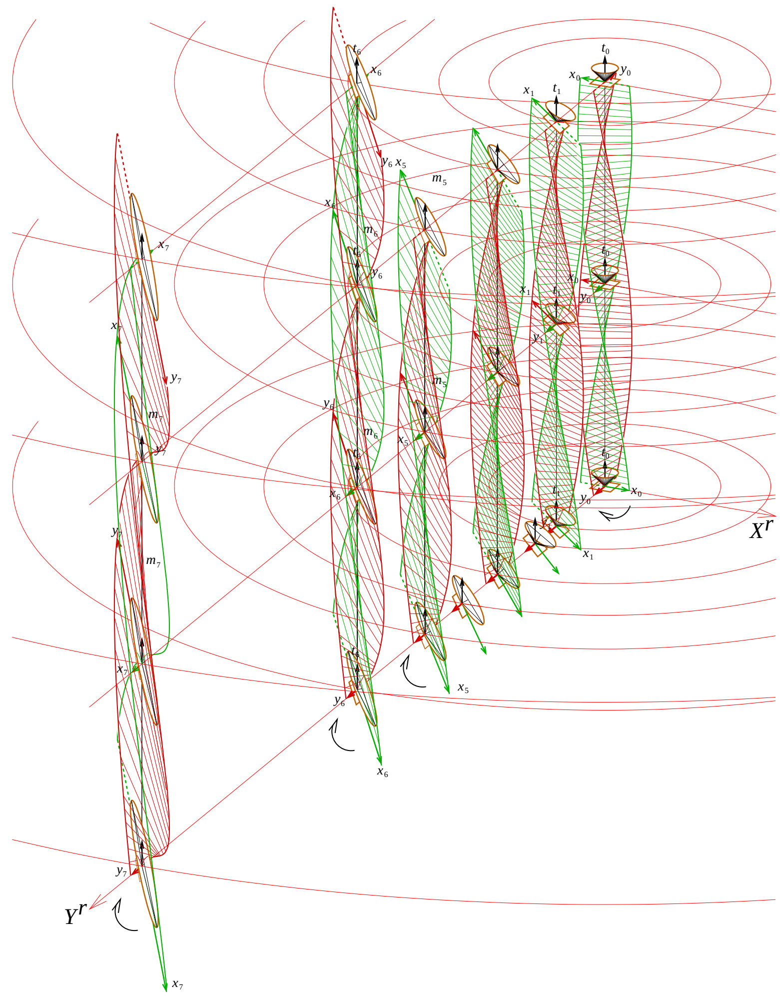

Figure 24: Tilted Dervish World.

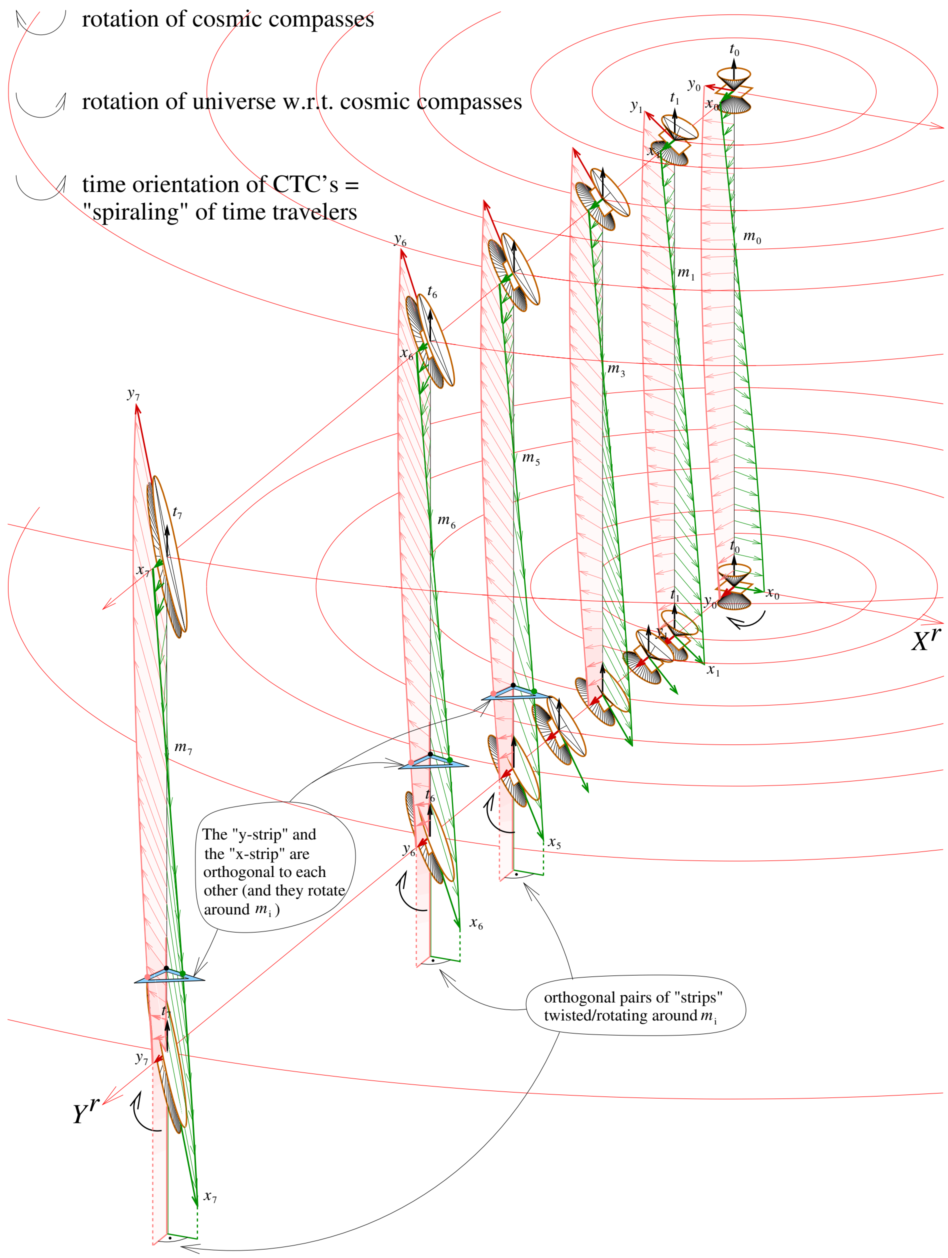

Figure 25: Tilted dervishes with original angular velocity.

5. Tilted Spiral World, i.e. Choice 1 Spiral World.

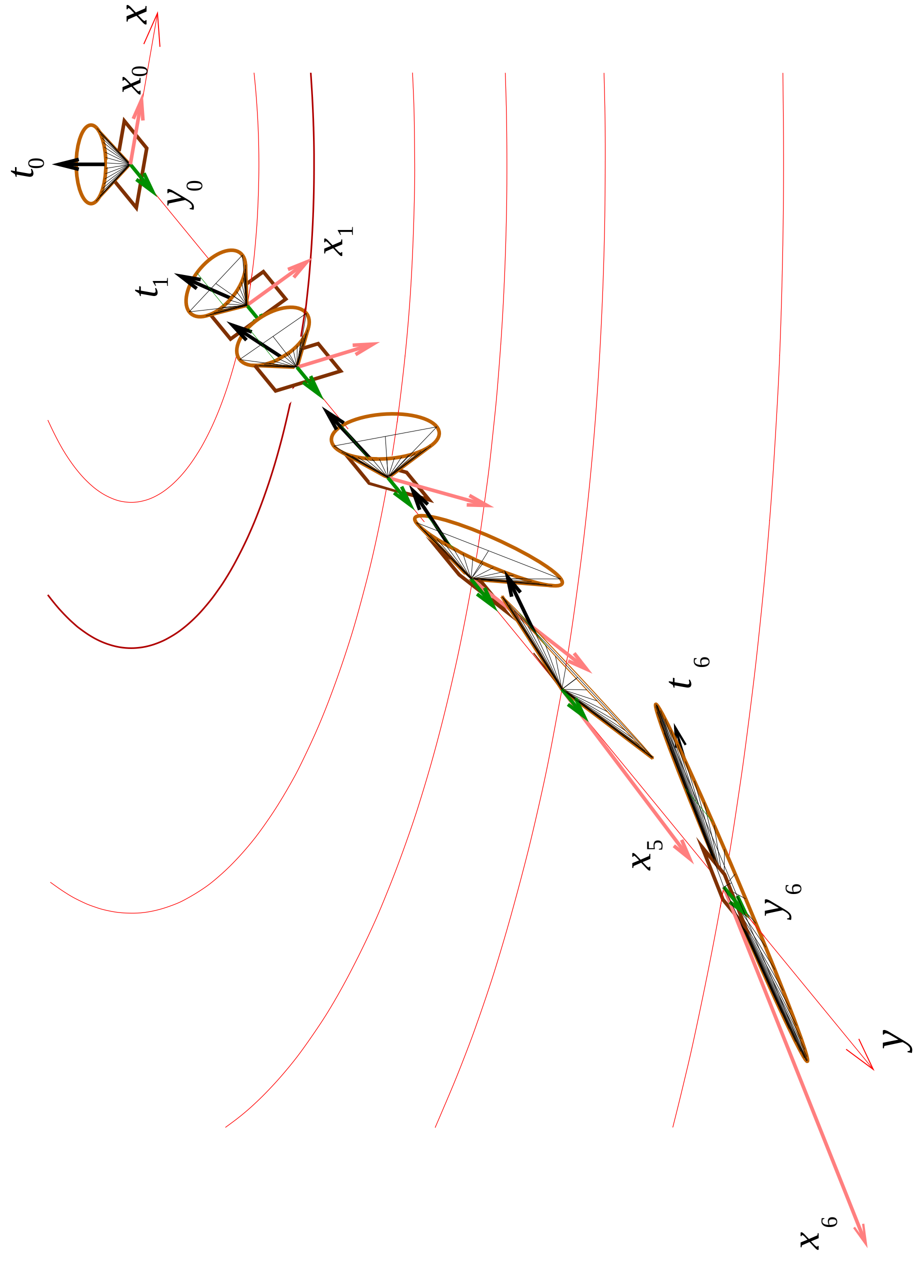

Figure 26: Tilted spiral world, i.e. Choice 1 Spiral World. Light-cones, unit-vectors along the y-axis. Compare with Figures 8,16 on pp.11,20.

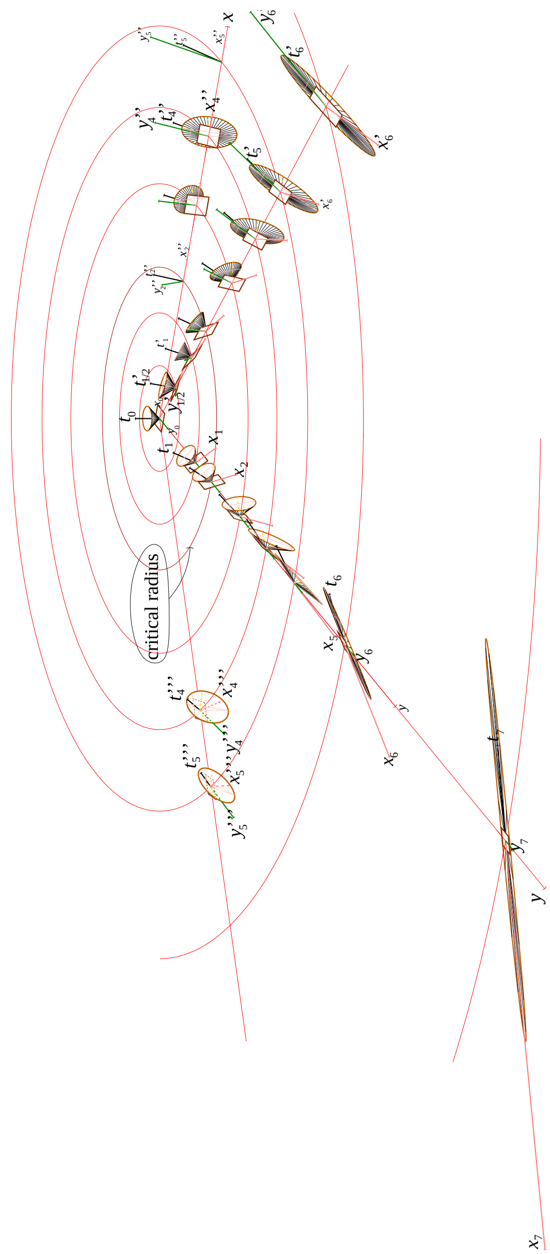

Figure 27: Tilted Spiral World.

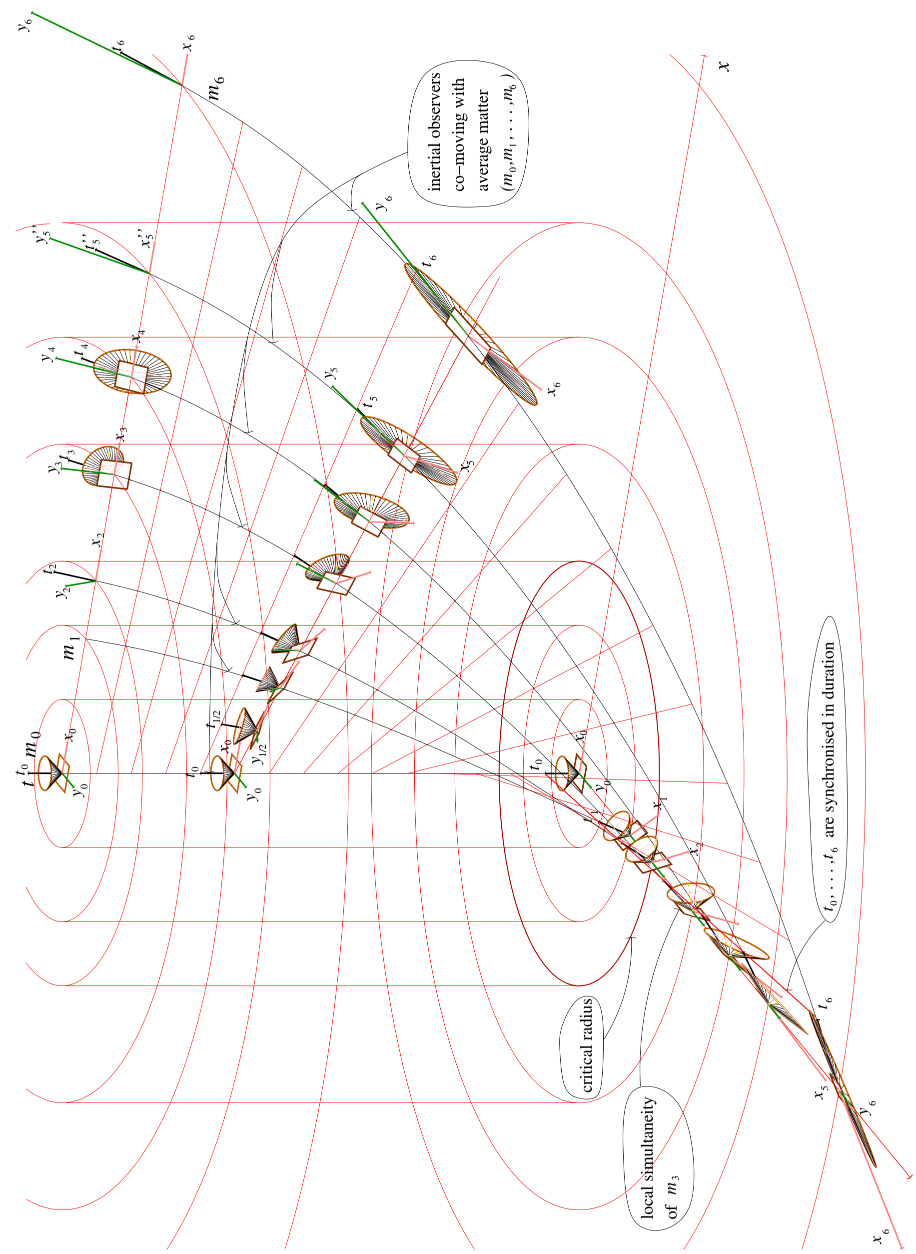

Figure 28: Tilted Spiral World, full view. Light-cones, worldlines, unit-vectors etc.

6. Giving physical meaning to cosmic compasses. What rotates in which direction (relative to whom).

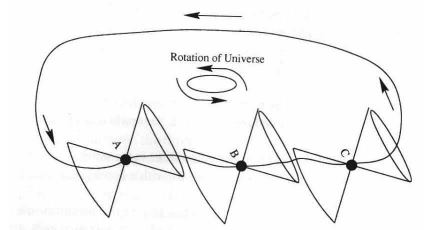

Figure 29: What rotates in which direction? The above is a picture from Pickover [**Pic98**, p.185] from the chapter on Gödelian Universe implicitly offering a natural answer to this question. This is also Figure 7.5 in Gribbin [**Gri98**, p.215].

In our "Tilted Spiral World" (Figures 27,28) the light cones are very strongly tilted forwards with increasing radius r. Therefore, if m_0 throws a ball, say in the y direction, the ball will start moving in the y direction but with increasing radius it will *have* to turn in the φ direction because the worldline of the ball has to stay inside the light-cones (i.e. it has to be a time-like curve). The same applies even to a photon in place of the ball. This effect is called the *gravitational drag effect* [12] and is illustrated e.g. in our Figure 1 or equivalently in Figure 31 of Hawking–Ellis [**HE73**] as the curving of the light-like geodesics. The drag effect affects those and only those inertial bodies which are not at rest relative to one of the m_i's. This drag effect is present in the Naiv GU, too, but in a less dramatic way. To study the drag effect in our Tilted GU (in Figures 28, 23), we notice that our Tilted Dervish World (Figure 23) is structurally very close to Gödel's original universe described and studied in Gödel [**Göd95b**], Hawking–Ellis [**HE73**, pp.168–170] and later papers. Hence the results about the drag effect in Gödel's universe obtained in these works are applicable to our version of GU in Figure 23. The drag effect can be analyzed and described by studying the behavior of geodesics. Indeed, Figure 1 represents "dragging" of some characteristic geodesics. Let us be in dervish world. Then Figure 1 indicates the following. A ball thrown by m_0 will start out radially, then will make a big circle and will come back to m_0 from a new direction. From now on, we will call the circular motion or rotation traced out by this circle the *drag rotation*. In Figure 1 the direction of the drag rotation coincides with the φ-direction which in turn coincides with the direction of CTC's. All this remains true in our Tilted Dervish World (Figure 23). In the Tilted Spiral World, matter (the m_i's) is seen to rotate in the same direction φ. Therefore in the Tilted Spiral World what we said above about the drag rotation, CTC's etc. remains true. Hence, in the Tilted Spiral World the drag rotation is even stronger than in the dervish world and points in the same direction φ in which the matter content of the universe rotates. Hence in the Tilted Spiral World, we have an *increased drag effect*. As a curiosity we note that in the Tilted Spiral World everything rotates in the same direction φ.

---

[12] What we call drag effect is often called *dragging of inertial frames*. For references on gravitational drag effect see p.42.

Next we turn to replacing our cosmic compasses which were "abstract directions" so far with physically tangible compasses of an "observational" kind (i.e. subject to testing by thought experiment). In general relativity, the devices used for this purpose are called *gyroscopes* or *compasses of inertia*. The nonspecialist reader does not need to recall the definition, what we write below is amply enough for the present paper. The most important property (for us) of gyroscopes is that their working is based on inertial motion, hence the behavior of geodesics will also influence the behavior of gyroscopes. For the non-physicist reader we note the following.

In Newtonian physics it is provable that certain devices called gyroscopes preserve their directions despite of our moving them around, in other words, they behave like "cosmic compasses".[13] We do not recall the definition of gyroscopes in detail. However we note that they can be made smaller and smaller in some sense such that their Newtonian property of preserving direction (whatever this means) remains true in general relativity (here the basic idea is that general relativity agrees with Newtonian mechanics for small enough speeds [with sufficient precision]). The essential idea behind gyroscopes is that a rigid body rotating fast enough tends to preserve its axis of rotation (in Newtonian physics). If we make the body small enough, then the tangential velocities of its parts will tend to zero. Hence the tangential velocities involved can be made small enough for the Newtonian approximation to be satisfactory.

It is natural to assume that the increased drag effect in Tilted GU described above will "drag" the gyroscopes, too, in the φ direction. Indeed, an analysis of the geodesics of Gödel's universe in Lathrop–Teglas [**LT78**] suggests that this is so.

Our next goal is to find a new coordinatization C^+ for our Tilted GU in which the gyroscope directions do not rotate. [14] One needs not regard this new coordinatization C^+ superior in some sense to e.g. our Tilted Spiral World or more "real" than Tilted, instead, C^+ is a coordinatization with some interesting and useful properties. C^+ will be a (new) spiral world. We will call this new spiral world Refined (or Choice 2) Spiral World. After constructing C^+, it will be worthwhile to reconstruct the dervish world in such a form that the *new local frames* (i.e. "veils" or "hands" of the whirling dervishes) will be frames co-rotating with the gyroscopes. Then the local frames will be what are called *local inertial frames* in general relativity. A representation of the dervish world with these new local inertial frames represented as the "veils" of the dervishes will be called Refined (or Choice 2) Dervish World. The two tilted spiral worlds (Choices 1,2) and the two tilted dervish worlds (Choices 1,2) represent the same space-time in different coordinates.

In the Refined Dervish World all the mass-carrier observers m_i are at rest, they are evenly distributed and they are completely alike, yet their compasses of inertia are rotating. This violates Mach's principle which says that the state of zero rotation of an inertial frame should coincide with the state of zero rotation with respect to the distribution of matter in the universe. For Mach's principle see e.g. Barbour [**Bar89**] and [**BP95**]. For more references on the drag effect and its connection with Mach's principle see page 42.

[13]See e.g. Epstein [**Eps81**, p.128] for nice illustration.

[14]Below by gyroscopes we always mean gyroscopes of m_0.

Above (p.33) we recalled a picture from Pickover [**Pic98**] because it "addresses" the question of what rotates in which direction. (E.g. does the universe rotate in the same direction as the time-travelers (CTC's) do?) To make the question meaningful, one has to tell relative to what coordinate system is the question understood. [15] Of course, one would like to name an "observable" coordinate system for asking such a question. A possibility is to choose that coordinate system in which the gyroscopes do not rotate. [16] This is C^+ of our Choice 2 Spiral World. We will see that in C^+ the directions of the various rotations are essentially different from the ones in Pickover's picture. If one looks at C^+ without any preparation, then the directions of rotations appear as ad hoc, almost counter-intuitive. However, at least in our opinion, the train of thought outlined in this paper may provide an explanation for the arrangement of these directions. For more on this question of counter-rotation in the case of rotating (Kerr–Newman) black holes see [**ANW08**].

Let us return to our goal of finding a coordinatization C^+ of our Spiral World in which gyroscope directions do not rotate. [17] We have already observed that gyroscopes do rotate in our Tilted Spiral World (Figure 28). There are two equivalent ways for finding C^+:

(i) We analyze the rotation of gyroscopes as seen from the Tilted Dervish World, we observe that they rotate in the φ-direction. This means that in the spiral world gyroscopes rotate faster than the dervish world itself does (i.e. faster than ω). We choose the *refined spiral coordinates* to co-rotate with these gyroscopes. Hence the "gyroscope"-directions will be fixed when viewed from the Refined Spiral World as we wanted.

(ii) The following turns out to be equivalent with what we outlined in (i) above. Let us go back to Section 4 p.24, where we refined our Naive GU to get Tilted GU. There, on p.24, we found two possible choices (Choices 1,2) for the desired fine-tuning. Of the two, so far we took the simpler one, Choice 1. Choice 2 consists of tilting the light-cones in the dervish world *backwards* i.e. in a direction *opposite* to that of φ (in Choice 1 we tilted them forwards). What we claim here is that the result of choosing Choice 2 in Section 4 is equivalent with the result of the refinements outlined in item (i) above. This is the reason why we call our newest refined spiral and dervish worlds outlined in item (i) above *Choice 2* worlds as well as Refined worlds.

The new Choice 2 spiral and dervish worlds are illustrated and elaborated (constructed) in the figures below. A natural question comes up: If we had to refine our Choice 1 worlds because the drag effect made the gyroscope directions rotate, how do we know that the same problem will not come up in the new Choice 2 worlds? The answer is two-fold. (1) The extremely strong drag effect in Choice 1 Spiral World was caused by tilting the light-cones forwards extremely with increasing radius r. Cf. Figure 27 for this effect. Now, in our Choice 2 Spiral World the light-cones are not tilted forwards so much, actually recall that Choice 2 was obtained from Choice 1 by tilting light-cones backwards (relative to our naive GU). So, this very strong drag effect affecting even the gyroscopes need not arise (more precisely,

[15] E.g. *relative* to the coordinates of our Tilted Spiral World everything rotates in the same direction φ.

[16] Technically, we have Fermi coordinates in mind.

[17] This means that in C^+, gyroscopes of m_0 preserve their directions (relative to the coordinate system).

need not be strong enough for affecting the gyroscopes). Indeed, as we said earlier, our dervish world is very close structurally to Gödel's original space-time (GU). Therefore results about the original GU are applicable to our versions (calibrated slightly differently). Now, the results in Lathrop–Teglas [**LT78**] can be used to conclude that in our Choice 2 Spiral World gyroscope directions are fixed, i.e. they do not rotate. This can be seen by their characterization of geodesics in basically Choice 2 Spiral World, as well as from their claim that Choice 2 Spiral coordinates are so called Fermi coordinates.

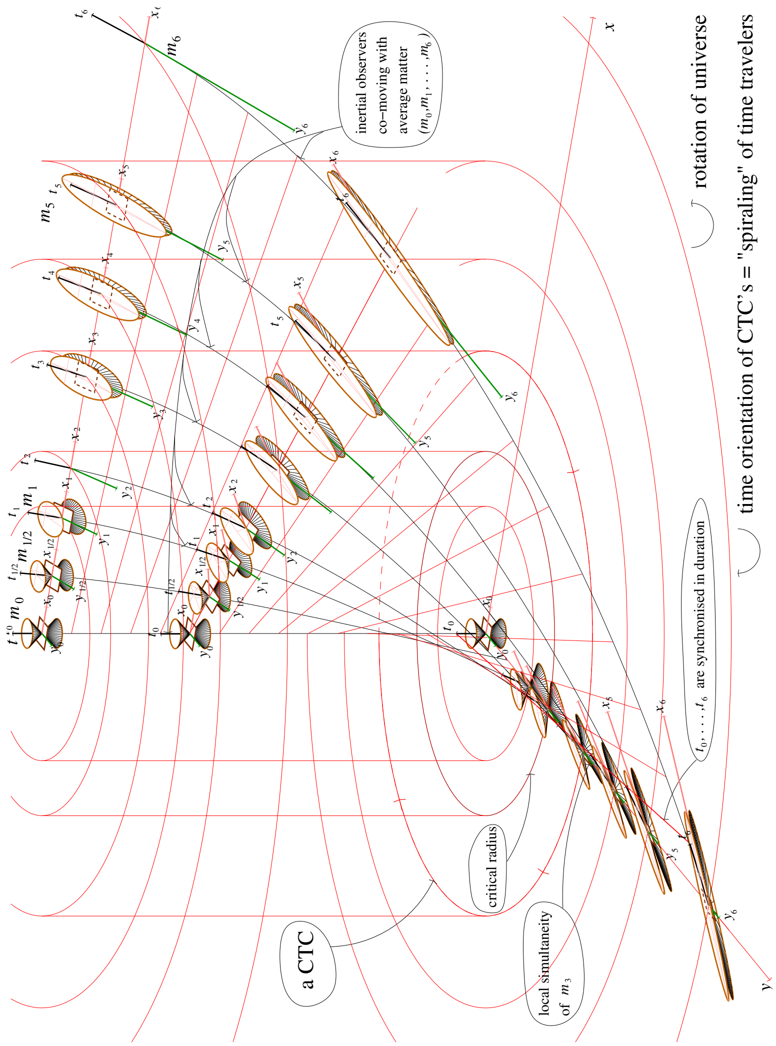

Figure 30: Choice 2 GU spiral view (i.e., Refined Spiral World). Here gyroscope directions are fixed (they do not change). (We are in Fermi coordinates in the sense of e.g. Lathrop–Teglas [**LT78**].)

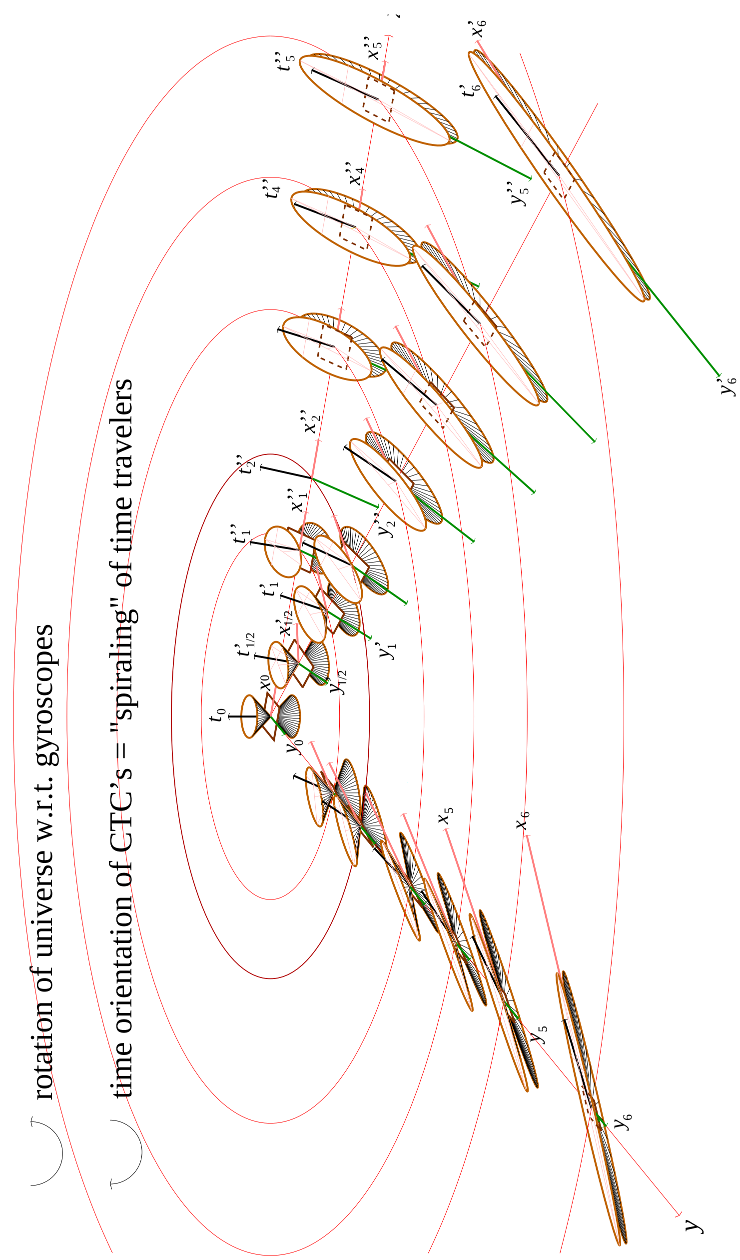

Figure 31: Choice 2 spiral view. Cf. Fig.30 for more information.

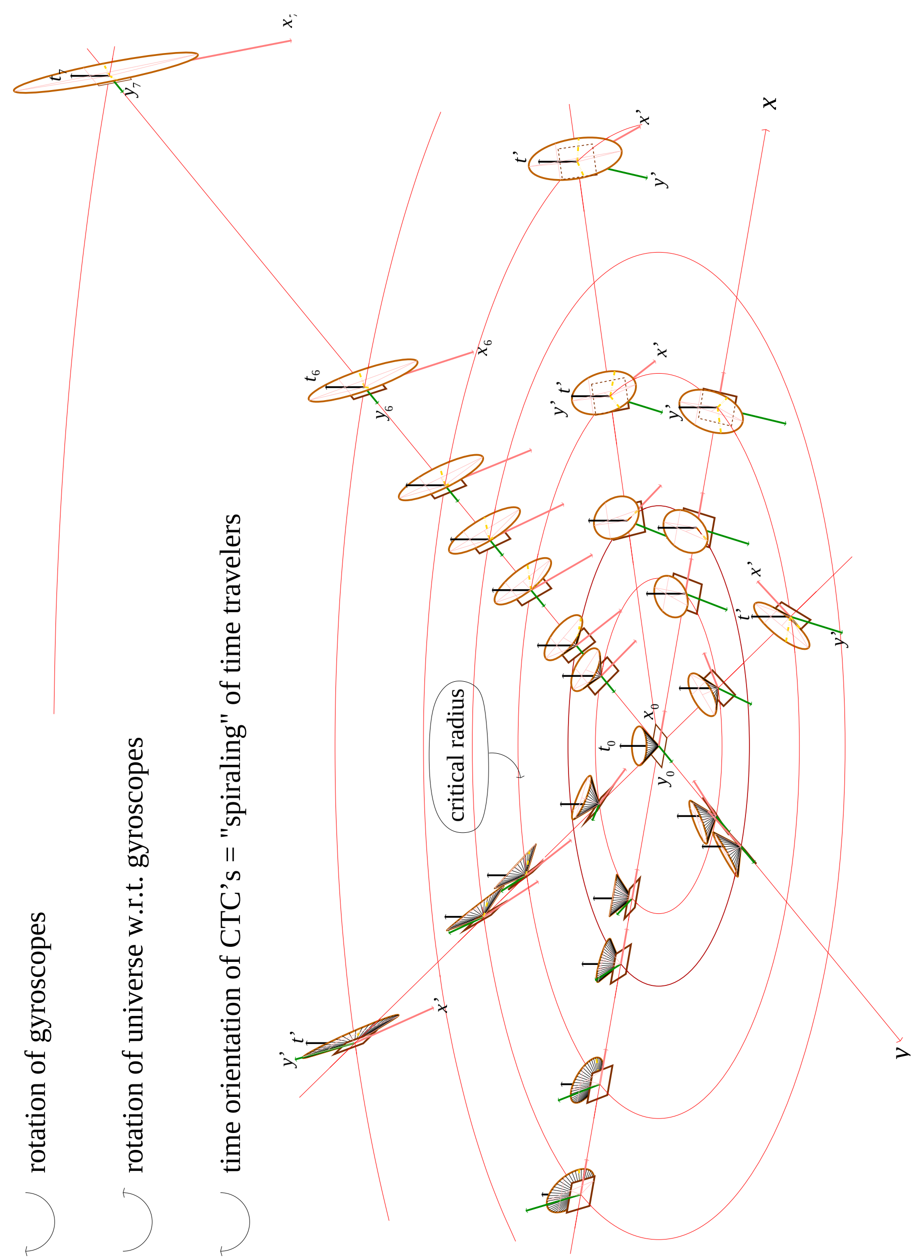

Figure 32: Dervish view in dual GU (Choice 2). Compare with Figure 61 on p.169 in Hawking–Ellis [**HE73**] (cf. also Fig.1 herein).

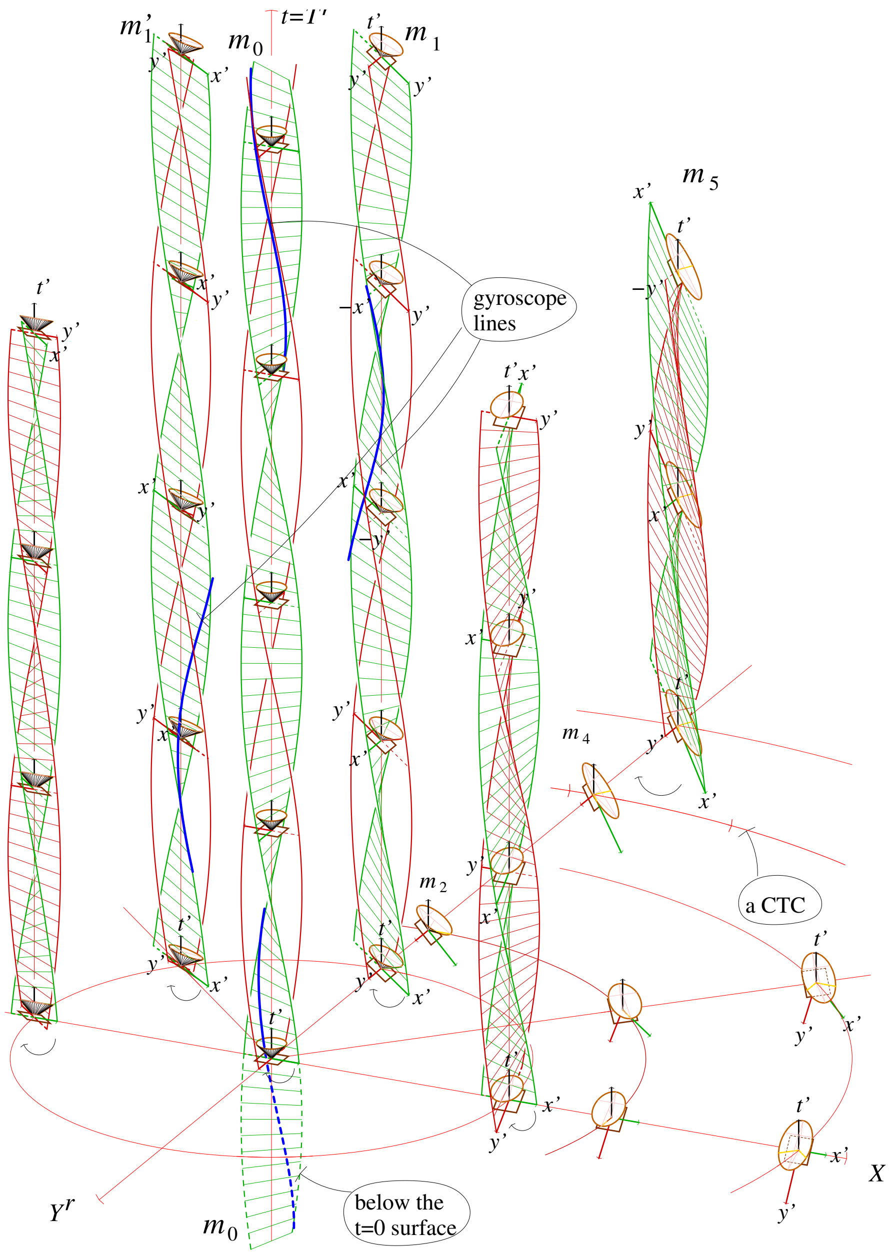

Figure 33: Choice 2 dervish view. “Fast” gyroscope lines. Spinning dervishes are artificially sped up (“artificial ω of dervishes” $= \pi/15$).

7. Metric tensors and some literature.

7.1. The metric tensor of the Naive GU. The line element in the Naive Spiral World is

$$\mathsf{ds}^2 = -\frac{1-r^2\omega^2}{(1+r^2\omega^2)^2}\,\mathsf{dt}^2 + \mathsf{dr}^2 + \mathsf{dz}^2 + \frac{r^2(1-r^2\omega^2)}{(1+r^2\omega^2)^2}\,\mathsf{d}\varphi^2 - \frac{4r^2\omega}{(1+r^2\omega^2)^2}\,\mathsf{d}\varphi\mathsf{dt}\,.$$

Thus the components of the metric tensor g of the Naive GU in the Naive Spiral World are

$$\mathsf{g}_{tt} = -\frac{1-r^2\omega^2}{(1+r^2\omega^2)^2}, \qquad \mathsf{g}_{rr} = \mathsf{g}_{zz} = 1,$$
$$\mathsf{g}_{\varphi\varphi} = \frac{r^2(1-r^2\omega^2)}{(1+r^2\omega^2)^2}, \qquad \mathsf{g}_{\varphi t} = \mathsf{g}_{t\varphi} = -\frac{2r^2\omega}{(1+r^2\omega^2)^2}\,,$$

and the rest of the g_{ij}'s are 0.

The nonzero Christoffel symbols Γ^i_{jk} are

$$\Gamma^r_{tt} = \frac{r\omega^2(r^2\omega^2-3)}{(1+r^2\omega^2)^3}\,, \quad \Gamma^t_{tr} = \frac{(1-r^2\omega^2)r\omega^2}{(1+r^2\omega^2)^2}\,, \quad \Gamma^\varphi_{tr} = \frac{-2\omega}{(1+r^2\omega^2)^2 r}\,,$$
$$\Gamma^r_{t\varphi} = \frac{2r\omega(1-r^2\omega^2)}{(1+r^2\omega^2)^3}\,, \quad \Gamma^t_{r\varphi} = \frac{2r^3\omega^3}{(1+r^2\omega^2)^2}\,, \quad \Gamma^\varphi_{r\varphi} = \frac{1-r^2\omega^2}{(1+r^2\omega^2)^2 r}\,,$$
$$\Gamma^r_{\varphi\varphi} = \frac{r(3r^2\omega^2-1)}{(1+r^2\omega^2)^3}\,, \quad \text{and the } \Gamma^i_{kj} = \Gamma^i_{jk} \text{ for the nonzero } \Gamma^i_{jk} \text{ listed above.}$$

The scalar curvature is

$$R = 2\omega^2\frac{(2r^2\omega^2-7)}{(r^2\omega^2+1)^2}\,.$$

Now, $\Gamma_{rr} = \bar{0} = \langle 0,0,0,0\rangle$ shows that the radial straight lines in the xy-planes (i.e., the lines with direction "dr") are geodesics. The worldlines of the galaxies are of direction $\omega\mathsf{d}\varphi + \mathsf{dt}$, hence

$$\omega^2\Gamma_{\varphi\varphi} + 2\omega\Gamma_{\varphi t} + \Gamma_{tt} = \bar{0}$$

shows that the worldlines of the distinguished observers m_i are geodesics in the Naive GU.

Gödel wanted the distinguished observers $m_0, \ldots, m_i$ to be fully "equivalent" with each other. This means that m_i and m_0 should be indistinguishable for any choice of m_i. This means that there should exist an automorphism $h_{i,0} : \langle\mathbb{R},\mathsf{g}\rangle \longrightarrow \langle\mathbb{R},\mathsf{g}\rangle$ such that $h_{i,0}$ takes the worldline of m_i to that of m_0. Since the scalar curvature is preserved by automorphisms, this implies that the scalar curvature should not depend on r (as it really does not depend on r in Gödel's universe as we will see soon). This implies that in the Naive GU, the distinguished observers m_i are not fully equivalent with each other, because the scalar curvature depends on r.

We note that the line element in the Naive Dervish World is

$$\mathsf{ds}^2 = -\,\mathsf{dt}^2 + \mathsf{dr}^2 + \mathsf{dz}^2 + \frac{r^2(1-r^2\omega^2)}{(1+r^2\omega^2)^2}\,\mathsf{d}\varphi^2 + \frac{2r^2\omega}{(1+r^2\omega^2)}\,\mathsf{d}\varphi\mathsf{dt}\,.$$

7.2. The metric tensor of Gödel's universe GU. Gödel in [**Göd95b**, p.275], [**Göd90**, p.195] and elsewhere defines his universe by presenting the "line element" (i.e. the "metric tensor field") as

$$(\star)\qquad \mathsf{ds}^2 = \frac{2}{\omega^2}\left[-\mathsf{dt}^2 + \mathsf{dr}^2 + \mathsf{dz}^2 + (\sinh^2 r - \sinh^4 r)\mathsf{d}\varphi^2 + 2\sqrt{2}\sinh^2 r\mathsf{d}\varphi\mathsf{dt}\right].$$

This is understood in the *cylindric-polar coordinates* $\langle t^d, r^d, \varphi^d, z^d\rangle$ of the dervish world we discussed in Sections 3,8. Cf. Figure 35. Instead of $\frac{2}{\omega^2}$, Gödel writes $4a^2$ but in our notational system these two constants are basically the same. (One can interpret Gödel's a as $a = \frac{1}{\sqrt{2}}\omega$. [18] Anyway, a and ω are only "parameters".) Other differences are that Gödel used the $+ - --$ sign-convention and we also made a $\varphi \to -\varphi$ coordinate transformation so as to use the same form of Gödel's metric that Lathrop–Teglas [**LT78**] uses. In tensorial form, $(\star)$ can be written by specifying that Gödel's metric tensor field $\frac{2}{\omega^2}\mathsf{g}$ is defined by

$$\mathsf{g}_{tt} = 1\,,\qquad \mathsf{g}_{rr} = \mathsf{g}_{zz} = -1\,,\qquad \mathsf{g}_{\varphi\varphi} = \sinh^4 r - \sinh^2 r\,,\qquad \mathsf{g}_{\varphi t} = \sqrt{2}\sinh^2 r\,,$$

$\mathsf{g}_{t\varphi} = \mathsf{g}_{\varphi t}$, and the rest of the g_{ij}'s are 0.

Clearly, $\mathsf{g}(p)$ is a function of $p = \langle t, r, \varphi\rangle$, but only $\mathsf{g}_{\varphi\varphi}$ and $\mathsf{g}_{\varphi t}$ depend on p. Further, of the parts of p, they depend only on r_p and on φ_p. This is caused by the symmetries of our space-time, i.e. rotation along φ and translation along t are automorphisms of GU (both for all versions of GU herein as well as in Gödel's quoted [19] papers). Notice that in the Naive Dervish World, both $\mathsf{g}_{\varphi\varphi}$ and $\mathsf{g}_{t\varphi}$ tend to constants as r tends to infinity while in Gödel's Dervish World they both tend to infinity as r tends to infinity. This is why we refined our Naive GU to obtain the Tilted GU.

Lathrop–Teglas [**LT78**] presents Gödel's universe in so-called Fermi coordinates. This means that the t axis as well as the radial lines are geodesics and the gyroscopes (i.e., compasses of inertia) of m_0 are not rotating. This is a spiral world where the cosmic compasses are replaced with compasses of inertia. It is very similar to Refined (Choice 2) Spiral World depicted in Figure 30. Indeed, [**LT78**] obtains this metric from $(\star)$ above by the following coordinate transformation. Below t', r', z', φ' are the new coordinates, t, r, z, φ are the coordinates used in $(\star)$ and $c = \frac{\sqrt{2}}{\omega}$.

$$t' = ct,\qquad r' = cr,\qquad z' = cz,\qquad \varphi' = \omega t' - \varphi\,.$$

This is the transformation from forward tilted (Choice 1) Dervish World to backward tilted (Choice 2) Spiral World (apart from multiplying with a constant c). From now on, for simplicity, we write t, r, φ, z for t', r', φ', z'.

Let us use the notation

$$\mathrm{sh} = \sinh(\frac{\omega}{\sqrt{2}}\,r)\qquad\text{and}\qquad \mathrm{ch} = \cosh(\frac{\omega}{\sqrt{2}}\,r)\,.$$

Now, the "line element" (i.e. the "metric tensor field") of Gödel's universe in Fermi coordinates is

$$\mathsf{ds}^2 = -(1 + 2\,\mathrm{sh}^2\,\mathrm{ch}^2)\mathsf{dt}^2 + \mathsf{dr}^2 + \mathsf{dz}^2 + \frac{2}{\omega^2}\,\mathrm{sh}^2(1 - \mathrm{sh}^2)\mathsf{d}\varphi^2 + \frac{4}{\omega}\,\mathrm{sh}^4\,\mathsf{d}\varphi\mathsf{dt}\,.$$

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[18] Cf. item (9) on p.191 in Gödel [**Göd90**].

[19] There are papers of Gödel in which these symmetries fail (for rotating universes), cf. e.g. [**Göd90**, p.208].

The nonzero Christoffel symbols Γ^i_{jk} are

$$\Gamma^r_{tt} = \omega\sqrt{2}\,\mathrm{sh}\,\mathrm{ch}(2\,\mathrm{ch}^2-1)\,,\quad \Gamma^t_{tr} = \omega\sqrt{2}\,\mathrm{sh}\,\mathrm{ch}\,,\quad \Gamma^\varphi_{tr} = \omega^2\sqrt{2}\,\mathrm{sh}\,\mathrm{ch}\,,$$

$$\Gamma^r_{t\varphi} = -2\sqrt{2}\,\mathrm{sh}^3\,\mathrm{ch}\,,\qquad \Gamma^t_{r\varphi} = \frac{\sqrt{2}sh^3}{\mathrm{ch}}\,,\qquad \Gamma^\varphi_{r\varphi} = \frac{-\omega(2\,\mathrm{ch}^4-4\,\mathrm{ch}^2+1)}{\sqrt{2}\,\mathrm{sh}\,\mathrm{ch}}\,,$$

$$\Gamma^r_{\varphi\varphi} = \frac{\sqrt{2}\,\mathrm{sh}\,\mathrm{ch}(2\,\mathrm{ch}^2-3)}{\omega}\,,$$ and the $\Gamma^i_{kj} = \Gamma^i_{jk}$ for the nonzero Γ^i_{jk} listed above.

The scalar curvature is

$$R = 2\omega^2\,.$$

A sample of papers investigating Gödel's universe is Chakrabarti–Geroch–Liang [**CGL83**], Chandrasekhar–Wright [**CW61**], Dorato [**Dor**], Gödel [**Göd95a**], [**Göd95b**], Heckmann–Schücking [**HS55**], Kundt [**Kun56**], Lathrop–Teglas [**LT78**], Malament [**Mal84**], Obukhov [**Obu00**], Plaue–Scherfner–de Sousa [**PSd**], Sklar [**Skl84**], Stein [**Ste70**]. A sample of books about general relativity and time (especially relevant to the present paper) is Earman [**Ear95**], Gibilisco [**Gib83**], Gott [**Got01**], Horwich [**Hor87**], Novikov [**Nov98**], O'Neill [**O'N95**], Pickover [**Pic98**], Yourgrau [**You99**].

For more on the drag effect and its connections with Mach's principle cf. e.g. Wald [**Wal84**, p.89 item 3.(c), p.187 Problem 3(b), p.319 immediately below item (12.3.17)]. For more detail on "drag" and Mach cf. Misner–Thorne–Wheeler [**MTW73**, §21.12 (entitled "Mach's...") and especially pp.546–548, also item B on p.879, pp.1117, 699, 893, 1120]. Cf. also d'Inverno [**d'I83**, §9.2 (pp.121–124)], Gibilisco [**Gib83**, pp.19–123 (subtitle: Alone in the universe)].

For the gravitational drag effect we refer to Rindler [**Rin77**, pp.10–13, §§1.15, 1.16], Wald [**Wal84**, pp.9,71,89,183,319], Wald [**Wal77**, pp.32–33], together with Misner–Thorne–Wheeler [**MTW73**, §40.7 (pp.1117–1120), §33.4 (p.892), §21.12 (in particular p.547), p.1120 (footnote)]. The gravitational drag effect is related to Mach's principle as is explained e.g. in [**MTW73**, §21.12] and in [**Rin77**, §1.15 (e.g. p.12)].

Figure 1 is a slightly corrected version of Figure 31 in Hawking–Ellis [**HE73**]. This picture can also be found in Yourgrau [**You99**]. Malament [**Mal84**, p.99] pointed out that the light-cones on that figure are tilted so much that they do not contain the vertical lines which are the worldlines of the distinguished observers in the dervish-world (which the figure represents). Below we include the Figure from Malament's paper (in which the light-cones are corrected already).

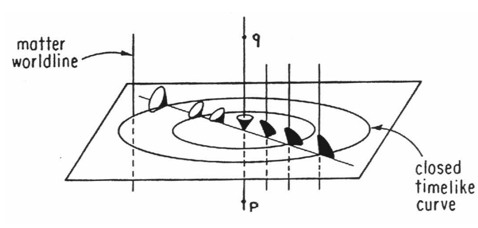

Figure 34: Figure from Malament's paper [**Mal84**].

The present work is part of a broader effort for what we could bluntly call demystifying general relativity theory and its relatives like wormhole-theory and cosmology. More concretely, we try to provide a purely logic based conceptual analysis for general relativity and its relatives. One of the aims is to provide a technically correct but easily understandable introduction to general relativity including its most exotic reaches for the questioning mind of the nonspecialist. A sample of works in this general direction is [**AMN07**], [**ANW08**], [**Mad02**], [**Szé09**], [**Szé09a**].

8. Appendix: technical details for the constructions.

Connections between our spiral coordinate system $\langle t, x, y, z\rangle = \langle t^s, \dots, z^s\rangle$ and co-rotating (dervish) coordinate system $\langle t', x', y', z'\rangle = \langle T^r, X^r, Y^r, Z^r\rangle$:

By definition, $t' = t$ and $z' = z$. Throughout we suppress the irrelevant spatial coordinate z. Below, instead of the Cartesian systems $\langle t, \dots, y\rangle, \langle t', \dots, y'\rangle$ we use their cylindric-polar-coordinates variants $\langle t, \varphi, r\rangle$ and $\langle t', \varphi', r'\rangle$. [20] The connections are the usual standard ones, e.g. $r = \sqrt{x^2 + y^2}$, $y = r \cdot \cos(\varphi)$, $x = r \cdot \sin(\varphi)$, $\varphi = \arctan(x/y)$. In more detail, $r(p) = \sqrt{x(p)^2 + y(p)^2}$ etc. $\langle t^s, \varphi^s, r^s\rangle := \langle t, \varphi, r\rangle$ and $\langle T^r, \varphi^r, r^r\rangle = \langle t^d, \varphi^d, r^d\rangle = \langle t', \varphi', r'\rangle$. Here s abbreviates "spiral" and "d" abbreviates "dervish".

The "galaxies" $m_1, m_2, \dots, m_i$ appear as rotating around m_0 in direction φ with angular velocity ω in $\langle t^s, \dots\rangle$ while their cosmic compasses x_i, y_i appear fixed (non rotating). As a contrast, $\langle T^r, \dots\rangle$ shows $m_1, \dots, m_i$ as motionless, while it shows their cosmic compasses as rotating in direction $-\varphi$ with angular velocity ω. We use p to denote an arbitrary point which has coordinates $t(p), \varphi(p), r(p)$ etc. We represent these simple connections in Figures 35–37. As we said, we suppress coordinate z. In Figure 35 below (p.44) we regarded only such points p which are on the cylinder $r(p) = 1$. Generalizing to arbitrary points is trivial since r does not change. As it is obvious from the picture, the transformation "spiral" $\mapsto$ "dervish" is

$$\varphi^d(p) = \varphi^s(p) - \omega \cdot t^s(p)$$
$$r^d(p) = r^s(p)$$
$$t^d(p) = t^s(p)$$
$$z^d(p) = z^s(p).\ \text{Clearly,}$$
$$\varphi^s(p) = \varphi^d(p) + \omega \cdot t^d(p).$$

The angular velocity of the rotation of the universe as seen by $\langle t^s, \dots\rangle$ is ω.

[20] Cf. e.g. d'Inverno [**d'I83**, Fig.19.2 (p.253)].

View from the spiral coordinate system $\langle t^{\mathrm{s}}, \varphi^{\mathrm{s}}, r^{\mathrm{s}} \rangle$:

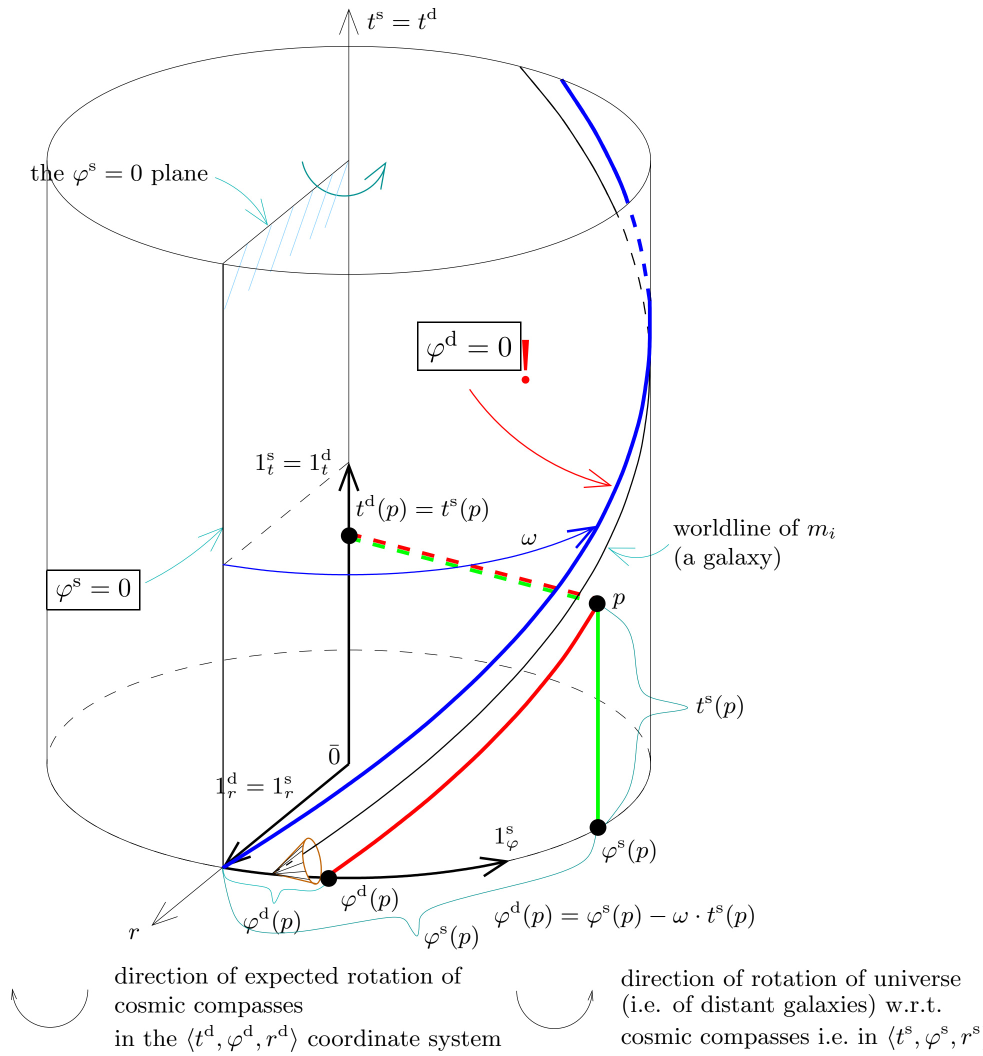

Figure 35: As throughout this work, here too, the irrelevant spatial coordinates $z^{\mathrm{d}} = z^{\mathrm{s}} = z_{\mathrm{i}} = z$ are suppressed.

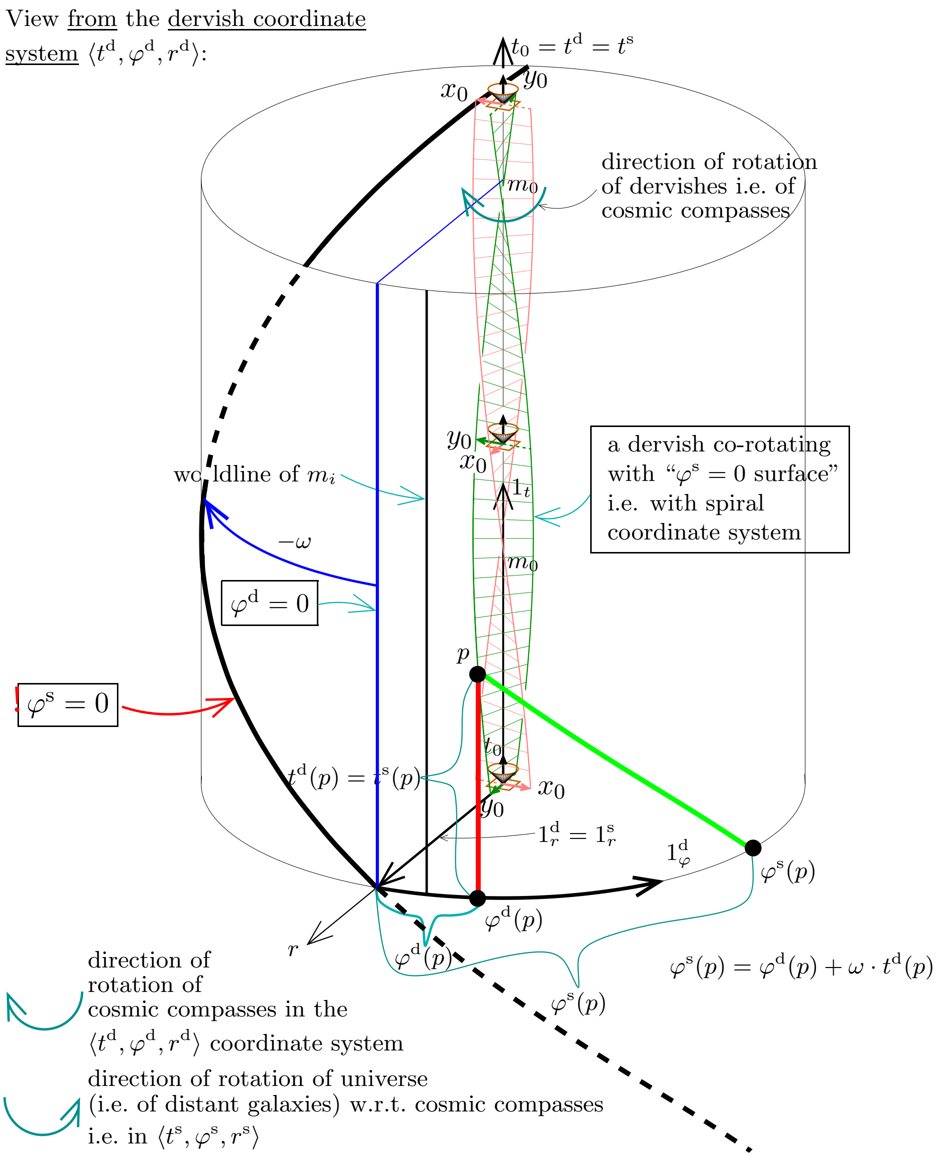

Figure 36: Dervish view of spiral world, i.e. backward transformation $\langle t^{\rm d}, \ldots\rangle \longrightarrow \langle t^{\rm s}, \ldots\rangle$. Notice that the $t = 0$ plane in this figure coincides with that of previous figure (e.g. marked points are the same on the two).

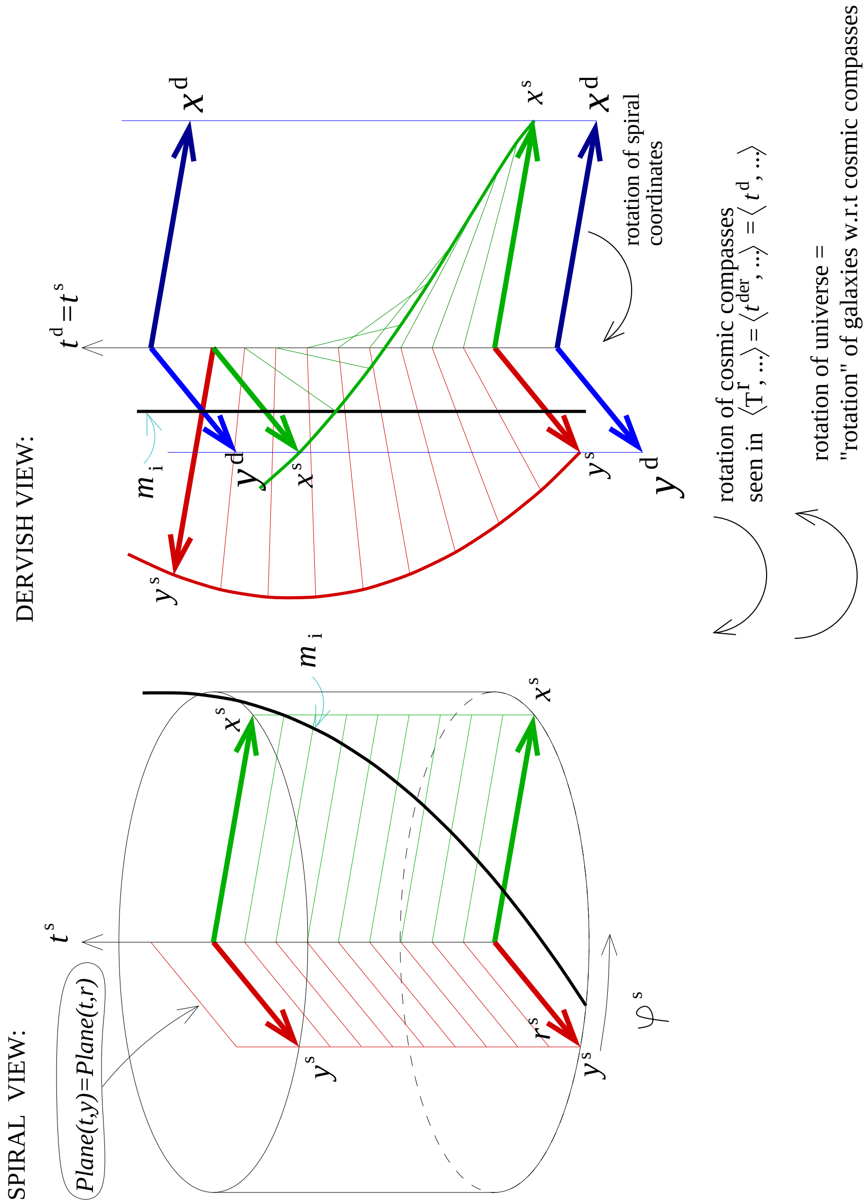

Figure 37: In the Spiral World, the "galaxies" $m_1, m_2, \ldots, m_i$ appear as rotating around m_0 in direction φ with angular velocity ω while their cosmic compasses x_i, y_i appear fixed (non rotating). As a contrast, the Dervish World shows $m_1, \ldots, m_i$ as motionless, while it shows their cosmic compasses as rotating in direction $-\varphi$ with angular velocity ω.

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Rényi Institute of Hungarian Academy of Sciences, Reáltanoda street 13-15, H-1053 Hungary

E-mail address: inemeti@gmail.com

Rényi Institute of Hungarian Academy of Sciences, Reáltanoda street 13-15, H-1053 Hungary

E-mail address: madarasz@renyi.hu

Rényi Institute of Hungarian Academy of Sciences, Reáltanoda street 13-15, H-1053 Hungary

E-mail address: andreka@renyi.hu

RIKEN, BSI, Amari Research Unit, 2–1, Hirosawa, Wako, Saitama 351-0198, Japan.

E-mail address: andaia@math.bme.hu