RELATIVISTIC COMPUTERS AND THE TURING BARRIER

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Abstract. We examine the current status of the physical version of the Church-Turing Thesis (PhCT for short) in view of latest developments in spacetime theory. This also amounts to investigating the status of hypercomputation in view of latest results on spacetime. We agree with Deutsch et al [17] that PhCT is not only a conjecture of mathematics but rather a conjecture of a combination of theoretical physics, mathematics and, in some sense, cosmology. Since the idea of computability is intimately connected with the nature of Time, relevance of spacetime theory seems to be unquestionable. We will see that recent developments in spacetime theory show that temporal developments may exhibit features that traditionally seemed impossible or absurd. We will see that recent results point in the direction that the possibility of artificial systems computing non-Turing computable functions may be consistent with spacetime theory. All these trigger new open questions and new research directions for spacetime theory, cosmology, and computability.

"Of all the entities I have encountered in my life in physics, none approaches the black hole in fascination. And none, I think, is a more important constituent of this universe we call home. The black hole epitomizes the revolution wrought by general relativity. It pushes to an extreme—and therefore tests to the limit—the features of general relativity (the dynamics of curved spacetime) that set it apart from special relativity (the physics of static, "flat" spacetime) and the earlier mechanics of Newton. Spacetime curvature. Geometry as part of physics. Gravitational radiation. All of these things become, with black holes, not tiny corrections to older physics, but the essence of newer physics."
—John Archibald Wheeler (2000).

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1. Aims, perspective

We discuss the perspectives and scope of applicability of the Physical Church-Turing Thesis (PhCT). Roughly, PhCT is the conjecture that whatever physical computing device (in the broader sense) or physical thought experiment will be designed by any future civilization, it will always be simulatable by a Turing machine. We carefully defined what we understand by PhCT in Etesi-Németi [26], here we do not recall that definition in detail.

In this paper we discuss the issue of whether in the light of latest developments in theoretical physics and cosmology there is likely to be a theoretical possibility for going beyond PhCT or not. By going beyond PhCT we do not mean a perhaps "cheap" or easy negation of PhCT, i.e. we do not visualize a beyond-Turing computer like a pocket calculator or a laptop, but we mean it as a perhaps extremely expensive, physical experiment which needs the latest, most exotic results of theoretical physics or cosmology (as its theoretical foundation). By beyond-Turing computer or hypercomputer we refer to situations where an artificial system or physical thought experiment performs a computation which is beyond the Turing limit (i.e. implements a non-Turing computable function).

For such a discussion we have to clarify the nature of PhCT. We agree with Deutsch et al [17] in that PhCT is not a purely mathematical conjecture, but rather it is a combination of physical, mathematical and in some sense cosmological conjectures. The main emphasis is on the part which says that PhCT is partly a physical-cosmological conjecture. (It also has some connections with the mathematical foundations of Artificial Intelligence research as was pointed out in Leeuwen and Wiedermann [43], [87] but for brevity we omit these aspects while completely agreeing with what Leeuwen and Wiedermann say.) This view of Deutsch et al [17] that PhCT is mostly a physical conjecture has been recently advocated by many authors.

The PhCT was formulated and generally accepted in the 1930's. At that time a general consensus was reached declaring PhCT valid, and indeed in the succeeding decades the PhCT was an extremely useful and valuable maxim in elaborating the foundations of theoretical computer science, logic and related areas. As an exception, we would like to mention that László Kalmár, one of the leading logicians of that time, expressed occasionally his hope that sometime in the future mankind will be able to supersede the PhCT, [40].

But since PhCT is partly a physical, cosmological conjecture, we emphasize that the general consensus of the 1930's was based on the physical world-view of the 1930's. Moreover, many thinkers considered PhCT as being based on mathematics + common sense. But "common sense of today" means "physics of 100 years before". Therefore we claim that the general consensus accepting PhCT in the 1930's was based on the world-view deriving from Newtonian mechanics. Einstein's equations became known to a narrow circle of specialists around 1920, but around that time the consequences of these equations were not even guessed at. In other words, the world-view of modern black hole physics was very far from being generally known until much later, until after 1970. Summing up, PhCT became generally accepted on the basis of the world-view of, basically, Newtonian mechanics.

Our main point is that in the last few decades (well after 1970) there has been a major paradigm shift in our physical world-view as well as our cosmological one. This started in 1970 by Hawking's and Penrose's singularity theorem firmly establishing black hole theory and putting general relativity into a new perspective. After that, discoveries and new results have been accelerating. About 10 years ago astronomers obtained firmer and firmer evidence for the existence of larger and more exotic black holes, not to mention evidence supporting the assumption that the universe is not finite after all. Nowadays the whole field is in a state of constant revolution.

What does this tell us about the PhCT? Roughly, it tells us that the background world-view on the basis of which PhCT was generally accepted (even formulated) is not valid any more. (Actually, our world has changed so much that no one can bring back that kind of world-view ever in the future.) If the background foundation on which PhCT was based has changed so fundamentally, so radically, then it is desirable to re-examine the status and scope of applicability of PhCT in view of the new evidence, in view of the change of our general world-picture.

Indeed, in [26] we prove that it is consistent with Einstein's equations, i.e. with general relativity, that by certain kinds of relativistic experiments, future generations might find the answers to non-computable questions like the halting problem of Turing machines or the consistency of Zermelo Fraenkel set theory (the foundation of mathematics, abbreviated as ZFC set theory from now on). For brevity, we call such thought experiments relativistic computers. Moreover, the spacetime structure we assume to exist in these experiments is based in [26] on huge slowly rotating black holes the existence of which is made more and more likely (almost certain) by recent astronomical observations.

Before going more into this, let us step back for a second and ask ourselves what the general idea behind this kind of developments is. Why would the switch to general relativity and new cosmology help us in designing beyond-Turing computing devices?

A special feature of the Newtonian world-view is the assumption of an absolute time scale. Indeed, this absolute time has its mark on the Turing machine as a model for computer. As a contrast, in general relativity there is nothing even similar to absolute time. Kurt Gödel was particularly interested in the exotic behavior of time in general relativity (GR). Gödel [31],[32] was the first to prove that there are GR spacetimes (models of GR) which, technically speaking, do not admit a foliation. Foliation of a spacetime $\langle M, g\rangle$ means that a "global time" (or global temporal preordering) satisfying certain natural properties can be put on $\langle M, g\rangle$. In particular, various observers at various points of spacetime in different states of motion might experience time radically differently. Therefore we might be able to speed up the time of one observer, say O_c, relatively to the other observer, say O_p. Thus O_p may observe O_c computing very fast. The difference between general relativity and special relativity is (roughly) that in general relativity this speed-up effect can reach, in some sense, infinity assuming certain conditions are satisfied. Of course, it is not easy to ensure that this speed-up effect happens in such a way that we could utilize it for implementing some non-computable functions. Actually, there were many false starts before the arrangement elaborated in e.g. [26] was arrived at. Very strongly related positive results, similar in spirit to [26] were arrived at e.g. in Hogarth [37], [36], [38], Malament [46], Earman-Norton [21],[22], Earman [20, Chap. 4], Tipler [80, pp.447-448], Barrow [7], Bacon [3], Brun [10]. Nowadays there is an ever broadening circle of researchers including Hogarth, Earman, Etesi, Andréka, Sági, Shagrir, Pitowsky, the present authors and others (cf. also the references) who are working on refining the idea of beyond-Turing computers based on the latest findings of spacetime theory.

The purpose of the present paper is to discuss whether it is consistent with general relativity, GR, that future generations might be able to design beyond-Turing computers. So, technically, we are working inside GR. Whenever we claim that something is possible, the safe interpretation of this claim is that there is a GR spacetime $\langle M, g\rangle$ in which the claimed arrangement is possible. To save space, we will not restate this ramification whereever we should. Occasionally we will venture beyond GR (like black holes emitting radiation, acceleration of the expansion of the universe) but e.g. because of the nonexistence of a decisive theory of quantum gravity, these ventures remain on the level of speculation designed to trigger new research in interesting directions. In these parts we tried to use the latest developments in cosmology, astronomy, spacetime theory, but all the same, they remain on the level of speculations designed to trigger research interest in contrast with the firmer conclusions of the pure GR parts. In particular, in section 5 we discuss some ideas about the physical realizability of our relativistic computers. Section 2 below intends to illustrate the general idea of relativistic computers (on an intuitive but logically coherent level), without paying attention to the above indicated distinction between what is real and what is only "mathematical imagination". The precise presentation comes in section 4.

We will be careful to avoid basing the beyond-Turing power of our computer on "side-effects" of the idealizations in our mathematical model/theory of the world. For example, we will avoid relying on infinitely small objects (e.g. pointlike test particles, or pointlike bodies), infinitely elastic balls, infinitely (or arbitrarily) precise measurements, or anything like these. Moreover, we devote the whole of section 5 to discussing physical realizability and realism of our design for a computer. In other words, we will make efforts to avoid taking advantage of the idealizations which were made when GR (or whatever theory we use) was set up.

2. An intuitive idea for relativistic hypercomputers

In this section we would like to illuminate the ideas of why relativistic computers work, why they can work in principle at least, without going into the mathematical details. We will return to the details in section 4, and also the details have been elaborated among others in [26] and in [37]. To make our narrative more tangible, here we use the example of huge slowly rotating black holes for our construction of relativistic computers. However, as it was emphasized in [26] and [38], any one of the many different kinds of the so-called Malament-Hogarth (MH) spacetimes is suitable for carrying through essentially the same construction. These MH-spacetimes will be defined in section 4.

Let us start out from the so-called Gravitational Time Dilation (GTD)[1] (or gravitational redshift). What is the GTD? The GTD is a theorem of relativity which says that gravity makes time run slow. More sloppily: gravity slows time down. Clocks that are deep within gravitational fields run slower than ones that are farther out. We will have to explain what this means but before explaining what this means we would like to mention that this is not only a theorem of general relativity. This theorem, GTD, can be already proved in (an easily understandable logic-based version of) special relativity in such a way that we simulate gravity by acceleration. For this direction we refer to [44], [45]. So one advantage of GTD is that actually why it is true can be traced down by using only the simple methods of special relativity. Another advantage of GTD is that it has been tested several times, and these experiments are well known. Roughly, GTD can be interpreted by the following thought experiment. Choose a high enough tower on the Earth, put precise enough (say, atomic) clocks at the bottom of the tower and the top of the tower, then wait enough time, and compare the readings of the two clocks. Then the clock on the top will run faster

[1]In the popular literature GTD is sometimes referred to as timewarp.

(show more elapsed time) than the one at the basement, at each time one carries out this experiment. Therefore we will often refer to GTD as the "Tower Paradox".[2] Actually, the experiment done at Harvard was carried out in a way simpler than this. Namely, they simply measured the redshift of photons emitted in the basement of the tower and received at the top. For more detail cf. [64] or [81].[3]

How could we use the Tower Paradox for hypercomputation? In the above outlined situation, by using the gravity of the Earth, it is difficult to make practical use of the Tower Paradox. However, instead of the Earth, we could choose a huge black hole. A black hole is a region of spacetime from which even light cannot escape (cf. Cambridge homepage on Relativity). There are several types of black holes, we will distinguish between them later. An excellent source is Taylor and Wheeler [78]. All the kinds of black holes we will use in this paper have an *outer event horizon*. The outer event horizon is a bubblelike hypersurface surrounding the black hole from which even light cannot escape (because of the gravitational pull of the black hole). From points outside the outer event horizon light can escape, this is the reason for the adjective "outer".

For simplicity, at the beginning we will restrict attention to the simplest kind of black holes which have only one event horizon. These are called Schwarzschild black holes. We will introduce more complex black holes when we need them, but we note that all what we will say about Schwarzschild black holes remain true for the more general ones if we replace "event horizon" with "outer event horizon" (everywhere in our sentences). So, for a while we will write "event horizon" for "outer event horizon".

As we approach the event horizon from far away outside the Schwarzschild black hole, the gravitational "pull" of the black hole approaches infinity as we get closer and closer to the event horizon. This is rather different from the Newtonian case, where the gravitational pull also increases but remains finite even on the event horizon. On the other hand, the event horizon also exists in the Newtonian case, namely, in the Newtonian case, too, the event horizon is the "place" where the escape velocity is the speed of light (hence light cannot escape to infinity from inside this event horizon "bubble").[4]

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[2]The word "Paradox" here does not refer to a logical impossibility. Instead, it only refers to contradicting Newtonian intuition. This is similar to the use in Twin Paradox.

[3]More direct tests of the Tower Paradox (gravity causes slow time) were carried out several times by comparing atomic clocks in high orbit around the Earth and comparing them with similar clocks on the Earth e.g. by NASA, cf. [5, chap.26.2] or [35], or see the literature on GPS (global positioning system).

[4]The Newtonian event horizon was discovered by e.g. Laplace 1799 (and by J. Mitchell 1784).

Let us study observers suspended over the event horizon. Here, suspended[5] means that the distance between the observer and the event horizon does not change. (Since the black hole has a gravitational pull, the world-lines of these suspended observers are not geodesics.) Assume one suspended observer H is higher up and another one, L, is suspended lower down. So, H sees L below him while L sees H above him. Now the gravitational time dilation (GTD) effect discussed above will cause the clocks of H run faster than the clocks of L. Moreover, they both agree on this if they are watching each other e.g. via photons. Let us keep the height of H fixed. Now, if we gently lower L towards the event horizon, then this ratio between the speeds of their clocks increases. Moreover, as L approaches the event horizon, this ratio approaches infinity. This means that for any integer n, if we want H's clocks to run n times as fast as L's clocks, then this can be achieved by lowering L to the right position.

Let us see what this means for computational complexity. This means that if the programmer wants to speed up his computer with an arbitrarily large ratio, say n, then he can achieve this by putting the programmer to the position of L and putting the computer to the position of H. Already at this point we could use this situation, the arrangement with the black hole, for making computers faster. The programmer goes very close to the black hole, leaving his computer far away. Then the programmer has to wait a few days and the computer does a few million year's job of computing and then the programmer knows a lot about the consequences of, say, ZFC set theory or whatever mathematical problem he is investigating. So we could use this for just speeding up computation which means dealing with complexity issues. However, we do not want to stop at complexity issues. Instead, we would like to see whether we can attack somehow the "Turing barrier".

At this point our assumption that the black hole is huge becomes useful since this ensures that the programmer does not experience too big tidal forces at the event horizon. We note that astronomical evidence suggests the existence of black holes much bigger than what we need, e.g. black holes of $10^{10}m_\odot$ ($m_\odot$ refers to solar mass) seem to exist whose size is roughly that of the solar system. We will return to this issue later.

The above arrangement for speeding the computer up raises the question of how the programmer avoids consequences of the fact that the whole manoeuver will slow down the programmer's own time relative to the time on his home planet, e.g. on the Earth. We will deal with this problem later. So the reader is kindly asked to believe for a while that this effect will be circumnavigated somehow. Let us turn now to

[5]Equivalently, instead of suspended observers, we could speak about observers whose spaceship is hovering over the event horizon, using their rockets for maintaining altitude.

the question of how we can use this effect of finite (but unbounded) speed-up to achieve an infinite speed-up, i.e. to breaking the Turing barrier.

If we could suspend the lower observer L on the event horizon itself then from the point of view of H, L's clocks would freeze, therefore from the point of view of L, H's clocks (and computers!) would run infinitely fast, hence we would have the desired infinite speed-up upon which we could then start our plan for breaking the Turing barrier. The problem with this plan is that it is impossible to suspend an observer on the event horizon. As a consolation for this, we can suspend observers arbitrarily close to the event horizon.

To achieve an "infinite speed-up" we could do the following. We could lower and lower L again towards the event horizon such that L's clocks slow down (more and more, beyond limit) in such a way that there is a certain finite time-bound, say b, such that, roughly, throughout the whole history of the universe L's clocks show a (proper) time smaller than b. More precisely, by this we mean that whenever H decides to send a photon to L, then L will receive this photon before time b according to L's clocks. This is possible.

Are we done, then? Not yet, there is a remaining task to solve. As L gets closer and closer to the event horizon, the gravitational pull or gravitational acceleration tends to infinity. Since L has to approach the event horizon very slowly, it has to withstand this enormous gravity (or equivalently acceleration). The problem is that this increasing gravitational force (or acceleration) will kill L before his clock shows time b, i.e. before the planned task is completed. To solve this problem, we would like to achieve slowing down the "fall" of L not by brute force (e.g. rockets), but by an effect coming from the structure of spacetime itself. Let us see if there is a variant of our originally simplistic black hole in which besides the gravitational pull of the black hole (needed to achieve the time dilation effect) there is a counteractive repelling effect which would cause L to slow down in the required rate. So the idea is that instead of the rockets of L, we would like to use for slowing the fall of L a second effect coming from a second feature of the black hole.

As it turns out, there are at least two kinds of black holes with this secondary repelling effect (or force). One is the slowly rotating Kerr black hole where the centrifugal force (coming from the rotation) provides this repelling effect[6], while the other is the electrically charged black hole where, very roughly, the electrostatic repelling force provides

[6]The rotational effect is transferred from the rotating ring (source) to γ_p via the so-called drag effect [or "dragging of inertial frames"]. This drag effect helps γ_p to achieve a large enough angular momentum (around the rotational axis of the black hole) which yields the centrifugal force (acting on γ_p) needed for the above outlined plan. In passing we note that the orbital motion characterized by this angular

this effect. The latter are also called Reissner-Nordström black holes or RN spacetimes.

In some black holes with such a repelling force, two event horizons form, see Figures 1,2. The outer one is the result of the gravitational pull and behaves basically like the event horizon of the Schwarzschild hole, i.e. as described above. The inner horizon marks the point where the repelling force overcomes the gravitational force. So inside the inner horizon, it is possible again to "suspend" an observer, say L, i.e. it becomes possible for L to stay at a constant distance from the center of the black hole (or equivalently from the event horizons).

Let us turn to describing how a slowly rotating black hole implements the above outlined ideas, and how it makes possible to realize our plan for "infinite speed-up". Figure 1 represents a slowly rotating huge Kerr black hole and Figure 2 represents its spacetime structure. Figure 5 is the "causal diagram" of this spacetime.

As we said, there are two event horizons, the inner one surrounded by the outer one. The source of gravity of the black hole is a ring shaped singularity situated inside the inner horizon. The path of the infalling observer L can be planned in such a way that the event when L reaches the inner horizon corresponds to the time-bound b (on the wristwatch of L) mentioned above before which L receives all the possible messages sent out by H. In Figures 1,2 the world-lines of L and H are denoted as γ_p and γ because we think of L as the programmer and we think of H as L's computer.

By this we achieved the infinite speed-up we were aiming for. This infinite speed-up is represented in Figure 2 where γ_p measures a finite proper time between its separation from the computer γ and its touching the inner horizon at proper time b. On the other side, whenever γ decides to send a photon towards γ_p, that photon will reach γ_p before γ_p meets the inner horizon. A more detailed but also more abstract representation of this is in Figure 5. The above outlined intuitive plan for creating an infinite speed-up effect is elaborated in more concrete mathematical detail in section 4.

Let us see how we can use all this to create a beyond-Turing computer, in particular, to decide whether ZFC set theory is consistent. I.e. we want to learn whether from the axioms of set theory one can derive the formula FALSE. (This formula FALSE can be taken to be $\exists x(x \neq x)$.) This means that we can start a computer which checks one by one the theorems of set theory, and as soon as the computer finds a contradiction in set theory, i.e. a proof of the formula FALSE, from the axioms of set theory, the computer sends a signal to the programmer indicating that set theory is inconsistent. (This is a special example only. The general idea is that the computer enumerates a

momentum of γ_p "rotates" in the same direction as the rotation of the black hole. The relevance of this is illustrated by [2].

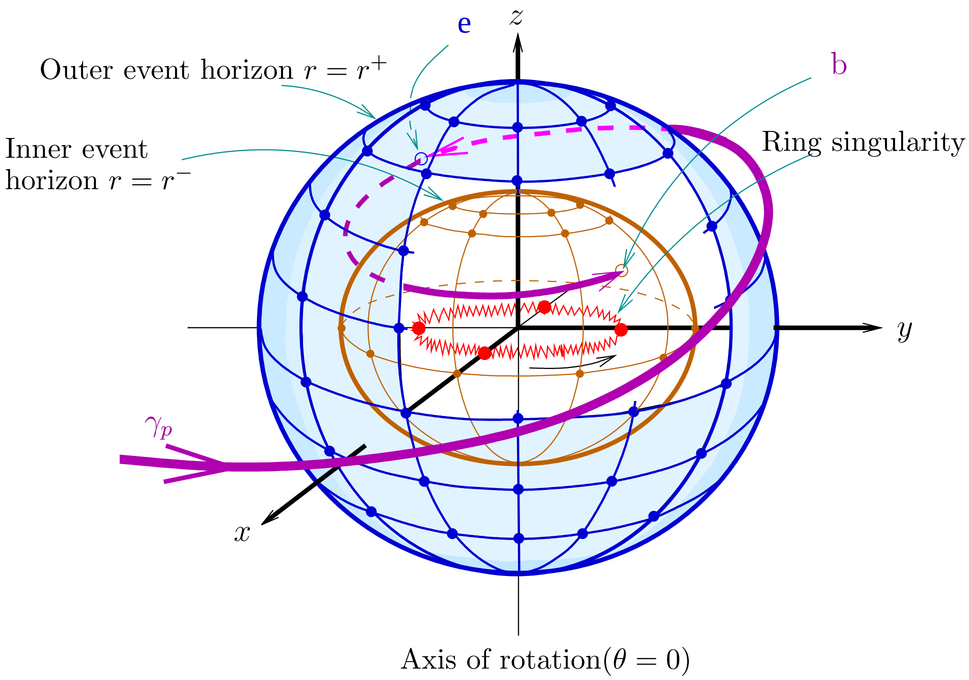

FIGURE 1. A slowly rotating (Kerr) black hole has two event horizons and a ring-shape singularity. The ring singularity is inside the inner horizon $r = r^-$ in the "equatorial" plane of axes x, y. Time coordinate is suppressed. See Figure 2 for a spacetime diagram with x, y suppressed. (Figure 2 denotes z as r.) Rotation of ring is indicated by an arrow. Orbit of infalling programmer γ_p is indicated, it enters outer horizon at point e, and meets inner horizon at point b.

recursively enumerable set and, before starting the computer, the programmer puts on the tape of the computer the name of the element which he wants to be checked for belonging to the set. The computer will search and as soon as it finds the element in question inside the set, the computer sends a signal.) If it does not find the thing in the set, the computer does nothing.

How can the programmer use this? What happens to the programmer γ_p from the point of view of the computer γ? This is represented in Figure 2. Let γ's coordinate system be the one represented in Figure 2. By saying "from the point of view of γ" we mean "in this particular coordinate system (adjusted to γ) in Fig.2". In this coordinate system when the programmer goes closer and closer to the inner horizon of the black hole, the programmer's clock will run slower and slower and slower, and eventually on the inner event horizon of the black hole the time of the programmer stops. Subjectively, the programmer does not

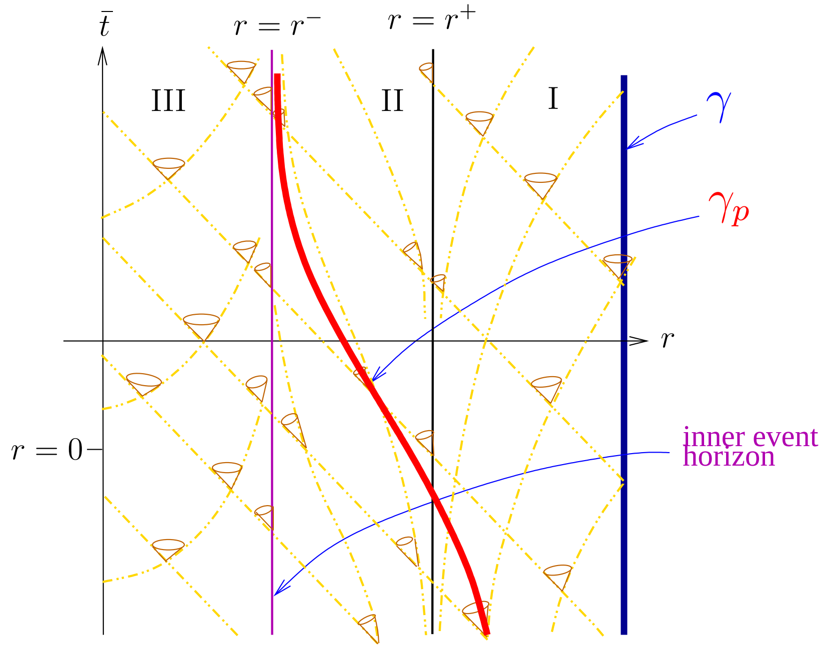

FIGURE 2. The "t-r slice" of spacetime of slowly rotating (i.e. slow Kerr) black hole in Eddington-Finkelstein coordinates where r is the axis of rotation of black hole. The pattern of light cones between the two event horizons r^- and r^+ illustrates that γ_p can decelerate so much in this region that he will receive (outside of r^-) all messages sent by γ. Compare with Figures 1,5. r^+ is the outer event horizon, r^- is the inner event horizon, $r = 0$ is the "center" of the black hole as in Figure 1. The tilting of the light cones indicates that not even light can escape through these horizons. That there is an outward push counteracting gravity can be seen by the shape of the light-cones in region III (central region of the black hole). The length of γ_p is finite (measured between the beginning of the experiment and the event when γ_p meets the inner event horizon at b) while the length of γ is infinite.

experience it this way, this is how the computer will coordinatize it in the distance, or more precisely, how the coordinate system shown in Figure 2 represents it. If the computer thinks of the programmer, it will see in its mind's eye that the programmer's clocks stop and the programmer is frozen motionless at the event horizon of the black hole. Since the programmer is frozen motionless at the event horizon of the black hole, the computer has enough time to do the computation, and as soon as the computer has found, say, the inconsistency in set theory, the computer can send a signal and the computer can trust that the programmer—still with his clock frozen—will receive this signal.

What will the programmer see? The programmer will see that as he is approaching the inner event horizon, his computer is running faster and faster and faster. Then the programmer falls into the inner event horizon of the black hole. Since the black hole is enormous, the programmer will feel nothing when he passes either event horizon of the black hole—one can check that in case of a huge black hole the so-called tidal forces on the event horizons of the black hole are negligibly small.[7] So the programmer falls into the inner event horizon of the black hole and either the programmer will experience that a light signal arrives from the direction of the computer, of an agreed color and agreed pattern, or the programmer will observe that he falls in through the inner event horizon and the light signal does not arrive. After the programmer has crossed the inner event horizon, the programmer can evaluate the situation. If a signal arrives from the computer, this means that the computer found an inconsistency in ZFC set theory, therefore the programmer will know that set theory is inconsistent. If the light signal does not arrive, and the programmer is already inside the event horizon, then he will know that the computer did not find an inconsistency in set theory, did not send the signal, therefore the programmer can conclude that set theory is consistent. So he can build the rest of his mathematics on the secure knowledge of the consistency of set theory.

The next question which comes up naturally is whether the programmer can use this new information, namely that set theory is consistent, or whatever he wanted to compute, for his purposes, continue research in mathematics, and so on. He could take a huge spaceship while he goes into the black hole, he takes all his mathematical and scientist friends with himself, and after he crossed the inner event horizon he wants to process the information he obtained (from the above outlined experiment with the computer) and base their future research on this. But a pessimist could say that OK they are inside a black hole, so—now we are using common sense, we are not using relativity theory—common sense says that the black hole is a small unfriendly area and the programmer will sooner or later fall into the middle of the black hole where there is a singularity and the singularity will kill the programmer. This is why some authors, for example Pitowsky in 1990, concluded this story by saying that now the programmer disintegrates with a happy smile on his face because he knows the solution to the problem in question, e.g. whether ZFC set theory is consistent or not. But this disintegration need not be the case. It is suggested only by common sense, reality may be different.

[7]We will return to this in subsection 5.2.1.

The reason why we emphasized at the beginning that we wanted to choose our black hole to be a huge slowly rotating one, say of mass $10^{10}m_\odot$, is the following. If the programmer falls into a black hole which is as big as this and it rotates slowly, then the programmer will have quite a lot of time inside the black hole because the center of the black hole is far from the event horizon, relatively far. Such a black hole might be roughly of the size of the solar system. More precisely, this size[8] is in the range of 3×10^{10} km ~ 200 AU (Astronomical Units). But this is not the key point. The key points are that the black hole is big and it rotates slowly. If it rotates, then the "matter content", the so-called singularity which "keeps the black hole together" so to speak, which is the source of the gravitational field of the black hole, is not a point. It is a ring. (This "matter", this source of gravitational field is technically called the singularity.) So if the programmer chooses his route in falling into the black hole in a clever way, say, relatively close to the north pole instead of the equatorial plane, so his motion is roughly perpendicular to the plane of the ring, then the programmer can comfortably pass through the middle of the ring, never get close to the singularity and happily live on forever.[9] We mean, the rules of relativity will not prevent him from happily living forever. He may have descendants, he can found society, he can use the so obtained mathematical knowledge.

So the key point is that in this arrangement (which is described e.g. in [26]) based on a huge slowly rotating black hole the programmer may not get even close to the singularity and therefore the above mentioned usual common sense argument saying that eventually the black hole will destroy the programmer is not true. There is enough room (both space and time) for the programmer to stay there, and actually the extended theory of the so-called Kerr black holes says that the programmer can come out on the other side of the ring. Moreover, he may decide to stay in the central, ring dominated region indefinitely or he might try to come out at the "other side" of the black hole. If he succeeds to come out, it might be a different universe maybe not ours, or it might be a different part of our universe, and he might be able to go on to do interesting things.[10] But now the key point is not

[8]By the size of a black hole we mean how big the spheroid of the event horizon is for a distant observer outside the black hole.

[9]In order to have all the beneficial effects on his side, the programmer will have to plan his approach of the rotating hole quite carefully. In [26, pp.355-356] we described in detail the path the programmer has to choose. For completeness we note that in the textbook O'Neill [53, pp.245-247] such paths are called "timelike long flyby orbits of type B". Cf. also Fig.4.19 therein.

[10]As it is discussed in [26], embarking on such an adventure (involving unknown universes) does present risks (dangers) for the programmer. But this does not render the project impossible (only risky . . . somewhat). One can make preparations for reducing the risks. We will come back to this later, in section 5.

coming out to other universes and exploring exciting things which is definitely interesting, now our point is only to do this computation, that is, deciding whether Zermelo-Fraenkel set theory is consistent or not and surviving the consequences of this.

It is like the original sin when the knowledge was grabbed by Adam when he took the apple from Eve and had a bite of it. Reaching this sacred knowledge, the question is whether it destroys the scientist who seeks the knowledge like for example in the story of Prometheus the gods punished him because of his knowledge, seeking this sacred knowledge, whether it kills the programmer or not. We claim that it does not necessarily kill the programmer, the programmer has to choose a suitable enough black hole which roughly means a huge slowly rotating black hole like the ones in the centers of galaxies—which have recently been discovered year by year by astronomers—and navigate inside the black hole in a clever enough way and then he will not ever even feel strong tidal forces we mentioned earlier.

This is the general idea for how the relativistic computer works, and what the essential ingredients of the computer are.

3. A brief history of the ideas outlined above

The paper [13] in New Scientist credited the idea to Etesi, Németi, Malament, Hogarth (as independent sources) and it traced the idea back to Herman Weyl. However, Weyl never suggested anything like the idea of a relativistic computer, the only speculation which he made was the observation that if we could speed up a Turing machine indefinitely, e.g. doubling its speed after each step, then this imaginary device could in principle compute a non-Turing-computable function, cf. Weyl [86, p.42]. This consideration is basically a reformulation of the Achilles and Tortoise paradox from antiquity and it is explained in e.g. Earman [20, Chap.4] that this does not yet involve a significant step in the direction of physics-based beyond-Turing computers like e.g. relativistic computers. The general relativistic idea as outlined in section 2 was found independently by Németi in 1987 [52], Pitowsky in 1990 [63], Malament in 1988 [46] and Hogarth in 1992 [36]. Németi's idea used large slowly rotating black holes (slow Kerr spacetimes) but the careful study of feasibility and transversability of these was done later in Etesi-Németi [26].

Pitowsky 1990 used a simpler spacetime (special relativistic spacetime with accelerated observers; or even Schwarzschild spacetime) in which the idea cannot completely pushed through for the reasons we mentioned in section 2 when discussing why we needed a more complex black hole than the simplest kind, i.e. Schwarzschild. About this, Earman [20, p.107] writes "Malament (1988) and Hogarth (1992) sought to solve the conceptual problem with Pitowsky's example by utilizing a different spacetime structure". Malament and Hogarth elaborated a very general approach—of which Kerr black holes form only a specific example—exhibiting a large family of solutions of Einstein's equations in each of which possibly there exist relativistic beyond-Turing computers. These spacetimes are called MH (for Malament and Hogarth) spacetimes and will be introduced in the next section. An excellent and convincing work elaborating the details and realizability of relativistic computers is Hogarth [37] to which Hogarth [36], [38] are valuable additions. Tipler [80, pp.447-448] also describes a general relativistic computer which can compute a non-Turing-computable function. In this respect Tipler's argument is similar to ours, and it points in the same direction. Tipler also discusses the physical realizability aspects of our kind of non-Turing computers. Barrow and Tipler belong to the early proponents of relativistic beyond-Turing computability, see e.g. [7]. The argument in Penrose [61, section 7.10, especially p.383 line 3] points in the same direction as ours. Earman [20, Chap. 4], Earman-Norton [22], van Leeuwen-Wiedermann [43], [87], Etesi [25], Shagrir-Pitowsky [72] contain important contributions to the theory of relativistic computers, to mention a few.

Sections 3.7 - 3.12 of Hogarth [37, pp.88-113] contain an extremely careful, scholarly, highly valuable re-evaluation/re-thinking of Church's Thesis including PhCT. It also puts PhCT into a new perspective taking into consideration the 1995 world-view of modern physics and cosmology which of course was not available to the founding fathers in the 1930's. Hogarth's just quoted work also provides a careful historical analysis of the emergence of PhCT. Therefore it would be fruitful to take this excellent piece of highly relevant work more into account in the debates about hypercomputation. There is also a very useful new perspective on these issues (PhCT etc) in Cooper [15]. The analogy with "artificial horses" at the end of [15] is particularly nice and illuminating.

4. More formal definition of relativistic computers

By a Malament-Hogarth spacetime (MH-spacetime) we understand a general relativistic spacetime $\langle M, g\rangle$ in which there is a point $q \in M$ and a future-directed infinite timelike half-curve $\gamma : R^+ \to M$ such that the whole of γ lies in the causal past of q. The definition of MH-spacetimes in more detail[11] goes as follows.

Definition 4.1. *By a general relativistic spacetime we mean a pair* $\langle M, g\rangle$ *where* M *is a smooth, oriented, and time-oriented 4-manifold*

[11] We try to be as self-contained here as possible. The few concepts not introduced here can be found in any textbook on general relativity, e.g. in Wald [85]. For Einstein's equations we refer to [85, p.72], as well as to [74, sec.4.4] in this volume. A very elementary introduction to the basic concepts of relativity theory can be found in [1]. In this volume [28, section 3], [4, section 2.4], [75], [54, section:Infinite time], [50, end of section 6] also touch upon Malament-Hogarth spacetimes.

while g is a smooth Lorentzian metric on M which is a solution to Einstein's equations, w.r.t. a physically reasonable matter field represented by a smooth stress-energy tensor T on M (i.e. T satisfies one of the standard energy conditions).

In the above, M represents the set of events, and g represents the "local metric". In particular, for a vector v (in the tangent space T_q of a point $q \in M$), we think of $\sqrt{|g_q(v, v)|}$ as the "length" of the vector v. We usually omit the index q. When v is timelike, this length means roughly "rate of time passing at q in direction v".

Definition 4.2. *The length of an at least once continuously differentiable timelike half-curve $\gamma : R^+ \to M$ from a to b, where $a \in R^+$ and either $b \in R^+$ or b is "infinity", is the integral*

$$\|\gamma\|_a^b = \int_a^b \sqrt{|g(\dot\gamma(t), \dot\gamma(t))|}\,\mathrm{d}t.$$

We say that the curve γ is well-parameterized if $\|\gamma\|_a^b = b - a$ for all $a \leq b$, $a, b \in R^+$. We say that the curve γ is upward-infinite if $\|\gamma\|_0^\infty = \infty$.

Intuitively, the integral $\|\gamma\|_a^b$ is the length of the curve γ from a to b according to the metric g. As usual, we interpret a future-directed, timelike, well-parameterized curve γ as the world-line of an observer (living in the spacetime $\langle M, g\rangle$). $\mathrm{Im}\gamma$ is the collection of the events happening to γ during his life, and we imagine that t is the time showed on the wristwatch of γ at the event $\gamma(t)$. From now on we always assume that the curves are well-parameterized, and we say that t shows "proper" time, or "wristwatch time" of observer γ.

Definition 4.3. *The causal past of the event $q \in M$ is defined as*

$J^-(q) := \{x \in M :$ *there is a future-directed nonspacelike continuous curve joining x with q*$\}$.

Intuitively, $J^-(q)$ consists of those events $x \in M$ from which one can send signals to q. Summing up:

Definition 4.4. *A spacetime $\langle M, g\rangle$ is called a Malament–Hogarth spacetime if there is a future-directed timelike half-curve $\gamma : R^+ \to M$ such that $\|\gamma\|_0^\infty = \infty$ and there is a point $q \in M$ satisfying $\mathrm{Im}\gamma \subseteq J^-(q)$. The event $q \in M$ is called a Malament–Hogarth event.*

Before going on, we give two examples. As the first example, take $\langle M, g\rangle$ where $M = R^4$ and the metric tensor g at each $p \in R^4$ is given by the 4×4 matrix

$$\begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}.$$

This is Minkowski spacetime, it is not a MH–spacetime, see Figure 3.[12]

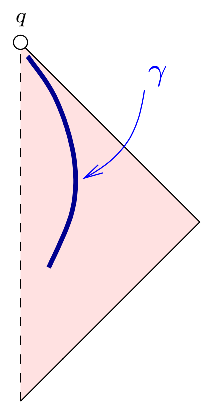

FIGURE 3. Conformal diagram of Minkowski spacetime. Minkowski spacetime is not MH! All upward-infinite timelike curves γ converge to points like q which are not in the spacetime.

For the second example take the so-called vacuum Kerr spacetime $\langle M, g\rangle$ with parameters $m > 0$ and a. Below we use the so-called Boyer–Lindquist coordinates $(t, \varphi, r, \vartheta)$. Here, $(t, \varphi, r, \vartheta)$ are kind of polar-cylindric coordinates, r being radius[13] and φ, ϑ being angles. The metric tensor g at $p = (t, \varphi, r, \vartheta)$ is given by the 4×4 matrix

$$\begin{vmatrix} -1+\mu & -\mu a \sin^2\vartheta & 0 & 0 \\ -\mu a \sin^2\vartheta & g_{\varphi\varphi} & 0 & 0 \\ 0 & 0 & \Sigma/\Delta & 0 \\ 0 & 0 & 0 & \Sigma \end{vmatrix},$$

where $\Sigma = r^2 + a^2\cos^2\vartheta$, $\Delta = r^2 - 2mr + a^2$, $\mu = 2mr/\Sigma$, and $g_{\varphi\varphi} = (r^2+a^2+\mu a^2 \sin^2\vartheta)\sin^2\vartheta$. This spacetime is called the spacetime of a rotating Kerr black hole of zero electric charge; m is thought of as the mass of the black hole (one can visualize it as hiding in the ring-singularity) and a is thought of as the angular momentum per unit mass. The values of $r \neq 0$ yielding $\Delta = 0$ represent the

[12]Figures 3-5 are so-called conformal diagrams or Penrose diagrams of spacetimes. As opposed to “ordinary” spacetime diagrams like Figure 2, conformal diagrams intend to represent causal relations between events (disregarding metric ones). On conformal diagrams photon world-lines are always straight lines of slope 45^o. Hence no light-cones are tilted (helping to represent causality).

[13]To be precise, r is the logarithm of the radius.

locations of the event horizons. When $|a| < m$, there are two event-horizons, an outer one and an inner one. The assumption $|a| < m$ means that we are in the slow (Kerr) case, by definition. Figure 2 represents such a slow Kerr hole in Eddington-Finkelstein coordinates, while Figure 5 shows the Penrose-diagram or conformal diagram for a slow Kerr hole. Figure 5 is more informative than (the more intuitive) Figure 2. Figure 5 reveals that this is indeed a MH-spacetime, there are MH-events on the inner event horizon as represented in the figure. For more on the Kerr spacetime with the above metric g we refer to the textbook O'Neill [53, sec.2.1, pp.58-59]. The fact that this is a MH-spacetime is proved both in [26] and in [25, Prop.2.4]. The proof can be reconstructed e.g. on the basis of [53, pp.246-7(cases B and S)].

The so-called Kerr-Newman black holes or spacetimes are obtained from the Kerr case described above by adding an extra parameter e for electric charge in an appropriate way. These, too, are MH-spacetimes, assuming $|a| + |e| \neq 0$. If $a = 0$ but $e \neq 0$ this becomes the so-called Reissner-Nordström black hole. If $a = e = 0$, this becomes the Schwarzschild spacetime, which does not have the MH-property, see Figure 4.

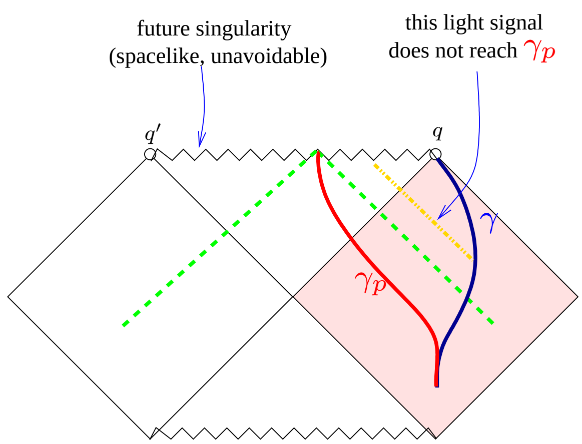

FIGURE 4. Penrose diagram of Schwarzschild black hole. There is no point in the spacetime whose causal past contains all of an upward-infinite future-directed curve. Hence, MH property fails.

Another example of physically realistic MH-spacetimes is the anti de Sitter spacetime (illustrated in [28, Fig.1], this volume). Earman and Norton [21],[22] investigate MH-spacetimes in general, and show their importance from the point of view of the cosmic censor hypothesis. Etesi [25] contains a classification of MH-spacetimes satisfying appropriate energy conditions and points out another interesting relationship between such MH-spacetimes and the strong cosmic censorship

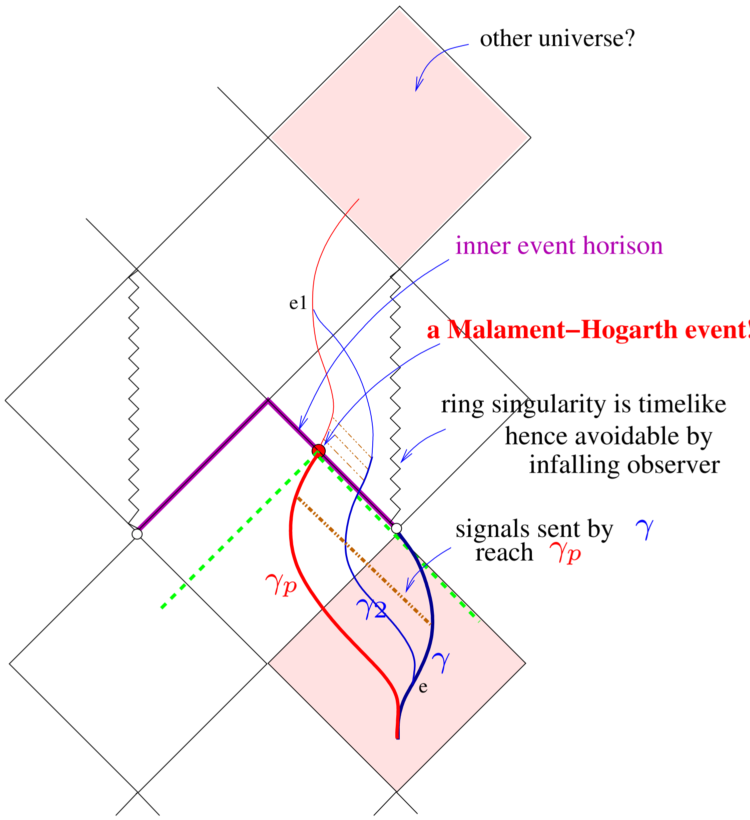

Figure 5. Penrose diagram of slowly rotating black hole along the symmetry axis. This is a Malament-Hogart spacetime. The length of γ_p is finite, while the length of γ is infinite. (γ_2 will be used later, in section 5.3.2.)

hypothesis. Gödel's rotating cosmological models [31], [32], Gott's elegant spacetime with two cosmic strings [30], are all MH-spacetimes and there are many more.[14] For computing a non-Turing computable function, Hogarth uses anti de Sitter spacetimes in [37], Etesi and Németi use slow Kerr spacetime or Kerr-Newman spacetimes in [26], and Earman uses Reissner-Nordström spacetimes in [20].

The way a MH-spacetime is used for defining relativistic computers is the following. We add an extra timelike curve γ_p such that q lies on the curve γ_p and an initial segment of γ_p coincides with that of γ. The latter means that there is a bound $d \in R$ such that for all $d > r \in R^+$ we

[14]Some of the above involve CTC's (closed timelike curves), but CTC's are not necessary for MH, there are many MH-spacetimes without so-called "strong causality violations", cf. Etesi [25]. On the other hand, existence of CTC's implies the MH-property.

have $\gamma(r) = \gamma_p(r)$.[15] We regard γ_p as the world-line of the programmer, γ as the world-line of the computer. The event q happens at a fixed finite time according to the proper time of γ_p, say $q = \gamma_p(b)$. As a contrast, every event on the curve γ (i.e. every event occurring to the computer) is in the causal past of q. Below the bound d the computer γ and the programmer γ_p are together, not moving relative to each other and their proper times (wristwatch times) agree. After "timepoint" d they move on separate world-lines. The programmer uses the time-period before d for transferring input data to the computer γ as well as for programming γ. Suppose the task for the pair $\langle \gamma_p, \gamma \rangle$ is to decide whether ZFC set theory is consistent. (The case when the task is to find whether a number, say n, is in a recursively enumerable set H of the integers is completely analogous.) Then γ starts checking whether the theorems derivable from the axioms of ZFC contain the contradictory formula FALSE. So γ derives the theorems of ZFC one by one and checks whether FALSE is among them. If γ finds FALSE among the consequences of ZFC, it sends a signal towards γ_p. Suppose this happens at proper time t of γ. Then since, by definition of MH-spacetimes, $\gamma(t)$ is in the causal past of $q = \gamma_p(b)$, we know that we can arrange that γ_p receives this signal latest at the event q. I.e. the signal arrives at $t_0 \leq b$ on the world-line of the programmer γ_p. So if ZFC is inconsistent, then γ_p will receive a signal latest at event q, i.e. latest at proper time b (which is a fixed number). On the other hand, if γ_p never finds an inconsistency in ZFC, then it never sends a signal to the programmer γ_p, hence at proper time b, γ_p will know that no signal was sent. I.e. at proper time b, the programmer γ_p will know whether or not ZFC is consistent.

Definition 4.5. *By a relativistic computer in a MH-spacetime $\langle M, g \rangle$ we understand a triple $\langle \gamma_p, \gamma, q \rangle$ such that γ is an upward-infinite future-directed timelike curve lying in the causal past of the event $q \in M$, γ_p is a timelike curve such that q lies on γ_p and an initial segment of γ_p coincides with that of γ.*

By the above we described how the relativistic computer $\langle \gamma_p, \gamma, q \rangle$ decides the set of theorems provable from ZFC (or from any other recursively axiomatizable theory). This task is well known to be "beyond the Turing barrier", i.e. non-Turing-computable.

For any other recursively enumerable set, say H, of the integers a relativistic computer $\langle \gamma_p, \gamma, q \rangle$ deciding whether any given number is in H (i.e. deciding H) is constructed completely analogously. The case, when the task for $\langle \gamma_p, \gamma, q \rangle$ is computing a usual Turing computable function

[15] Here, for simplicity, we do not worry about showing the existence of such a γ_p, we may assume that it is included in the definition of a MH-spacetime. In e.g. [26] we prove the existence of γ_p whenever needed, moreover we prove many feasibility conditions on γ_p. Hogarth [37, p.73] proves that such a curve γ_p always exists.

is discussed in [26], in Hogarth [37], in Leeuwen-Wiedermann [87]. For brevity, we do not recall this case here. Here, it is not our purpose to investigate how far one can push the limits of relativistic computability along these lines (i.e. along the so-called degrees of unsolvability or, in other words, along the arithmetical hierarchy $\Sigma_n, \Pi_n, n \in \omega$). To some extent this was discussed in [26] and this limit was pushed very-very far in works of Hogarth [36], [38] and Wischik [88]. Leeuwen-Wiedermann [43] gives a characterization for the class of sets decidable, and the class of functions computable, by a relativistic computer.

Instead, we would like to concentrate on the question whether the above outlined idea of relativistic computers $\langle \gamma_p, \gamma, q \rangle$ going beyond the Turing limit is physically realistic. In other words, we are interested in finding out whether one really can "break the Turing limit" by using new physics. This direction was also the main thrust of [26], but we hope that by now we can add a little to the degree of confidence achieved there.

5. On physical realizability of beyond-Turing relativistic computers

As we already indicated, we are not aiming for making our relativistic computers routinely realizable "cheap" devices like a laptop or a PC. Instead, we are aiming to show that, in principle, if a beyond-Turing task becomes extremely important for (a future generation of) mankind like e.g. deciding whether the foundation of mathematics, ZFC, is consistent or not, then with sufficient concentration of effort, resources, time and energy, it can be made physically realizable (under perhaps extremely high costs) as opposed to something which is absolutely impossible (like e.g. building perpetuum motion machines, or finding a Turing machine which decides the halting problem).

In [26] we start out from the spacetime of a huge slowly rotating black hole like the ones in the centers of galaxies, like the Milky Way. Such a black hole exists in the center of the Milky Way according to [49], cf. also [24],[27],[69] and astronomical evidence reviewed in section 5.2.2 below. In [26], starting out from such a spacetime we construct a relativistic computer $\langle \gamma_p, \gamma, q \rangle$ as above. Then we make certain that in addition to the properties described in Def.4.5, this configuration has certain further realizability properties. E.g. we prove that γ_p and γ have only very strictly bounded acceleration, so that realizing them requires only a finite amount of energy. (Actually, the acceleration of γ in [26] is uniformly 0.) Also, from any point p of the world-line γ, a light signal can be sent such that it arrives at γ_p *strictly before* the MH-event q (on the inner horizon).

A large part of [26] is devoted to ensuring/studying the physical realizability of the relativistic computer $\langle \gamma_p, \gamma, q \rangle$ based on a so-called galactic size slowly rotating black hole. Earman [20, sec.4.8, p.119] is also devoted to the discussion of physical realizability of relativistic computers. Here we address some problems left open in [26] and in [20]. We do not aim at completeness. As we discussed at the end of section 1, and as pointed out in [26, bottom of p.343], there are two different kinds of realizability issues here. The first issue concerns realizability of computation by some idealized device with respect to some concrete physical theory (such as some concrete spacetime of classical general relativity). The second issue concerns realizability by taking into account all of our present day physical, cosmological, etc., knowledge about the universe we are living in. We will not carefully indicate below which of the two issues is being addressed at which point, but we hope context will help. In the answers we will concentrate on the (slow Kerr-based) relativistic computer outlined in [26] and in section 4 herein, but occasionally we will mention relativistic computers based on some other (than Kerr) MH-spacetimes. If not indicated otherwise, by a black hole we will always mean a slowly rotating (Kerr) black hole.

5.1. **Do we need to implement a so-called supertask?** The answer is definitely “no”. Realizability of our kind of relativistic computer is a strictly weaker assumption than realizability of a proper supertask in the sense of e.g. Earman-Norton [21],[22] or Earman [20, Chap.4]. The reason for this is the following. Relativistic computers do not perform infinitely many steps in finite time. This is so because the computer performs its infinitely many steps in infinite time, and the programmer implements only finitely many steps (namely, detecting and decoding the signal sent by the computer) in finite amount of time. Therefore e.g. eventual discreteness of spacetime in some versions of quantum-gravity (or Planck scale physics for that matter) does not interfere with the functioning of this relativistic computer. This was already mentioned in Etesi-Németi [26, item 4 on p.367], but recent experience tells us that this point needs to be spelled out with more emphasis. To emphasize that we do not need a proper supertask for relativistic computers, Barrow [7] distinguished pseudo-supertasks from supertasks. Our relativistic computer involves only a pseudo-supertask because γ_p need not observe all the infinity of events in its causal past. Instead, γ_p needs only to decode a single prearranged message coming from γ and may ignore the rest of events happening with γ. See also [22, sec.11, p.251].

5.2. **Will the programmer survive?** The first group of questions concerns whether the programmer will survive after getting the answer from the computer. We already discussed this issue in section 2. There we discussed two questions, namely whether the programmer survives passing the event horizon, and whether he can avoid falling into the singularity.

5.2.1. *Tidal forces.* We said that if the black hole is big enough then the tidal forces on the event horizon will be small. Black holes are known to exist between $10^7 m_\odot$ and $10^{10} m_\odot$. E.g. to ensure safe traversability of such a black hole for a humanlike traveler, it is amply enough that the size of the black hole reaches $10^7 m_\odot$ (the bigger the safer because the tidal forces on the event horizon of a bigger black hole are smaller). These tidal forces and similar effects were checked in [26] and were also recalled from the literature yielding reassuring results.

For example, a careful analysis of the situation was carried out by Ori [55][56] in the case of the Reissner–Nordström black hole, and partially in the case of the Kerr–Newman black hole. In accordance with his calculations (accepting the validity of certain technical assumptions) it seems that despite the existence of the scalar curvature divergence, the tidal forces remain finite moreover negligible in the case of realistic slow Kerr black holes when crossing the inner horizon. Though the inner horizon (which contains the Malament–Hogarth event) is a real curvature singularity, it is only a so-called *weak singularity* since the tidal forces still remain finite on it [55], [56]. As an example, [56] computes that for a Kerr black hole of mass $M = 10^7 m_\odot$ the relative distortion of an object of typical size l crossing the inner horizon is

$$\frac{\Delta l}{l} \leq 10^{-55}.$$

Thus, in theory at least, the MH-event can be approached by the observer γ_p safely, although it is situated in a "dangerous" region of the Kerr–Newman spacetime.

5.2.2. *Existence of supermassive slowly rotating black holes.* The existence of supermassive (or galactic) black holes of mass approximately $10^{10} m_\odot$ is made likely by many recent astronomical observations, e.g. in [69], [27], [77, section 25.5 "Supermassive black holes"]. In this last work, the second sentence writes "…is a rotating supermassive black hole of order a billion solar masses …". Cf. also [41], [42], Melia [48].

5.2.3. *Not the programmer travels.* For the case the reader should feel uncomfortable about the arrangement in section 2 that it is the programmer γ_p who takes the journey to the exotic regions of the universe (e.g. into a huge and "tame" Kerr black hole) and it is the computer γ who stays safely away from the black hole (actually the computer may move farther and farther away from the black hole e.g. in order to not disturb the hole's equilibrium), we note the following. This division of labor is not necessary for MH computers. The roles can be switched, e.g. by replacing the black hole with an anti de Sitter spacetime as in Hogarth [37], Earman [20, p.113] or we can use setting appropriate values for the cosmological constant Λ (which needs not be really constant according to latest findings) amounting to a repulsive kind of gravity which may be responsible for the acceleration of the expansion of the universe, in order to make the same relativistic idea work with the roles of γ_p and γ interchanged.[16]

5.3. **Can the programmer receive and understand the signal sent by the computer?**

5.3.1. *The so-called blueshift problem.* This is the problem of communication between the computer γ and the programmer γ_p. This problem has been extensively discussed in Earman [20], Hogarth [37, p.87], Etesi-Németi [26, item 4 on p.367]. The problem was basically solved there but some technicalities were left open. We agreed in [26] that in order to avoid burning γ_p by an infinite amount of energy (cf. Lemma 4.2 in [20] or Prop.6 in [26]) the computer sends only a "yes" or "no" type signal. To be able to distinguish the signal from background "noise", γ_p and γ still have to agree on a long enough and complicated enough signal, but the point is that they know in advance how long the (finite) signal will be. The remaining problem was that although the signal is of a fixed finite length, the gravitational effects may make the signal's wavelength so short that γ_p cannot recognize it. A possible solution is the following. When γ finds the inconsistency in ZFC set theory, it calculates how close γ_p is to the inner event horizon and from this γ calculates the blueshift to be expected. So, γ knows what frequency it should use for the signal so that after the blueshift it will appear just right for γ_p. But how could γ generate an arbitrarily low frequency signal?[17] Well, γ sends a spaceship S in the direction opposite to the direction of the Kerr hole. Now if S moves fast enough, then any signal sent from S to γ_p will be redshifted because of the speed of S. Now, γ chooses the speed of S to be such that the redshift caused by this speed exactly cancels out the blueshift caused by the gravitational effects where γ_p is when he receives the signal. Since γ has enough time and enough data for making these calculations (and since γ_p is still outside the inner horizon when receiving the signal),

[16] For cosmological realism of a non-vanishing cosmological constant Λ we refer to the latest results in cosmology, cf. e.g. Sir Martin Rees [67],[68]. Also recent discussions of "dark energy" and "negative energy" cf. e.g. Kaku [39] are relevant here making relativistic computing easier in some sense. For recent ideas on negative energy and cosmic censor violations cf. e.g. [34]. For actuality of $\Lambda > 0$ and its variants cf. also item 5.4.3 way below.

[17] This question was originally posed in 1988 when one of the present authors proposed the present relativistic beyond-Turing computer at the Algebra Seminar of Prof. Ervin Fried (Eötvös Univ. Budapest 1988). The objection was that the computer γ will need longer and longer antennas for emitting the low frequency signal in the direction of γ_p. Since the blueshift tends to infinite, the length of the antenna would also tend to infinite, which seems to be a physical impossibility. We think that the arrangement proposed here will take care of this problem. It was this objection which resulted in postponing the publication of the present Kerr-based $\langle \gamma_p, \gamma, q \rangle$ machine from 1987 to 2000. We mention this as a curiosity of the dynamics of developments of ideas and effects of the Iron Curtain.

this arrangement is possible (in theory at least). If we want to avoid using too much fuel for speeding S, we could use a second black hole for redshifting the signal from S (sent towards γ_p) appropriately. Namely, we deposit a large enough Schwarzschild black hole far away from the Kerr black hole such that it should not disturb the working of the Kerr one. Then, when γ finds the inconsistency, γ "drops" the spaceship S into the Schwarzschild black hole in such a way that the message sent out by S gets redshifted in exactly such an extent that this redshift cancels the blueshift effect out at the receiving end, i.e. at γ_p.

A different solution to the blueshift problem is at the end of subsection 5.4.1. It is an interesting future research possibility to solve the communication problem between γ and γ_p by some quantum-information theoretic methods.

5.3.2. *Recognizability of the signal.* With the above solution to the blueshift problem, the problem remains to see the details of how γ_p can recognize the signal coming from γ.

In order to achieve our main goal, we managed to slow down the subjective time (or proper time) of γ_p relative to the computer γ such that on the inner event horizon, roughly, γ_p's clocks are frozen motionless from the point of view of γ. This gives γ sufficient time for computing the desired task. But when γ has obtained the result, there remains the engineering task of transferring the result to γ_p in such a way that γ_p can "notice" the result. If γ simply sends a stream of photons to γ_p whose clocks are frozen motionless, then this "frozen" γ_p will experience the presence of these photons for an infinitely short time period only as measured by the clocks of γ_p. But any measuring instrument needs some finite (nonzero) time for reacting to a change in the surrounding electromagnetic field or to a change in anything, i.e. to a signal. So if we do not do something, then γ_p might not be able to notice the signal.

In other words, to achieve our main purpose, we "transformed" γ_p into a new "world" in which time passes differently from that of the "world" of γ. To communicate the result to γ_p, γ encodes this result into a physical system S_2. If we just send S_2 to γ_p, then γ_p might not be able to decode the message in S_2 because the clocks of S_2 are not tuned to those of γ_p. A reasonable solution to this is that we "transform" S_2 from the "world" of γ to the world of γ_p. E.g. we can choose S_2 to be a so-called messenger spaceship. After γ obtained the result, it puts the result into the spaceship S_2 and sends S_2 after γ_p with the task of getting itself to the same "state of motion" or "same world" as γ_p is. Then S_2's clocks would be, roughly, synchronized to γ_p's ones. After this, S_2 is supposed to communicate the result to γ_p. In what comes below, the world-line of S_2 is denoted as γ_2.

For more detail let us look at Figure 5. Assume that γ finds the inconsistency of ZFC at a point e on its world-line. As can be seen in Figure 5, there is a timelike curve γ_2 starting from e and ending on a point $e1$ on γ_p somewhere beyond the MH-event q of γ_p. This new curve γ_2 intersects the inner horizon IH somewhere in the causal past of q. Now, a possibility is that after finding the inconsistency, the computer γ sends out a second spaceship S_2 towards the black hole, with world-line γ_2. When S_2 meets γ_p, it can safely transfer the message to γ_p. Here we need to worry about the time γ_p has to wait after the MH-event q for the arrival of S_2. To ensure that this time period is strictly bounded, S_2 can start sending light-signals to γ_p after it crosses the IH. These light-signals need not get blue-shifted, since γ_p and S_2 live in the same segment of the Kerr space-time, cf. Figure 5, and their world-lines are "roughly parallel". Now, S_2 can repeat the signal arbitrarily many times without a danger of "burning" γ_p.[18]

A different solution to the background noise problem (together with the blueshift one) is at the end of subsection 5.4.1 below.

5.4. **Can the computer "live" for an infinite amount of time?** As it turns out, this problem has much in common with latest investigations in the literature concerning whether mankind can survive forever. Cf. [39], [67], [80] or the Omega point argument in Barrow-Tipler [8, pp.676-677].

5.4.1. *Evaporation of black hole.* In [26, p.368] the theoretical worry is mentioned that if we take quantum effects also into account, then, many authors think, perhaps even the largest Kerr black holes might evaporate eventually, so if the computer finds the inconsistency in ZFC set theory, say, only after 10^{100} years, then it will not be able to notify the programmer[19]. There are many things to be said about this:

(1) This does not affect theoretical possibility of beyond-Turing computers in general relativity, and the GR version is our main aim here. So, our "first issue" is taken care of.

(2) The programmer γ_p can program his fleet of self-reproducing robots (serving the computer γ) so that these robots fly to farther and farther reaches of the universe and send matter into the Kerr hole, so as to prevent its evaporation. The amount of matter sent this way into the black hole need not be much, what is important is that it should never stop. The possibility of this seems to depend on the rate of expansion of our universe. There are two cases. (I) The expansion

[18] There is a different kind of solution to the same (recognizability of signal) problem which uses mirrors (surrounding γ_p) instead of messenger ships. The end-result is similar: the physical system S_2 carrying the message remains around γ_p for a long enough proper time of γ_p. This is available from the authors.

[19] An estimation for the evaporation time of a galactic black hole, i.e. one of mass $10^{10} m_\odot$ is between 10^{100} and 10^{150} years.

of our universe continues to accelerate forever, or (II) the acceleration of expansion slows down eventually, though the expansion itself may go on. In case (II) it seems possible, according to the calculations of the present authors, to feed the black hole forever. Hence our thought experiment with the relativistic computer can be carried through. In case (I) this might not be possible (unless there is some extra effect compensating for the eternal acceleration of expansion).

We would like to mention that even in the unlucky case if our concrete universe would be of the type described by case (I), in theory a positive solution is consistent with spacetime theory of today (which permits case (II) as a possible spacetime). As described in [39, Chap.11 (escaping the universe), p.304 –] even in case (I), mankind could experiment with finding a solution for beyond-Turing computation, but for lack of space we do not discuss this here.

For case (II), it will be outlined below how the present arrangement seems to lead to a new kind of solution for (i) the blueshift problem, (ii) the infinite noise problem, and (iii) for ensuring the programmer at the MH-event that nothing went wrong with the computer (while checking consistency of ZFC).[20] This idea seems to work if our universe corresponds to case (II) outlined above. (For case (I) it remains an open problem to see whether all these problems can be solved satisfactorily.) So, assume case (II). In accordance with item (2) above, the computer γ "directs" (or oversees) the maintenance or feeding of the Kerr hole such that it does not evaporate. Now, we make a new convention for sending messages from γ to γ_p. If and when γ finds an inconsistency in ZFC set theory, γ orders all its robots to stop feeding the black hole. This will cause the black hole to evaporate. Assume at first approximation that γ_p can survive such an evaporation. So, γ_p will notice that the black hole evaporated. On the other hand, if there is no inconsistency in ZFC, then γ_p will cross the inner event horizon and therefore will know that ZFC is consistent. So, it seems that after the time comes for γ_p to reach the MH-event, γ_p will know whether ZFC is consistent or not. This arrangement seems to rely only on the assumption that γ_p is capable of deciding whether the black hole he is falling into has evaporated or not, and that γ_p can survive such an evaporation. (Such aspects of black hole evaporation are discussed in [20, section 3.6, "black hole evaporation"], according to which it is consistent that γ_p might survive such.) Actually, making the black hole evaporate may be a too drastic tool for communication between γ and γ_p. A more refined version of this is if γ makes pre-agreed upon changes on the feeding process of the black hole making the black hole's behavior change in such a way which can be noticed by γ_p. (E.g. γ could cause the black hole shrink to, say, 3/4 of its original size and then stop

[20]Idea due to Attila Gohér [29].

shrinking, or change the angular momentum or electric charge of the black hole.)

We should note that for making the ideas about feeding the black hole (against evaporation) realizable, future research is needed into asking how the geometry of a Kerr hole changes if matter/energy is sent into the black hole. Actually, this research might tell us how (from what direction, in what form etc) we should feed the black hole in order to render our relativistic computer workable.

At this point the reader might be puzzled about how the programmer γ_p will notice that he will have crossed the inner event horizon or how he will notice that the black hole will have started to evaporate. In other words, so far we have spoken in the manner as if there were some "road signs" at the inner event horizon informing γ_p that "you have reached the inner event horizon". In this connection, roughly, we claim that a sufficiently advanced civilization, with sufficient data about the black hole they plan to use, can design measurements for γ_p by which he can decide the above questions (so instead of watching out for a road sign, γ_p can measure/observe "this and that effect"). In more detail, before starting the experiment, γ_p can calculate how much time it will take for him for reaching the event horizon assuming that the black hole will be fed according to plans. They can also calculate margins of error for this measurement, taking into account that feeding the black hole might go a bit "unevenly" for some unexpected reason. Assume ZFC is consistent. So γ will not stop feeding the black hole. Then by looking at his wristwatch and making some extra astronomical observations, γ_p will be able to conclude that he will have crossed the inner event horizon. More concretely, to conclude that he has indeed crossed the inner horizon, γ_p might check whether the outside universe in which γ lives disappeared completely from γ_p's view, cf. Figure 5. By this we mean that the Penrose diagram on p.19 reveals that after γ_p crossed the inner event horizon, no light signal comes to γ_p from the universe inhabited by γ. Informally, we will refer to this by saying that γ entirely disappeared from the sky of γ_p (but this is only a picturesque mannerism to refer to the above mentioned observations/experiments). So, roughly, if γ_p's wristwatch shows the pre-calculated time and γ disappeared completely from the sky of γ_p, then γ_p concludes that ZFC is consistent.

It needs some extra research in the theory of rotating black holes to nail down the criteria after observing which γ_p can conclude that ZFC is inconsistent, but finding these criteria should be, theoretically, possible. To avoid digression, we keep discussion of these criteria very sketchy: We note that e.g. if γ_p sees the pre-calculated time on his wristwatch and he still sees traces of γ in its sky, i.e. the universe inhabited by γ still did not disappear completely from γ_p's view, then he may conclude that ZFC is inconsistent. The details for the conditions under which γ_p will conclude that ZFC is inconsistent depend on prearranged conventions between γ and γ_p involving e.g. how γ continues influencing (e.g. feeding, not feeding, half-feeding) the black hole after discovering an inconsistency in ZFC. These pre-arranged conventions will depend in turn on future theoretical results concerning how exactly a “controlled” evaporation of a Kerr black hole proceeds, e.g. on what effects an observer “trapped” in between the two event horizons will experience. The details of this plan of action might also depend on what possible influences of γ can be noticed by γ_p without killing γ_p. Such an action might be e.g. changing the feeding pattern of the black hole as outlined above. Another theoretical possibility is that, after having found an inconsistency in ZFC, γ speeds up (or slows down) the spin of the black hole the consequences of which might again be observable for γ_p. Also, γ might change the electric charge of the black hole which will certainly be noticeable for γ_p assuming he is prepared for measuring this effect. Though the electric charge will probably be gradually lost in time (in physically realistic situations), γ might be able to maintain a noticeable nonzero charge.

We refer to the internet movie “falling into a black hole” (http://casa.colorado.edu/∼ajsh/schw.shtml by Andrew Hamilton, Univ. Colorado, Boulder, Dept. APS) for visual effects which might reveal for an infalling observer whether she has passed through an event horizon. Notice however that for the time being, this movie is elaborated only for the Schwarzschild case and we would need it for the Kerr case with an emphasis on the inner event horizon. Cf. [71].[21]

A crucial part of our above plan was the assumption that at a particular given wristwatch-time (proper time) point, γ_p can decide by making appropriate observations/measurements whether he has crossed the inner event horizon or not. The measurement we suggested was based on the complete “disappearance” of the whole universe in which γ lives. We note that the theory of Kerr black holes (e.g. [53]) provides γ_p and γ with further possible experiments for deciding this issue (i.e. for γ’s communicating to γ_p that an inconsistency was found). Since there are many such theoretical possibilities (for γ’s communicating to γ_p without the latter’s destruction), and since it is a future research task to find out which work and which do not, we do not go into more detail about this subject here.

We note that a safe alternative solution to the blueshift problem was described in section 5.4.1.

5.4.2. *Decay of protons.* A worry similar to the one discussed in item 5.4.1 above (evaporation of black holes) is that according to present day

[21]Actually, if we were interested in noticing the outer horizon, we could try to use the Schwarzschild movie with appropriate modifications, but the inner horizon of a Kerr black hole shows fewer analogies with the Schwarzschild horizon.

Grand Unified Theories (GUT's) of particle physics, matter in the form we know it today might decay by the time huge black holes evaporate (roughly after, say, 10^{35} years or so), cf. e.g. Kaku [39, pp.298-299].[22] This means that protons from which the computer γ is built may decay eventually and the "repair-servants" of γ may not be able to find enough protons in the vicinity of γ for re-building γ. Again, for the pure GR version, this problem disappears. So let us look at item (2) of 5.4.1. As stated there, the future expansion of the universe corresponds either to case (I) or case (II) outlined there. For case (I) this issue remains an open problem[23]. For case (II), we proceed analogously to 5.4.1, namely as outlined in item 5.4.1(2), γ sends a fleet of self-reproducing robots to distant parts of our universe to send in energy in the form of photons to γ as raw material for recreating γ. Now, the maintainers of γ first focus ("compress") the energy sent in by the distant sub-robots in order to obtain high energy photons. (The reason for this is that high enough energy photons are needed for creating proton-antiproton pairs, and this late stage of the expanding universe might not contain such high energy photons in sufficient number.) Next, the maintainers of γ use the so obtained high energy photons for creating proton-antiproton pairs. (This is part of matter-antimatter pair creation by high energy photons. Some of the so obtained matter will be protons. These are enough for our purposes.) Next, they send the antiprotons to feed the black hole as described in subsection 5.4.1 above and use the protons for rebuilding γ.

5.4.3. *Infinite amount of energy or matter.* Costa and Mycka [51, p.4] and [16, p.4] bring up the doubt that perhaps relativistic beyond-Turing computers might need an unbounded amount of energy and may therefore be not implementable. We will analyze this doubt more rigorously in the second part of the present subsection starting with "let us turn to the infinite time and space problem" (p.31). But first we note the

[22] In more detail, GUT's are unified theories of particle physics intended to unify the theory of electroweak interactions (electromagnetic and weak nuclear forces) and strong nuclear forces described by quantum chronodynamics QCD. So, GUT's are designed to unify the theories of all forces in nature with the exception of gravitation. In contrast with the so-called Standard Model, GUT's predict that protons eventually decay but this has not been confirmed by experiment yet. All the same, the theoretical motivation is strong. Details can be found on the Internet Wikipedia. Cf. also e.g. Penrose [62], Barrow and Tipler [8, pp.647-653] where the authors are pursuing goals analogous with relativistic computers. The ideas suggested there to overcome the problem of proton decay (and similar problems in our section 5) present a viable alternative to our suggestions. (The halflife 10^{35} years of protons is only an estimated number, its real value is unknown yet.)

[23] One might try to rebuild γ by using the remaining kind of matter, probably electrons, positrons, neutrinos, photons but this seems extremely hard, not necessarily impossible, though.

following. Fortunately, the presently discussed doubt (about realizability) had been voiced by Pitowsky in 1990. The rather careful book Earman [20, p.119] addresses this doubt and explains in detail why and how this difficulty can be avoided without impairing realizability of the relativistic beyond-Turing machines. Also, Hogarth [37, p.120] explains how to solve this kind of doubt. Later Earman reformulates the above doubt in the following strictly milder form. If the computer γ works for an indefinite amount of time producing an indefinite amount of auxiliary information to be (temporarily) stored, then the so accumulated mass (read "information") around the location of γ might perhaps form something like a new black hole destabilizing the original Kerr hole into which the programmer γ_p is falling. What Earman writes subsequently indicates that he is not taking this destabilization problem very seriously. Anyway, let us answer this destabilizing doubt. The solution is the following. The "space" available for the operation of the computer γ is the entire universe (except for the interior of the Kerr black hole). Now, let us implement γ as a fleet of self-replicating robots which (i) move away from the Kerr hole (in order not to disturb it) and (ii) spreads itself very thinly over the universe (external to the Kerr hole) avoiding formation of clumps of matter or energy. In particular, γ does not store auxiliary information locally at some fixed place, but γ sends the pieces of material encoding parts of its auxiliary data far away from γ by its sub-robots, e.g. to distant "stars or even galaxies" and when they are needed then they are carried back to the location where γ needs them. This is like a real computer of today which writes partial results on, say, disks, then sends these disks to distant locations and asks for bringing these disks back when needed. The point is that the information needed for carrying out the computations of γ need not be stored locally at a single space, but it might be spread out evenly in the universe as thinly as we want. So, a careful enough "distributed" organization of the system of robots performing the task of γ can be spread out so thinly that we avoid formation of matter-energy densities which could seriously interfere with the task assigned to the Kerr hole.

Let us turn to the "infinite time and space" problem. The results of current astrophysics and astronomy predict that the expansion of the universe will never stop, in particular, that Einstein's cosmological constant Λ is positive. The discovery of this started in 1998 when independent teams of astronomers (one headed by S. Perlmutter) found that the expansion of the universe is accelerating (instead of slowing). These findings of modern physics and cosmology are based on hard experimental data (taken together with classical theory). The experimental data come from 3 radically different experimental research directions. These are (i) observing many supernovae at various distances, their redshifts, apparent luminosities etc. (e.g. S. Pearlmutter),

(ii) precisional studies of anisotropies in the cosmic background radiation (by COBE, and WMAP [89], e.g. D. Spergel), and (iii) the study of the large-scale distribution of galaxies in the universe (e.g. by Sándor Szalay, cf. Sloan Digital Sky Survey [73]). Any two of these different kinds of experimental research directions select the same cosmological model from the many theoretically possible ones, with great precision. In the weight of all this evidence, this model has been accepted as the (new) standard model of cosmology. This model predicts with great experimental confidence that the universe is infinite in time, is also infinite in space, and contains an infinite amount of matter [60]. All the above is described in detail in e.g. Dodelson [19] and reinforced in the Scientific American papers [76], [79].

The above model exists both in a Case II and a Case I version discussed in section 5.4.1(2). It is generally conjectured, see e.g. [11], that Case II is more likely. The reasons for this are as follows. After evidence became available for $\Lambda > 0$ in 1998, theoretical physicists started to search for understanding the "cause" for Λ, i.e. they wanted to find the substance or something that causes $\Lambda \neq 0$. Such an enterprise was already successful for the explanation of the inflation of the early universe. In that case, using results from particle physics, a scalar field was found responsible for $\Lambda > 0$. Nowadays it is conjectured that a similar scalar field is behind the present value of Λ. If so, as the universe expands, this scalar field gets diluted, and hence Λ will decrease and tend towards 0. This leads to the situation outlined in section 5.4.1(2) as Case II when the acceleration of the expansion of the universe slows down in the future but the expansion itself does not stop.[24] In the literature, the new role of a changing Λ has been reformulated in terms of "dark energy", "inflatons", "quintessence", to mention only a few. On the Internet, more technical papers on the above can be found under the just mentioned keywords. Cf. Dodelson [19], [11], Kaku [39, pp.102-105], Penrose [61, pp.772-774], Sir Martin Rees [67], Melia [48], Veneziano [84].

Let us return to relativistic computers, in the Case II version of this model. In section 5.4.1(2) we explained how feeding the black hole with matter for an infinity of time can be organized via subrobots of the computer γ, but then part of the matter collected by these subrobots of γ can be used for creating extra tape for γ to write on (while the remaining part is used for feeding the hole). So, if we are in a Case II universe, then infinity of time and space (together with the predicted evolution of matter density) permits us to collect enough matter/energy to write on. Hence relativistic computers are realizable. For Case I universes, it remains an open problem to elaborate how a relativistic computer

[24]Roughly, as Λ tends to zero (in the distant future), the geometry of our universe may approximate what is called Einstein - de Sitter spacetime in the textbook [18, p.335].

can be realized, but we referred to promising research initiatives in this direction in section 5.4.1(2) and the beginning of section 5.4.

This takes care of the doubt of Costa-Mycka-Pitowsky. Before going on, we would like to mention a further consideration undermining this doubt: it is not clear that to store a certain amount of information how much matter/energy is really needed. Actually, one of the main directions in quantum computing can be interpreted as undermining the belief that for storing one bit of information a certain fixed minimum amount of matter/energy is needed.

In passing, about the unbounded energy need doubt discussed above we would also mention a certain aspect of our perspective on relativistic computers: In agreement with Leeuwen and Wiedermann [43] we regard relativistic computers as "open systems". This means that analogously to a human intelligence (forming the model for an Artificial Intelligence) who uses his unbounded environment as a kind of resource, the relativistic computer $\langle \gamma_p, \gamma, q \rangle$ uses the entire universe (or even universes or multiverse) in which it is situated as a potential background resource into which e.g. γ can spread if desired.

5.5. **Instability of the inner horizon.** Because of the exotic properties of MH-spacetimes, the problem came up whether the overall geometric properties of the Kerr black hole might get lost if we add to the Kerr spacetime the world-line of some particle of a very small mass. Since the inner horizon (IH in the following) is the most interesting part of this spacetime, this problem became known as the (perturbational) instability problem of IH.[25]

In our case there are two world-lines, γ and γ_p, added to the Kerr spacetime. Of these, γ seems to be the more dangerous one since this is the one which is connected to the MH property. The present worry is based on the assumption that γ remains in the vicinity of the black hole while carrying out its task. If ZFC is consistent, γ will carry out an infinite number of "steps" in the vicinity of the black hole whose effect on the IH might build up in an analogous fashion as described in connection with the blueshift problem in [20, pp.111-112]. However, this problem admits a simple solution: the computer γ does not remain close to the black hole while carrying out its task, γ could move away farther and farther from the black hole such that its gravitational effects on the black hole become negligible. This does not interfere with our thought experiment. So, it seems that the destabilizing effect of γ can be avoided, and it is only the effect of γ_p which we need to think about.

Partial results about so-called RN black holes (these are electrically charged, non-rotating ones) pointed in the direction that a perturbation e.g. caused by γ_p crossing the inner horizon might cause the IH unstable

[25]The literature sometimes calls this "perturbation problem", or "instability of Cauchy horizon problem".

(in the RN case). It was conjectured that this instability will eventually lead to a spacelike singularity blocking the way of γ_p, making the originally traversable black hole non-traversable.[26] Cf. e.g. Wald [85, p.318, lower half of the page]. (Roughly, a singularity is spacelike iff it is not avoidable for a traveler, cf. [33].) Then it was conjectured that because of certain similarities between RN black holes and slow Kerr black holes, Kerr black holes (especially physically realistic rotating black holes) might inherit the same negative property leading to a spacelike singularity ([85, p.318, last 5 lines]). Later, it was proved in Ori [59] and also in related works that this is not necessarily the case. Namely, it might be true that the inner horizon IH gets unstable (because of the perturbation caused by γ_p) but the resulting singularity is not spacelike but null (i.e. photonlike), and weak in Tipler's sense which means that a small enough observer might approach the singularity unharmed. Ori [59] concludes that the IH singularities predicted for RN-holes in earlier works and IH singularities of rotating black holes are essentially different because the latter are null (not spacelike), weak, and of a rather simple asymptotic form. Ori's optimism is reinforced e.g. by Berger [6, sec.2.3.1] in 2002. So, Ori [59], Yurtsever [83], Berger [6] and related work quoted in [59] leave us with a hope that our relativistic computer can be realized by somehow circumnavigating this perturbation-caused instability of IH. It is emphasized in the quoted works that more research is needed for settling the issue satisfactorily either positively or negatively.

Independently of Ori [59], Kaku also suggests research plans for positive solutions to this perturbation caused instability of IH problem. Namely Kaku [39, p.322] recalls this perturbation problem caused by γ_p, under the subtitle "Are wormholes stable?". (He is discussing a project somewhat analogous to ours here.) Then Kaku goes on to outline a general plan for our future generations for overcoming this problem (for the case the research initiated in [59] would yield negative results for all possible directions, which seems not very likely). So both Ori [59] and [39, pp.322-327] leave us with an exciting research plan which might help us to obtain deeper understanding into the nature of both computation and cosmology.[27]

5.6. **Formation of supermassive black holes.** In the case of theoretic possibility (in general relativity) of relativistic beyond-Turing machines, the present issue causes no problem. So our "first issue"

[26] The idea of this perturbation-caused instability of IH originates with Penrose, who wanted to use this for reinforcing variants of his cosmic censor hypothesis.

[27] For the case the final outcome of the above research plan would yield only negative results (somewhat unlikely) we note that we can base our relativistic beyond-Turing computer on one of the many MH-spacetimes not involving black holes. These range from Gödel's rotating universes, through anti de Sitter spacetimes, to wormholes kept open by negative energy, or variants of M-theory.

is settled positively about this subject. However, if we study the formation of supermassive black holes in our given particular universe, then it is extremely difficult to compute in detail how infalling matter (probably pre-galactic cloud) forms eventually a Kerr type spacetime complete with its two event horizons, ring singularity and connections with other universes or other regions of our universe. Therefore it is safer to base our thought experiment (i.e. relativistic computer) on so-called primordial Kerr black holes which were created at the Big Bang (or earlier if there was no Big Bang). Though the existence of such primordial Kerr holes is theoretical only, it is a real possibility that such exist. Then this problem admits a (perhaps theoretical) solution. For primordial supermassive black holes cf. e.g. [66], for primordial black holes cf. e.g. Wald [85, p.306], and for recent developments about such black holes cf. Carr [12], [47].

5.7. **Relativistic computers based on other spacetimes.** Todd A. Brun [10] uses general relativity (GR) for "breaking complexity barriers" by designing computers which solve hard problems (like what are called NP complete problems) very fast. Let us call this kind of computers complexity-reducing computers. E.g. Brun [10] bases such a complexity-reducing computer on assuming the existence of a CTC (closed timelike curve). The question comes up naturally whether Brun's elegant method can be used also for designing beyond-Turing computers. In this connection we note that Brun's method is strictly different from ours, it is based roughly on the recent solutions for the grandmother paradox (related to CTC's) by Thorne, Novikov, Yurtsever, Morris, Gott, and others, cf. e.g. Earman [20, Chap.6] and/or Earman et al [23] or Gott [30, p.269]. We note that the research direction represented e.g. by Brun [10] using spacetimes with CTC's for complexity-reducing computers is active, cf. e.g. Dave Bacon [3]. It would be interesting to see whether these two approaches, i.e. the MH approach in section 4 of the present paper and the quantumgravitational CTC approach (Brun, Bacon) can be combined or connected.

5.8. **Further observational evidence for existence of real rotating black holes.** Below we include two kinds of Internet links:

(1) Observations based on the spinning rate (or rotational frequency) of infalling matter. This spinning rate they found was 450 times per second which when compared with the mass of the BH (hence the radius of the event horizon) [roughly 7 solar masses] implies a contradiction for nonrotating BH. From this it is inferred that the BH in question must be a rotating one. The astronomical name of this BH is GRO J1655-40. These findings were presented at an Amer. Phys. Soc. Meeting on April 30, 2001 Washington D.C. by Tod Strohmayer of NASA's Goddard S. F. Centrum in Maryland. The full paper is

available at the Los Alamos archives at the following code: arXiv:astro-ph/0104487 v1. It also appeared in the Astrophysical Journal Letters: Strohmayer, Tod E., Discovery of a 450 HZ Quasi-periodic Oscillation from the Microquasar GRO J1655-40 with the Rossi X-Ray Timing Explorer. The Astrophysical Journal, Volume 553, Issue 1, pp.L49-L53. May 2001. See also the Internet link http://adsabs.harvard.edu/cgi-bin/nph-bib_query?bibcode=2001ApJ...552L..49S&db_key=AST&high=3be05460e222702 . The homepage of Tod Strohmayer can be found at http://lheawww.gsfc.nasa.gov/users/stroh/ .

(2) A different kind of observational evidence for larger rotating BH's from Oct. 2003 (by Reinhard Genzel of Max Planck Institute et al) is summarized in the following link at Physics Web: http://physicsweb.org/articles/news/7/10/15/1 . The following seem to be also quite relevant to this: Last part of the page in Max Planck Institute, see http://www.mpe.mpg.de/ir/GC/index.php, and Black Hole Spin in AGN and GBHCs, by C. S. Reynolds, L. W. Brenneman, and D. Garofalo. ArXiv: astro-ph/0410116v1, Oct 5, 2004. http://arxiv.org/abs/astro-ph/0410116 .

5.9. Shagrir and Pitowsky [72] nicely and usefully complement the present paper in that they discuss and settle philosophical kinds of worries about relativistic computers which are not emphasized/addressed herein. Cooper's works, e.g. [14],[15] are also instructive in this connection.

6. Conclusion

In the introduction we wrote that this paper is written on two levels of abstraction, "pure GR level", and "level of physical realizability", roughly.

On the pure GR level, the investigations in the present paper and the quoted ones point in the direction that it is probably consistent with GR that relativistic beyond-Turing computers might, in principle, be constructed by future technology. (Even on this level we tried to avoid non-realistic assumptions like infinitely small test-bodies.) On the pure GR level, subsection 5.5 already seems to motivate interesting further reseach in central areas of spacetime theory.

On the level of studying physical realizability, the discussions in section 5 (and in the quoted works, e.g. in [37]) show that trying to put the PhCT into a new perspective in view of latest results of physics and cosmology leads to interesting and instructive questions about basic issues of spacetime theory. There are also connections to the foundation of mathematics and logic. These discovered interconnections between seemingly distant areas create a cross-fertilization which appear to us as mutually beneficial. This quest for a deeper understanding of our reality (both physical and logico-mathematical) is our main motivation for pursuing the present topic.

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