Can new physics challenge "old" computational barriers?

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Abstract. We discuss the impact of very recent developments of spacetime theory, black hole physics, and cosmology to well established foundational issues of computability theory and logic. Namely, we describe a physical device in relativistic spacetime which can compute a non-Turing computable task, e.g. which can decide the halting problem of Turing machines or whether ZF set theory is consistent or not. Connections with foundation of mathematics and foundation of spacetime theory will be discussed.

1 Introduction

We discuss here the impact of very recent developments in spacetime theory and cosmology on well established foundational issues (and interpretations) of logic and computability theory. The connections between computability theory, logic and spacetime theory (general relativity theory, GR) cut both ways: logic provides a tangible foundation for GR, cf. [1], while GR and its new developments might profoundly influence our interpretation of basic results of computability theory and logic, as we will see in this paper. The new computability paradigms, on the other hand, have influence on the foundation of mathematics and logic.

Two major new paradigms of computing arising from new physics are quantum computing and general relativistic computing. Quantum computing challenges complexity barriers in computability, while general relativistic computing challenges the physical Church-Turing Thesis itself. In this paper we concentrate on relativistic computers and on the physical Church-Turing Thesis.

The physical Church-Turing Thesis, PhCT, is the conjecture that whatever physical computing device (in the broader sense) or physical thought-experiment will be designed by any future civilization, it will always be simulateable by a Turing machine. The PhCT was formulated and generally accepted in the 1930's. At that time a general consensus was reached declaring PhCT valid, and indeed in the succeeding decades the PhCT was an extremely useful and valuable maxim in elaborating the foundations of theoretical computer science, logic, foundation of mathematics and related areas.[1] But since PhCT is partly

[1] As a contrast, one of the founding fathers of PhCT, László Kalmár, always hoped for a refutation of PhCT and to his students he emphasized that PhCT is meant to be a challenge to future generations, it is aimed at "teasing" researchers to put efforts into attacking PhCT. [21]

a physical conjecture, we emphasize that this consensus of the 1930's was based on the physical world-view of the 1930's. Moreover, many thinkers considered PhCT as being based on mathematics + common sense. But "common sense of today" means "physics of 100 years ago". Therefore we claim that the consensus accepting PhCT in the 1930's was based on the world-view deriving from Newtonian mechanics. Einstein's equations became known to a narrow circle of specialists around 1920, but about that time the consequences of these equations were not even guessed at. The world-view of modern black hole physics was very far from being generally known until much later, until after 1980.

Our main point is that in the last few decades (well after 1980) there has been a major paradigm shift in our physical world-view. This started in 1970 by Hawking's and Penrose's singularity theorem firmly establishing black hole physics and putting general relativity into a new perspective. After that, discoveries and new results have been accelerating. About 10 years ago astronomers obtained firmer and firmer evidence for the existence of larger and larger more exotic black holes [37],[34] not to mention evidence supporting the assumption that the universe is not finite after all [39]. Nowadays the whole field is in a state of constant revolution. If the background foundation on which PhCT was based has changed so fundamentally, then it is desirable to re-examine the status and scope of applicability of PhCT in view of the change of our general world-picture. Cf. also Cooper [10] for a related perspective. Cf. also [19], [16], [30], [35].

A special feature of the Newtonian world-view is the assumption of an absolute time scale. Indeed, this absolute time has its mark on the Turing machine as a model for computer. As a contrast, in general relativity there is no absolute time. Kurt Gödel was particularly interested in the exotic behavior of time in general relativity. Gödel [17] was the first to prove that there are models of GR to which one cannot add a partial order satisfying some natural properties of a "global time". In particular, in GR various observers at various points of spacetime in different states of motion might experience time radically differently. Therefore we might be able to speed up the time of one observer, say C (for "computer"), relatively to the other observer, say P (for "programmer"). Thus P may observe C computing very fast. The difference between general relativity and special relativity is (roughly) that in general relativity this speed-up effect can reach, in some sense, infinity assuming certain conditions are satisfied. Of course, it is not easy to ensure that this speed-up effect happens in such a way that we could utilize it for implementing some non-computable functions.

In sections 2 and 3 we present an intuitive idea of how this infinite speed-up can be achieved and how one can implement a computer based on this idea. More concrete technical details can be found in [16], [30] (and to some extent in the remaining parts of this paper). For brevity, we call such thought-experiments *relativistic computers*. We will see that it is consistent with Einstein's equations, i.e. with general relativity, that by certain kinds of relativistic experiments, future generations might find the answers to non-computable questions like the halting problem of Turing machines or the consistency of Zermelo Fraenkel set theory (the foundation of mathematics, abbreviated as ZFC set theory from now on). Moreover, the spacetime structure we assume to exist in these experiments is based in [16],[30] on huge slowly rotating black holes the existence of which is made more and more likely (practically certain) by recent astronomical observations [37],[34].

We are careful to avoid basing the beyond-Turing power of our computer on "side-effects" of the idealizations in our mathematical model/theory of the physical world. For example, we avoid relying on infinitely small objects (e.g. pointlike test particles, or pointlike bodies), infinitely elastic balls, infinitely (or arbitrarily) precise measurements, or anything like these. In other words, we make efforts to avoid taking advantage of the idealizations which were made when GR was set up. Actually, this kind of self-constraint is essential for the present endeavor as illustrated by [40, pp.446-447].

In sections 4 and 5 we discuss some essential questions of principle as well as some technical questions in connection with realizability of a relativistic computer, such as e.g. the so-called blue-shift problem, assuming infinity of time and space. Many of these questions come close to the limits of our present scientific knowledge, provoking new research directions or adding new motivations to already existing ones. We show that, at least, the idea of relativistic computers is not in conflict with presently accepted scientific principles. E.g. we recall that the presently accepted standard cosmological model predicts availability of infinite time and space. We also show that the principles of quantum mechanics are not violated, no continuity of time or space is presupposed by a relativistic computer. Discussing physical realizability and realism of our design for a computer is one of the main issues in [30, §5].

A virtue of the present research direction is that it establishes connections between central questions of computability theory and logic, foundation of mathematics, foundation of physics, relativity theory, cosmology, philosophy, particle physics, observational astronomy, computer science and AI [43]. E.g. it gives new kinds of motivation to investigating central questions of these fields like "is the universe finite or infinite (both in space and time) and in what sense", "exactly how do huge Kerr black holes evaporate" (quantum gravity), "how much matter is needed for coding one bit of information (is there such a lower bound at all)", questions concerning the statuses of the various cosmic censor hypotheses, questions concerning the geometry of rotating black holes [6], to mention only a few. The interdisciplinary character of this direction was reflected already in the 1987 course given by the present authors [28] during which the idea of relativistic hypercomputers emerged and which was devoted to connections between the above mentioned areas.

Section 6 is devoted to the impact of the "new computability paradigm" on spacetime theory. We discuss a different kind of motivation for studying relativistic computers. Namely, such a study may have applications to theoretical physics as follows. To GR, there is an infinite hierarchy of hypotheses called causality constraints which can be added to GR as outlined in [12, §6.3, pp.164-167]. Among these occur the various versions of the cosmic censor hypotheses (CCH) of which the basic reference book of relativity theory [41, p.303] writes "whether the cosmic censor conjecture is correct remains the key unresolved issue in the theory of gravitational collapse". On p.305 [41] writes "... there is virtually no evidence for or against the validity of this second version of CCH". These causality hypotheses play a role in GR analogous with the role formulas like GCH independent of ZF set theory play in set theory (or logic). These causality hypotheses are independent of GR (they are not implied by GR), and their status is the subject of intensive study as op. cit. illustrates this. Now, the study of relativistic computers could, in principle, reveal how the physical Church Thesis PhCT is situated in this hierarchy, in a sense which we will discuss in section 6. If we could find out which one of these constraints imply PhCT (or are implied by PhCT), that could be illuminating in why certain issues are difficult to settle about these constraints, cf. e.g. Etesi [15] and [41, p.303].

Tangible data underlying the above interconnections and also more history, references are available in [30]. The book Earman [12, p.119, section 4.9] regards the same interdisciplinary perspective as described above to be one of the main virtues of the present research direction. It is the unifying power of logic which makes it viable to do serious work on such a diverse collection of topics. One of the main aims of the research direction represented by [3], [2], [1], [23]–[25] is to make relativity theory accessible for anyone familiar with logic.

2 Intuitive idea for how relativistic computers work

In this section we would like to illuminate the ideas of how relativistic computers work, without going into technical details. The technical details are elaborated, among others, in [16], [19], [30]. To make our narrative more tangible, here we use the example of huge slowly rotating black holes for our construction of relativistic computers. There are many more kinds of spacetimes suitable for carrying out essentially the same construction. We chose rotating black holes because they provide a tangible example for illustrating the kind of reasoning underlying general relativistic approaches to breaking the "Turing barrier". Mounting astronomical evidence for their existence makes them an even more attractive choice for our didactic purposes.

Let us start out from the so-called Gravitational Time Dilation effect (GTD). The GTD is a theorem of relativity theory, it says that gravity makes time run slow. More sloppily: gravity slows time down. Clocks that are deep within gravitational fields run slower than ones that are farther out. We will have to explain what this means, but before explaining it we would like to mention that GTD is not only a theorem of general relativity. This theorem, GTD, can be proved already in special relativity in such a way that we simulate gravity by acceleration [23], [25]. So one advantage of GTD is that actually why it is true can be traced down by using only the simple methods of special relativity. Another advantage of GTD is that it has been tested several times, and these experiments are well known. Roughly, GTD can be interpreted by the following thought-experiment. Choose a high enough tower on the Earth, put precise enough (say, atomic) clocks at the bottom of the tower and the top of the tower, then wait enough time, and compare the readings of the two clocks. Then the clock on the top will run faster (show more elapsed time) than the one in the basement, at each time one carries out this experiment. Figure 1 represents how GTD can be proved in special relativity using an accelerated spaceship for creating artificial gravity and checking its effects on clocks at the two ends of the spaceship. The

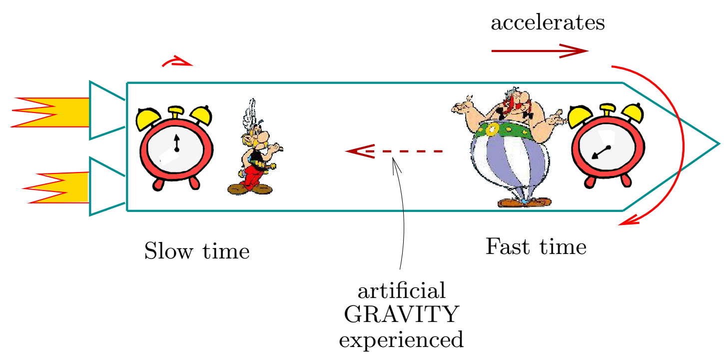

Fig. 1. GTD is a theorem of Special Relativity (SR) easily proved in first-order logic version of SR. Detailed purely logical formulation and proofidea are in [24].

next picture, Figure 2, represents the same GTD effect as before, but now using a tall tower on the Earth experiencing the same kind of gravity as in the spaceship. Gravity causes the clock on the top ticking faster. Therefore computers there also compute faster. Assume the programmer in the basement would like to use this GTD effect to speed up his computer. So he sends the computer to the top of the tower. Then he gets some speed-up effect, but this is too little. The next two pictures, Figure 3 and Figure 4, are about the theoretical possibility of increasing this speed-up effect.

How could we use GTD for designing computers that compute more than Turing Machines can? In the above outlined situation, by using the gravity of the Earth, it is difficult to make practical use of GTD. However, instead of the Earth, we could choose a huge black hole, cf. Figure 5. A black hole is a region of spacetime with so big "gravitational pull" that even light cannot escape from this region. There are several types of black holes, an excellent source is Taylor and Wheeler [38]. For our demonstration of the main ideas here, we will use huge, slowly rotating black holes. (These are called slow-Kerr in the physics literature.) These black holes have two so-called *event horizons*, these are bubble-like surfaces one inside the other, from which even light cannot escape (because of the gravitational pull of the black hole). See Figures 6–8. As we approach the outer event horizon from far away outside the black hole, the gravitational "pull" of the black hole approaches infinity as we get closer and closer to the event horizon. This is rather different from the Newtonian case,

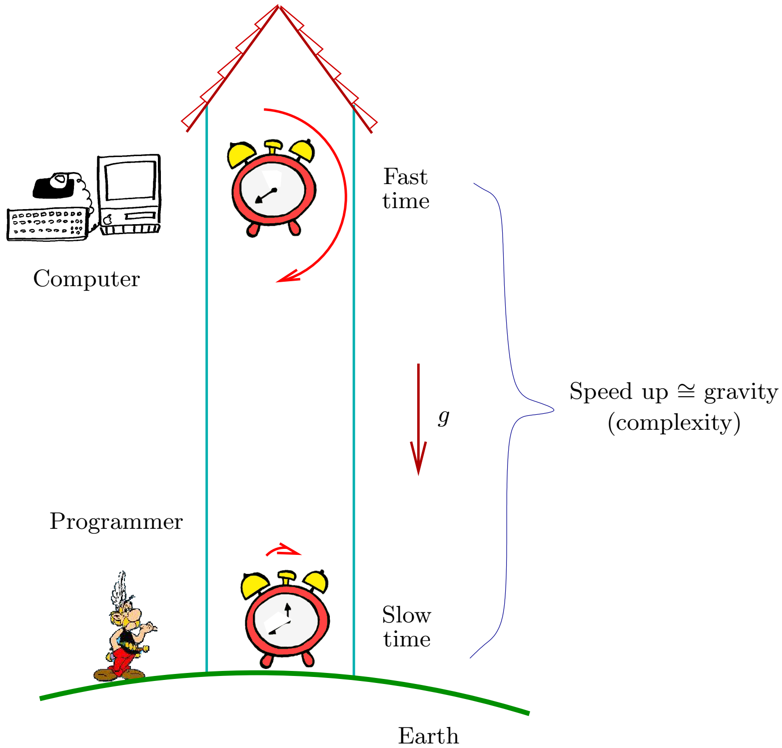

Fig. 2. TIME WARP (GTD, effects of gravity on time). Clocks higher in a gravitational well tick faster.

where the gravitational pull also increases but remains finite everywhere. For a while from now on "event horizon" means "outer event horizon".

Let us study observers suspended over the event horizon. Here, suspended means that the distance between the observer and the event horizon does not change. Equivalently, instead of suspended observers, we could speak about observers whose spaceship is hovering over the event horizon, using their rockets for maintaining altitude. Assume one suspended observer C is higher up and another one, P, is suspended lower down. So, C sees P below her while P sees C above him. Now the gravitational time dilation (GTD) will cause the clocks of C run faster than the clocks of P. Moreover, they both agree on this if they are watching each other e.g. via photons. Let us keep the height of C fixed. Now, if we gently lower P towards the event horizon, this ratio between the speeds of their clocks increases. Moreover, as P approaches the event horizon, this ratio approaches infinity. This means that for any integer n, if we want C's clocks to run n times as fast as P's clocks, then this can be achieved by lowering P to the right position.

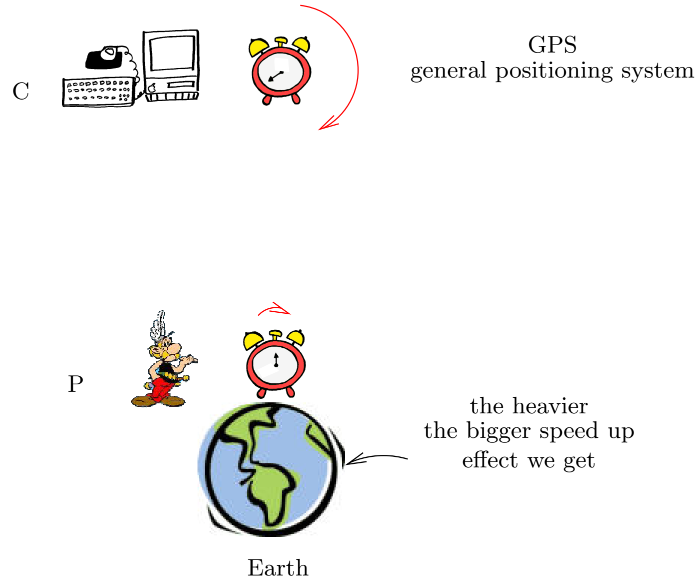

Fig. 3. Thought-experiment for fast computation: The programmer "throws" his slave-computer to a high orbit. Communicates via radio.

Already at this point we could use this arrangement with the black hole for making computers faster. The programmer goes very close to the black hole, leaving his computer far away. Then the programmer has to wait a few days and the computer does a few million year's job of computing and then the programmer knows a lot about the consequences of, say, ZFC set theory or whatever mathematical problem he is investigating.[2] So we could use GTD for just speeding up computation which means dealing with complexity issues. However, we do not want to stop at complexity issues. Instead, we would like to see whether we can attack somehow the "Turing barrier".

If we could suspend the lower observer P on the event horizon itself then from the point of view of C, P's clocks would freeze, therefore from the point of view of P, C's clocks (and computers!) would run infinitely fast, hence we would have the desired infinite speed-up upon which we could then start our plan for breaking the Turing barrier. The problem with this plan is that it is impossible to suspend an observer on the event horizon. As a consolation for this, we can suspend observers arbitrarily close to the event horizon. To achieve an "infinite speed-up" we could do the following. We could lower and lower again P towards the event horizon such that P's clocks slow down (more and more, beyond limit)

[2] The above arrangement for speeding the computer up raises the question of how the programmer avoids consequences of the fact that the whole manoeuver will slow down the programmer's own time relative to the time on his home planet, e.g. on the Earth. We will deal with this problem later.

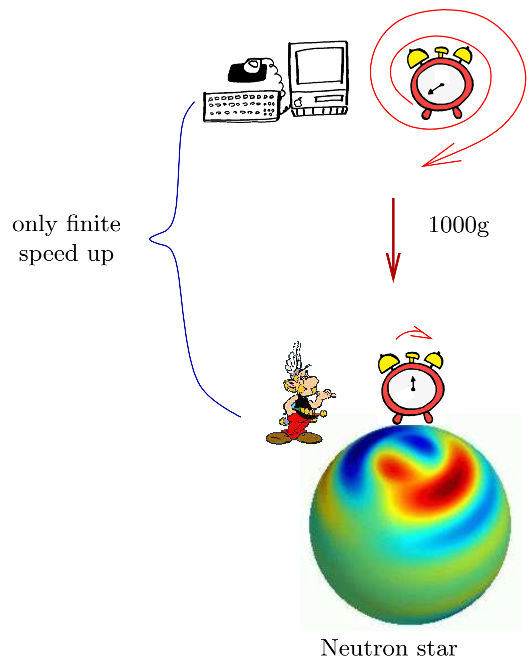

Fig. 4. By using a neutron star we still get only a finite speed-up.

in such a way that there is a certain finite time-bound, say b, such that, roughly, throughout the whole history of the universe P's clocks show a time smaller than b. More precisely, by this we mean that whenever C decides to send a photon to P, then P will receive this photon before time b according to P's clocks. This is possible. See Figure 8.

Are we done, then? Not yet, there is a remaining task to solve. As P gets closer and closer to the event horizon, the gravitational pull or gravitational acceleration tends to infinity. If P falls into the black hole without using rockets to slow his fall, then he does not have to withstand the gravitational pull of the black hole. He would only feel the so-called tidal forces which can be made negligibly small by choosing a large enough black hole. However, his falling through the event horizon would be so fast that some photons sent after him by C would not reach him outside the event horizon. Thus P has to approach the event horizon relatively slowly in order that he be able to receive all possible photons sent to him by C. In theory he could use rockets for this purpose, i.e. to slow his fall (assuming he has unlimited access to fuel somehow). Because P approaches the event horizon slowly, he has to withstand this enormous gravity (or equivalently acceleration). The problem is that this increasing gravitational force (or acceleration) will kill P before his clock shows time b, i.e. before the planned task is completed.

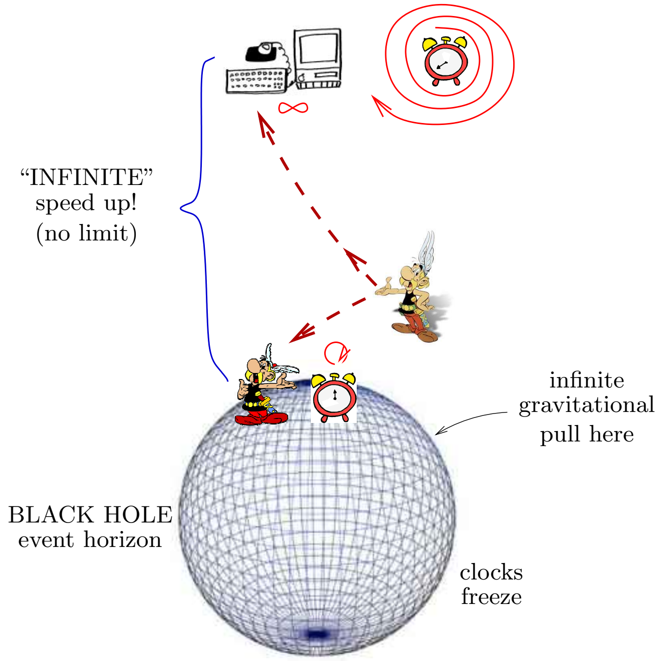

Fig. 5. Getting "infinite" speed-up.

At the outer event horizon of our black hole we cannot compromise between these two requirements by choosing a well-balanced route for P: no matter how he will choose his route, either P will be crashed by the gravitational pull, or some photons sent by C would not reach him. (This is the reason why we can not base our relativistic computer on the simplest kind of black holes, called Schwarzschild ones, which have only one event horizon and that behaves as we described above.)

To solve this problem, we would like to achieve slowing down the "fall" of P not by brute force (e.g. rockets), but by an effect coming from the structure of spacetime itself. In our slowly rotating black hole, besides the gravitational pull of the black hole (needed to achieve the time dilation effect) there is a counteractive repelling effect coming from the revolving of the black hole. This repelling effect is analogous to "centrifugal force" in Newtonian mechanics and will cause P to slow down in the required rate. So the idea is that instead of the rockets of P, we would like to use for slowing the fall of P this second effect coming from the rotation of the black hole. In some black holes with such a repelling force, and this is the case with our slowly rotating one, two event horizons form, see Figures 6–8. The outer one is the result of the gravitational pull and behaves basically like the event horizon of the simplest, so-called Schwarzschild hole, i.e. as described above. The *inner event horizon* marks the point where the repelling force overcomes the gravitational force. So inside the inner horizon, it is possible

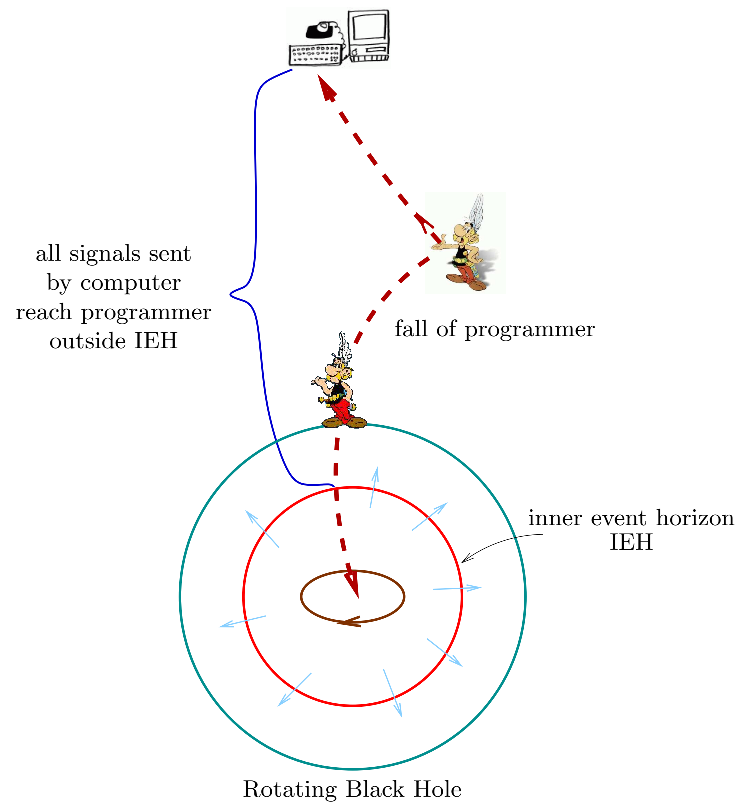

Fig. 6. Rotating Black Hole has two event horizons. Programmer can survive forever. (Ring singularity can be avoided.)

again to "suspend" an observer, say P, i.e. it becomes possible for P to stay at a constant distance from the center of the black hole (or equivalently from the event horizons).

Let us turn to describing how a slowly rotating black hole makes possible to realize our plan for "infinite speed-up". Figure 7 represents a slowly rotating huge Kerr black hole and Figure 8 represents its spacetime structure. As we said, there are two event horizons, the inner one surrounded by the outer one. The source of gravity of the black hole is a ring shaped singularity situated inside the inner horizon. The path of the in-falling observer P can be planned in such a way that the event when P reaches the inner horizon corresponds to the time-bound b (on the wristwatch of P) mentioned above before which P receives all the possible messages sent out by C.

By this we achieved the infinite speed-up we were aiming for. This infinite speed-up is represented in Figure 8 where P measures a finite proper time between its separation from the computer C (which is not represented in the figure) and its touching the inner horizon at proper time b (which point also is not rep-

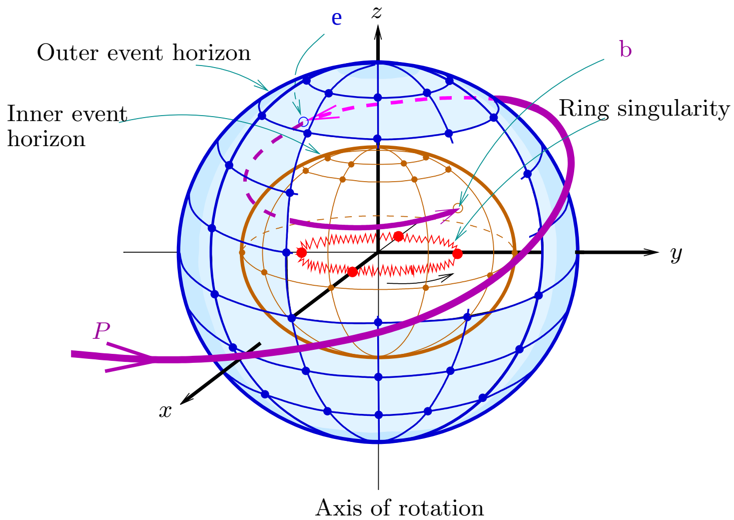

Fig. 7. A slowly rotating (Kerr) black hole has two event horizons and a ring-shape singularity (the latter can be approximated/visualized as a ring of extremely dense and thin "wire"). The ring singularity is inside the inner event horizon in the "equatorial" plane of axes x, y. Time coordinate is suppressed. Figure 8 is a spacetime diagram with x, y suppressed. Rotation of ring is indicated by an arrow. Orbit of in-falling programmer P is indicated, it enters outer event horizon at point e, and meets inner event horizon at point b.

resented in Figure 8). It can be seen in the figure that whenever C decides to send a photon towards P, that photon will reach P before P meets the inner horizon. The above outlined intuitive plan for creating an infinite speed-up effect is elaborated in more concrete mathematical detail in [16], [30].

3 Implementation for a relativistic computer

Let us see how we can use all this to create a computer that can compute tasks which are beyond the Turing limit. Let us choose the task, for an example, to decide whether ZFC set theory is consistent. I.e. we want to learn whether from the axioms of set theory one can derive the formula FALSE. (This formula FALSE can be taken to be $\exists x(x \neq x)$.) The programmer P and his computer C are together (on Earth), not moving relative to each other, and P uses a finite time-period for transferring input data to the computer C as well as for programming C. After this, P boards a huge spaceship, taking all his mathematical friends with him (like a Noah's Ark), and chooses an appropriate route towards a huge slowly rotating black hole, entering the inner event horizon when his wrist-watch shows time b. While he is on his journey towards the black hole, the computer checks one by one the theorems of set theory, and as soon as the computer finds a contradiction in set theory, i.e. a proof of the formula FALSE from the axioms

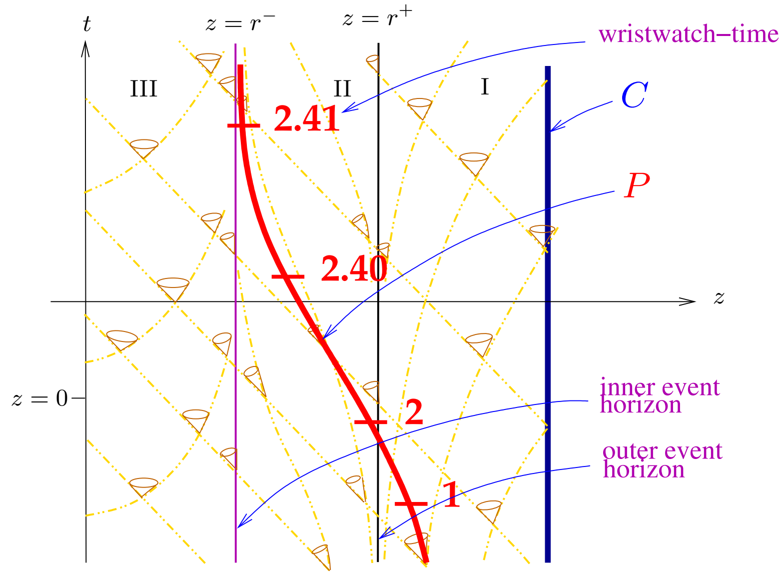

Fig. 8. The "tz-slice" of spacetime of slowly rotating black hole in coordinates where z is the axis of rotation of black hole. The pattern of light cones between the two event horizons r^- and r^+ illustrates that P can decelerate so much in this region that he will receive outside of r^- all messages sent by C. r^+ is the outer event horizon, r^- is the inner event horizon, $z = 0$ is the "center" of the black hole as in Figure 7. The tilting of the light cones indicates that not even light can escape through these horizons. The time measured by P is finite (measured between the beginning of the experiment and the event when P meets the inner event horizon at b) while the time measured by C is infinite.

of set theory, the computer sends a signal to the programmer indicating that set theory is inconsistent. If it does not find a proof for FALSE, the computer sends no signal.

So the programmer falls into the inner event horizon of the black hole and either the programmer will experience that a light signal arrives from the direction of the computer, of an agreed color and agreed pattern, or the programmer will observe that he falls in through the inner event horizon and the light signal does not arrive. After the programmer has crossed the inner event horizon, he can evaluate the situation. If a signal arrives from the computer, this means that the computer found an inconsistency in ZFC set theory, therefore the programmer will know that set theory is inconsistent. If the light signal does not arrive, and the programmer is already inside the inner event horizon, then he will know that the computer did not find an inconsistency in set theory, did not send the signal, therefore the programmer can conclude that set theory is consistent. So he can build the rest of his mathematics on the secure knowledge of the consistency of set theory.

The following questions come up in connection with realizability of this plan.

– can the programmer check whether the distant object he chose for a slowly rotating black hole is indeed one (whether it has the spacetime structure needed for his purposes)?
– can he check when he passed the event horizon?
– can he survive at all passing the event horizon?
– can he receive and recognize the signal sent by the computer?
– how long can he live inside the black hole?
– is there a way for the programmer to know that absence of signal from the computer is not caused by some catastrophy in the life of the computer?
– is it possible at all for a civilization to exist for an infinite amount of time?
– can the programmer repeat the computation or is this a once-for-a-lifetime computation for him?

Here we just assert that the answers to all these questions are in the affirmative, or at least do not contradict present scientific knowledge. These questions are discussed in detail in [30]. Below we address two of these questions.

On the question of traverseability of the event horizon: We chose the black hole to be large. If the black hole is huge[3], the programmer will feel nothing when he passes either event horizon of the black hole—one can check that in case of a huge black hole the so-called tidal forces on the event horizons of the black hole are negligibly small [31], [16].

On the question of how long the programmer can live after crossing the event horizon: The question is whether the programmer can use this new information, namely that set theory is consistent, or whatever he wanted to compute, for his purposes. A pessimist could say that OK they are inside a black hole, so—now we are using common sense, we are not using relativity theory—common sense says that the black hole is a small unfriendly area and the programmer will sooner or later fall into the middle of the black hole where there is a singularity and the singularity will kill the programmer and his friends. The reason why we chose our black hole to be a huge slowly rotating one, say of mass $10^{10}m_\odot$, is the following. If the programmer falls into a black hole which is as big as this and it rotates slowly, then the programmer will have quite a lot of time inside the black hole because the center of the black hole is relatively far from the event horizon. But this is not the key point. If it rotates, the "matter content", the so-called singularity, which is the source of the gravitational field of the black hole so-to-speak, is not a point but a ring. So if the programmer chooses his route in falling into the black hole in a clever way, say, relatively close to the north pole instead of the equatorial plane, then the programmer can comfortably pass through the middle of the ring, never get close to the singularity and happily live on forever. We mean, the rules of relativity will not prevent him from happily living forever. He may have descendants, he can found society, he can use and pass on the so obtained mathematical knowledge.

The above outlined train of thought can be used to show that any recursively enumerable set can be decided by a relativistic computer [16]. Actually, more

[3] this is a technical expression in observational astronomy

than that can be done by relativistic computers. Welch [42] shows that the arrangement described in section 3 using Kerr black holes can compute exactly Δ_2 problems in the arithmetical hierarchy (under some mild extra assumptions). Computability limits connected with relativistic computers are also addressed in [19], [20], [35], [43].

Relativistic computers are not tied to rotating black holes, there are other general relativistic phenomena on which they can be based. An example is anti-de-Sitter spacetime which attracts more and more attention in explaining recent discoveries in cosmology (accelerating expansion of the universe). Roughly, in anti-de-Sitter spacetime, time ticks faster and faster at farther away places in such a way that P can achieve infinite speed-up by sending away the computer C and waiting for a signal from her. This scenario is described and is utilized for computing non-Turing computable functions in [19]. This example shows that using black holes (or even singularities) is not inherent in relativistic computers.

Spacetimes suitable for an implementation (of relativistic computation) like in section 3 are called Malament-Hogarth spacetimes in the physics literature. A relativistic spacetime is called Malament-Hogarth (MH) if there is an event (called MH-event) in it which contains in its causal past a worldline of infinite proper length. The spacetime of ordinary Schwarzschild black hole is not MH, the spacetime of rotating Kerr black hole is MH and any event within the inner event horizon is MH, in anti-de-Sitter spacetime every event is an MH-event, the spacetime of an electrically charged BH (called Reissner-Nordström spacetime) is MH and there are many other examples for MH.

We note that using MH spacetimes does not entail some faith in some exotically "benevolent" global property of the whole of our universe. Instead, most of the MH spacetimes, like rotating BH's, can be built by a future, advanced civilization inside our usual "standard" universe of high precision cosmology. Namely, such MH spacetimes do not refer to the whole universe, but instead, to some "local" structure like a rotating ring of gravitationally collapsed matter in a "spatially finite part" of a more or less usual universe involving no particular global "whichcraft", so-to-speak. We are writing this because the word "spacetime" in the expression "MH spacetime" might be misleading in that it might suggest to the reader that it is an exotic unlikely property of the whole of God's creation, namely, the whole universe. However, in most MH spacetimes this is not the case, they are (in some sense) finite structures that can be built, in theory, by suitably advanced civilizations in a standard kind of universe like the one which is predicted by the present-day standard version of cosmology. In other words, nothing fancy is required from the whole universe, the "fancy part" is a structure which can, in theory, be manufactured in an ordinary infinite universe. Therefore in the present context it would be more fortunate to talk about MH regions of spacetime than about MH spacetimes.

4 Two sides of the coin

A relativistic computer as we described in section 3 is a team consisting of a Computer (C, for Cecil) and a Programmer (P, for Peter).

How does the computer C experience the task of this computing? C will see (via photons) that the programmer P approaches the black hole (BH), and as he approaches it, his wristwatch ticks slower and slower, never reaching wristwatch time b. C will see the Programmer approaching the BH in all her infinite time. For C, the Programmer shines on the sky for eternity. The only effect of C's time passing is that this image gets dimmer and dimmer, but it will never disappear. Under this sky, C computes away her task consisting of potentially infinitely many steps, i.e. checking the theorems of ZFC one by one, in an infinite amount of time.

How does the Programmer experience the task of this computing? He is travelling towards the black hole, and he only has to check whether he received a special signal from the Computer or not. For this task, which consists of finitely many steps, he has a finite amount of time.

What would he see would he watch his team-member, the Computer? He would see the Computer computing faster and faster, speeding up so that when his (P's) wristwatch time reaches b, C would just flare up and disappear. Well, this flare-up would burn P, because it carries the energy of photons emitted during the whole infinite life of C, thus the total amount of this energy is infinite. In fact, we have to design a shield (or mirror) so that only intended signals from C can reach P. This means that we have to ensure that P does not see C! P's task is to watch whether there is one special kind of signal coming through this shield. All in all, P's task is to do finitely many steps in a finite amount of time.

A task in the literature is called supertask if it involves one to carry out infinitely many steps in a finite amount of time [13]. Therefore, by the above, we think that the relativistic computer need not implement a supertask.

The above led us to the so-called blue-shift problem [12]. This is the following. The frequency of light-signals (photons) sent by C to P get increased (i.e. blue-shifted) by the time they reach P because of the infinite speed-up we worked so hard to achieve! Thus, if we do nothing about this, the one signal that C sends, can kill P. Further, P may not be able to recognize the blue-shifted signal. There are many solutions for this problem, two such solutions can be found in sections 5.3.1 and 5.4.1 of [30]. E.g., C can arrange sending the signal to P such that C asks her sister C' to embark a spaceship S which speeds up in the direction opposite to the direction of the Kerr hole, and send the signal from this spaceship. If S moves fast enough, then any signal sent from S to P will be red-shifted because of the speed of S. Then C chooses the speed of S to be such that the red-shift caused by this speed exactly cancels out the blue-shift caused by the gravitational effects at the event when P receives the signal.

5 Can we learn something about infinity?

The relativistic computer as we implemented it in section 3 assumes that an infinite amount of time is available for computing. This seems to be essential for breaking the Turing barrier (by our construction). We are in a good position here, because of the following. As a result of very recent revolution in cosmology, there is a so-called standard model of cosmology. This standard model is based on matching members of a family of GR spacetimes against a huge number of observational data obtained by three different astronomical projects. This huge number of measurements (made by using computers) all point amazingly to one specific GR spacetime. This spacetime is called the standard cosmological model, and in accordance with the so far highly successful scientific practice of the last 2500 years, we regard this standard model of the latest form of high-precision cosmology as the model best suited to explain observations and experience collected so far. According to this standard model, our universe is infinite both in regard of time and space, moreover there is an infinite amount of matter-energy available in it. We will see soon that the latter infinity is not needed for our construction. For more on this see Dávid [11], [30] and the references therein. Our point here is not believing that our universe indeed has infinite time or not. The point is that assuming availability of an infinite amount of time for computing is not in contradiction with our present-day scientific knowledge.

We would like to say some words on the question of how much matter/energy is needed for storing, say, 10 bits of information. Although this question is not essential for the realizability of the relativistic computer (because of availability of infinite energy in the standard model of cosmology), we still find this question interesting for purely intellectual/philosophical reasons.

Is information content strongly tied to matter/energy content? Is there a lower bound to mass which is needed to store 10 bits of information? This is a question which has nagged one of the authors ever since he wrote his MsC thesis [27] where a separate section was devoted to this issue. The question is: "If I want to write more, do I need more paper?" Right now it seems to us that the answer is in the negative. Matter and information might be two independent (orthogonal) "dimensions". The reason for this is the following. One might decide to code data by photons. Then the amount of matter/energy used is the energy total of these photons. But the energy of a photon is inversely proportional with its wavelength. So, one might double the wavelength of all photons and then one halved the energy needed to carry the same information one coded originally. If this is still too much energy expense, then one can double the wavelength again. Since there is no upper bound to the wavelengths of photons, there is no lower bound for the energy needed for storing 10 bits of data.

So, it seems to us that energy and information are not as strongly linked entities as energy and mass are (via $E = mc^2$). In the above argument when we said that there was no upper bound to the wavelength of possible photons, we used that according to the standard cosmological model the Universe is infinite in space. We note that Einstein when inventing photons did not say that there is a smallest nonzero value for energy. He said this only for light of a fixed color, i.e. fixed wavelength.

We would like to emphasize that we did not use that space is continuous. We seem to have used that time is continuous, but we can avoid that assumption by refining the implementation of relativistic computer. Constructions for this are in [30]. Thus, no contradictions with the principles of quantum mechanics seems to be involved in the idea of relativistic computer.

In the above, we argued that in principle, one can even build a relativistic computer sometime in the future. However, a fascinating aspect of relativistic computers for us is that they bring up mind-boggling questions about the nature of infinity. These questions would be worth thinking over even if our present-day science would predict a finite universe. We seem to understand and be familiar with the use of potential infinity in science. However, the above thought-experiment seems to use the notion of actual infinity. Is infinite a mental construction only or does it exist in a more tangible way, too? Can we learn something about actual infinity by making physical experiments? This leads to questions inherent in foundational issues in mathematics and physics. For more about this and about connection with Hilbert's Programme for mathematics we refer to [4].

6 Relativistic computers and Causality Hypotheses in Physics

Let us consider the hierarchy of causality hypotheses $C0, \dots, C9$ summarized in Earman [12, §6.3]. None of these follow from GR (cf. e.g. [1]), they function as extra possible hypotheses for narrowing the scope of the theory. The strongest of these is $C9$ saying that spacetime is globally hyperbolic. Roughly, this means that the temporal-causal structure of spacetime is basically the same as that of the Newtonian world in that it admits a "global time" associating a real number $t(p)$ to every point p of spacetime. In other words, $C9$ says that spacetime admits a "global foliation", i.e. it is a disjoint union of global time-slices or "global now"-s. This is a quite extreme assumption and its role is more of a logical status (i.e. investigate questions of what follows if $C9$ is assumed) rather than assuming that it holds for the actual universe (recall that Wald [41] wrote that "there is virtually no evidence for or against the validity of this").

Question 1. It would be interesting to know whether PhCT follows from GR+$C9$.

Since PhCT has not been formalized precisely yet, a more careful version of this question is asking for a natural and convincing, realistic formulation of PhCT which would follow from GR+$C9$. In other words, we are asking if there are some natural and convincing extra conditions on physically realistic computability which would yield PhCT from GR+$C9$. The need for such extra realisticity assumptions is demonstrated by e.g. [40, pp.446-447].

Next we note that $C9$ implies that spacetime is not Malament-Hogarth (NoMH for short), but NoMH does not imply $C9$. Hence NoMH is a strictly weaker causality hypothesis than $C9$. (Again, we have no reason to believe NoMH.)

Question 2. Under what natural (extra) conditions is NoMH equivalent with PhCT?

Comments on this question: Though in theory the MH property implies failure of PhCT (i.e. PhCT $\Rightarrow$ NoMH), cf. [12, §4], [19], there is a reason why in the works [16], [30], [29] we chose to implement our relativistic computer via a huge rotating black hole. Namely, huge-ness of the rotating BH was used to ensure that the tidal forces at the event horizons do not kill the programmer. It seems to be possible to construct a toy-example of a MH spacetime in which our kind of relativistic computer is not realistic physically. By physical realisticity we mean requirements that we do not use infinitely small computers (objects), infinitely precise measurements, or the like in designing our beyond-Turing computer, cf. [30] for more detail. We note that if we do not insist on physical realisticity, then already in Newtonian Mechanics PhCT would fail ad demonstrated e.g. in Tipler [40, pp.446-447].

The spacetime of a tiny slowly rotating BH is already MH but the tidal forces of this BH would render our design for a relativistic computer not realistic physically. This motivates the question of what natural assumptions would ensure PhCT $\Rightarrow$ NoMH or equivalently MH $\Rightarrow$ NotPhCT in a physically realistic way. This is part of Question 2 above.

The other direction of Question 2 seems to be the harder one: Under what natural conditions (if any) does NotPhCT imply MH. I.e. under what conditions is

$$(\star) \qquad \text{NotPhCT} \Rightarrow \text{MH}$$

true? One way of rephraising $(\star)$ is to conjecture that if there is a physically realistic beyond-Turing computer then there must be one which is built up in the style of the present paper utilizing MH property of spacetime. (By beyond-Turing computer we mean a physical computer that can compute beyond the Turing barrier.) This seems to be a daring conjecture. But let us remember that the question was: under what conditions is statement $(\star)$ true. So one possibility is that we start by assuming GR. In particular, if the physical beyond-Turing computer "designed" in the book Pour-El and Richards [33] turns out to be physically realizable, then our conjecture (that under some reasonable conditions $(\star)$ might become true) might get refuted.

We note that the conjecture implicit in Question 2 was arrived at jointly with Gábor Etesi.

7 History of relativistic computation

The idea of general relativistic computing as described in section 2 was found at different parts of the globe, independently. It was discovered by Németi in 1987 [28], Pitowsky in 1990 [32], Malament in 1989 [26], and Hogarth in 1992 [18] independently. Németi's idea used large slowly rotating black holes (slow Kerr spacetimes) but the careful study of feasibility and transversability of these was done later in Etesi-Németi [16]. All this led to a fruitful cooperation between the parties mentioned above, e.g. between Cambridge (Hogarth et al), Budapest (Németi et al), Pittsburgh (Earman et al). The first thorough and systematic study of relativistic computation was probably Hogarth [19]. Related work on relativistic computing include [42], [35], [12, §4], [13], [14], [29].

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References

1. Andréka, H., Madarász, J. X. and Németi, I., Logic of Spacetime and Relativity Theory. In: Logic of Space, eds: M. Aiello, J. van Benthem, and I. Hartman-Pratt, Springer-Verlag. To appear.
2. Andréka, H., Madarász, J. X. and Németi, I., Logical axiomatizations of spacetime. In: Non-Euclidean Geometries, János Bolyai Memorial Volume. Ed. A. Prékopa and E. Molnár, Mathematics and its Applications Vol. 581, Springer 2006. pp.155-185.
3. Andréka, H., Madarász, J. X. and Németi, I. with contributions from Andai, A., Sain, I., Sági, G., Tőke, Cs. and Vályi, S., On the logical structure of relativity theories. Internet book, Budapest, 2000. http://www.math-inst.hu/pub/algebraic-logic/olsort.html
4. Andréka, H. and Németi, I., Relativistic computers and Hilbert's Program for mathematics. Manuscript.
5. Andréka, H. Németi, I. and Sain, I., A complete logic for reasoning about programs via nonstandard model theory. Theoretical Computer Science 17 (1982), Part I in No 2, pp.193-212, Part II in No 3, pp.259-278.
6. Andréka, H., Németi, I. and Wüthrich, A twist in the geometry of rotating black holes: seeking the cause of acausality. Manuscript, 2005. 15pp.
7. Barrow, J. D., How to do an infinite number of things before breakfast. New Scientist, 29 January 2005.
8. Barrow, J. D. and Tipler, F. J., The anthropic cosmological principle. Oxford University Press, 1986.
9. Chown, M., Smash and grab. New Scientist, 2002. April 6, pp.24-28.
10. Cooper, S. B., How can Nature Help Us Compute? In SOFSEM 2006: Theory and Practice of Computer Science - 32nd Conference on Current Trends in Theory and Practice of Computer Science, Merin, Czech Republic, January 2006 (Jiri Wiedermann, Julius Stuller, Gerard Tel, Jaroslav Pokorny, Maria Bielikova, editors), Springer Lecture Notes in Computer Science No. 3831, 2006, pp. 1-13.

11. Dávid, Gy., Modern cosmology - astronomical, phsical and logical approaches. Abstracts for the conference "Logic in Hungary 2005", http://atlas-conferences.com/cgi-bin/abstract/caqb-64
12. Earman, J., Bangs, crunches, whimpers, and shrieks. Singularities and acausalities in relativistic spacetimes. Oxford university Press, Oxford, 1995.
13. Earman, J. and Norton, J., Forever is a day: supertasks in Pitowsky and Malament-Hogarth spacetimes. Philosophy of Science 60 (1993), 22-42.
14. Earman, J. and Norton, J., Infinite pains: the trouble with supertasks. In: S. Stich (ed.), Paul Benacerraf: the Philosopher and his critics. Blackwell, New York, 1994.
15. Etesi, G., Note on a reformulation of the strong cosmic censor conjecture based on computability. Physics Letters B 550 (2002), 1-7.
16. Etesi, G. and Németi, I., Turing computability and Malament-Hogarth spacetimes. International Journal of Theoretical Physics 41,2 (2002), 342-370.
17. Gödel, K., Lecture on rotating universes. In: Kurt Gödel Collected Works, Vol. III. Eds.: Feferman, S., Dawson, J. S., Goldfarb, W., Parson, C. and Solovay, R. N., Oxford University Press, New York Oxford 1995. pp. 261-289.
18. Hogarth, M. L., Non-Turing computers and non-Turing computability. In: D. Hull, M. Forbes, and R. M. Burian (eds), PSA 1994, Vol. 1. East Lansing: Philosophy of Science Association. pp. 126-138.
19. Hogarth, M. L., Predictability, computability, and spacetime. PhD Dissertation, University of Cambridge, UK, 2000. http://ftp.math-inst.hu/pub/algebraic-logic/Hogarththesis.ps.gz
20. Hogarth. M. L., Deciding arithmetic using SAD computers. Brit. J. Phil. Sci. 55 (2004), 681-691.
21. Kalmár, L., An Argument Against the Plausibility of Church's Thesis. In Heyting, A. (ed.) Constructivity in Mathematics. North-Holland, Amsterdam, 1959.
22. van Leeuwen, J. and Wiedermann, J., The Turing Machine paradigm in comtemporary computing. In B. Enquist & W. Schmid (Eds.), Mathematics Unlimited - 2001 and Beyond. Springer-Verlag, Berlin, 2001. pp. 1139-1155.
23. Madarász, J. X., Németi, I. and Székely, G., Twin Paradox and the logical foundation of space-time. Foundation of Physics 36,5 (2006), 681-714. arXiv:gr-qc/0504118.
24. Madarász, J. X., Németi, I. and Székely, G., First-order logic foundation of relativity theories. In: Mathematical problems from applied logic II, International Mathematical Series, Springer-Verlag, to appear.
25. Madarász, J. X. and Székely, G., The effects of gravitation on clocks, proved in axiomatic relativity. Abstracts for the conference "Logic in Hungary 2005", http://atlas-conferences.com/cgi.bin/abstract.caqb-41.
26. Malament, D., Private communication to Earman, J., 1988. Cf. Earman's book in the present list of references, p.238.
27. Németi, I., Computing the disturbance of phone lines caused by high-voltage electrical power lines. Master Thesis, Dept. Electrodynamics, Technical University Budapest, 1965.
28. Németi, I., On logic, relativity, and the limitations of human knowledge. Iowa State University, Department of Mathematics, Ph. D. course during the academic year 1987/88.
29. Németi, I. and Andréka, H., Can general relativistic computers break the Turing barrier? In: Arnold Beckmann, Ulrich Berger, Benedikt Lwe, and John V Tucker (eds.): Logical Approaches to Computational Barriers, Second Conference on Computability in Europe, CiE 2006, Swansea, UK, July 2006, Proceedings, Lecture Notes in Computer Science 3988, Springer-Verlag Berlin Heidelberg 2006. pp.398-412.

30. Németi, I. and Dávid, Gy., Relativistic computers and the Turing barrier. Journal of Applied Mathematics and Computation 178 (2006), 118-142.
31. Ori, A., On the traversability of the Cauchy horizon: Herman and Hiscock's argument revisited, in: Internal Structures of Black Holes and Spacetime Singularitites (ed. A. Ori, L.M. Ori), Ann. Isra. Phys. Soc. 13, IOP (1997).
32. Pitowsky, I., The physical Church thesis and physical computational complexity. Iyyun A Jerusalem Philosophical Quarterly 39 (1990), 81-99.
33. Pour-El, M. B. and Richards, J. I., Computability in analysis and physics, Springer-Verlag, Berlin (1989).
34. Reynolds, C. C., Brenneman, L. W. and Garofalo, D., Black hole spin in AGN and GBHCs. Oct. 5, 2004. arXiv:astro-ph/0410116. (Evidence for rotating black holes.)
35. Sági, G. and Etesi, G., Transfinite computations and diameters of spacetimes. Manuscript.
36. Shagrir, O. and Pitowsky, I., Physical hypercomputation and the Church-Turing Thesis. Minds and machines 13 (2003), 87-101.
37. Strohmayer, T. E., Discovery of a 450 HZ quasi-periodic oscillation from the microquasar GRO J1655-40 wtih the Rossi X-Ray timing explorer. The Astrophysical Journal, 553,1 (2001), pp.L49-L53. (Evidence for rotating black holes.) arXiv:astro-ph/0104487.
38. Taylor, E. F. and Wheeler, J. A., Black Holes. Addison, Wesley, Longman, San Francisco, 2000.
39. Tegmark, M., Parallel Universes. Scientific American May 2003, pp.41-51.
40. Tipler, F. J., The physics of immortality. Anchor Books, New York, 1994.
41. Wald, R. M., General relativity. University of Chicago Press, Chicago 1984.
42. Welch, P. D., The extent of computation in Malament-Hogarth spacetimes. School of Mathematics, University of Bristol, September 2006. Submitted.
43. Wiedermann, J. and van Leeuwen, J., Relativistic computers and non-uniform complexity theory. In: Calude et al (eds.) UMC 2002. Lecture Notes in Computer Science Vol. 2509, Springer-Verlag, Berlin, 2002. pp.287-299.
44. Wischik, L., A formalization of non-finite computation. Dissertation for the degree of Master of Philosophy, University of Cambridge, 1997. www.wischik.com/lu/philosophy/non-finite-computation.html