Conceptual structure of spacetimes, and category of concept algebras

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In the first part of the talk, we explore the first-order logic conceptual structure of special relativistic spacetime: We describe the algebra of explicitly definable relations of Minkowski-spacetime, and draw conclusions such as “the concept of lightlike-separability can be defined from that of timelike-separability by using 4 variables but not by using three variables”, or “no non-trivial equivalence relation can be defined in Minkowski-spacetime”, or “there are no interpretations between the classical (Newtonian) and the relativistic spacetimes, in either directions”.

In the second part of the talk, we generalize the notion of a concept algebra from first-order language/logic to any language. A duality between algebra and category theory emerges here quite nicely. Namely, category theoretic properties of the category of all concept algebras shed light on definability properties of the language. For example, “all implicitly definable concepts are explicitly definable (Beth definability property) if and only if epimorphisms are surjective in the category of concept algebras”, or “all existence-requiringly implicitly definable concepts are explicitly definable (weak Beth definability property) if and only if there is no proper epi-reflective subcategory of the category of concept algebras that contain the so-called full concept algebras”. Connections with category theoretic injectivity logic will be pointed out.