List of axioms and axiom systems

oldalszámokat ellenőrizni!

Convention: In this list the axiom systems (i.e. theories) to be recalled will be boxed in. The only purpose of this is to make searching in the list easier.

(1) Main axiom systems

$\boxed{\textbf{Basax}} \stackrel{\text{def}}{=} \{\, \textbf{Ax1}, \textbf{Ax2}, \textbf{Ax3}, \textbf{Ax4}, \textbf{Ax5}, \textbf{Ax6}, \textbf{AxE} \,\}$ (cf. p.51), where:

Ax1 $G = \mathsf{Eucl}(n, \mathbf{F})$, p.45.

Ax2 $Obs \cup Ph \subseteq Ib$, p.48.

Ax3 $(\forall h \in Ib)(\forall m \in Obs)\ tr_m(h) \in G$, p.48.

Ax4 $(\forall m \in Obs)\ tr_m(m) = \bar{t}$, p.48.

Ax5 $(\forall m \in Obs)(\forall \ell \in G) \Big(\mathrm{ang}^2(\ell) < 1 \ \Rightarrow\ (\exists k \in Obs)\, \ell = tr_m(k)$ and

$$\mathrm{ang}^2(\ell) = 1 \ \Rightarrow\ (\exists ph \in Ph)\, \ell = tr_m(ph)\Big),$$ p.50.

Ax6 $(\forall m, k \in Obs)\ Rng(w_m) = Rng(w_k)$, p.50.

AxE $(\forall m \in Obs)(\forall ph \in Ph)\ v_m(ph) = 1$, p.51.

$\boxed{\textbf{Newbasax}} \stackrel{\text{def}}{=} (\textbf{Basax} \setminus \{\, \textbf{Ax6}, \textbf{Ax3}, \textbf{AxE} \,\}) \cup \{\, \textbf{Ax6}_{\mathbf{00}}, \textbf{Ax6}_{\mathbf{01}}, \textbf{Ax3}_{\mathbf{0}}, \textbf{AxE}_{\mathbf{0}} \,\} = \{\, \textbf{Ax1}, \textbf{Ax2}, \textbf{Ax3}_{\mathbf{0}}, \textbf{Ax4}, \textbf{Ax5}, \textbf{Ax6}_{\mathbf{00}}, \textbf{Ax6}_{\mathbf{01}}, \textbf{AxE}_{\mathbf{0}} \,\}$ (cf. p.191), where:

$\textbf{Ax6}_{\mathbf{00}}$ $(\forall m, k \in Obs)\ w_m[tr_m(k)] \subseteq Rng(w_k)$, p.190.

Intuitively, observer k sees all those events which are seen by another observer m on k's life-line.

$\textbf{Ax6}_{\mathbf{01}}$ $(\forall m, k \in Obs)\ Dom(\mathsf{f}_{mk}) \in Open$, p.190.

$\textbf{Ax3}_{\mathbf{0}}$ $(\forall h \in Ib)\ (tr_m(h) \in G \cup \{\emptyset\} \ \ \wedge\ \ (\exists k \in Obs) tr_k(h) \neq \emptyset)$, p.191.

$\textbf{AxE}_{\mathbf{0}}$ $(\forall m \in Obs)(\forall ph \in Ph)(tr_m(ph) \neq \emptyset \ \ \Rightarrow\ \ v_m(ph) = 1)$, p.191.

$\boxed{\textbf{Bax}} \stackrel{\text{def}}{=} (\textbf{Newbasax} \setminus \{\, \textbf{Ax5}, \textbf{AxE}_\mathbf{0} \,\}) \cup \{\, \textbf{Ax5}^\mathbf{Obs}, \textbf{Ax5}^\mathbf{Ph}, \textbf{AxE}_\mathbf{00}, \textbf{AxE}_\mathbf{01} \,\} = \{\, \textbf{Ax1}, \textbf{Ax2}, \textbf{Ax3}_\mathbf{0}, \textbf{Ax4}, \textbf{Ax5}^\mathbf{Obs}, \textbf{Ax5}^\mathbf{Ph}, \textbf{Ax6}_\mathbf{00}, \textbf{Ax6}_\mathbf{01}, \textbf{AxE}_\mathbf{00}, \textbf{AxE}_\mathbf{01} \,\}$ (cf. p.219), where:

$\textbf{Ax5}^\mathbf{Obs}$ $(\exists ph)(\forall \ell)\Big(m \stackrel{\odot}{\to} ph \ \wedge\ [ang^2(\ell) < v_m(ph) \ \Rightarrow\ (\exists k)\ell = tr_m(k)]\Big)$, p.218

$\textbf{Ax5}^\mathbf{Ph}$ $ang^2(\ell) = v_m(ph) \ \Rightarrow\ (\exists ph)\ell = tr_m(ph)$, p.219.

$\textbf{AxE}_\mathbf{00}$ $(m \stackrel{\odot}{\to} ph_1, ph_2) \ \Rightarrow\ v_m(ph_1) = v_m(ph_2)$, p.218.

$\textbf{AxE}_\mathbf{01}$ $v_m(ph) \neq 0$, p.218.

$\boxed{\textbf{Flxbasax}} \stackrel{\text{def}}{=} \textbf{Bax} + \textbf{AxE}_\mathbf{02} = \{\, \textbf{Ax1}, \textbf{Ax2}, \textbf{Ax3}_\mathbf{0}, \textbf{Ax4}, \textbf{Ax5}^\mathbf{Obs}, \textbf{Ax5}^\mathbf{Ph}, \textbf{Ax6}_\mathbf{00}, \textbf{Ax6}_\mathbf{01}, \textbf{AxE}_\mathbf{00}, \textbf{AxE}_\mathbf{01}, \textbf{AxE}_\mathbf{02}\}$ (cf. p.428), where:

$\textbf{AxE}_\mathbf{02}$ $(\forall m, k \in Obs)(\forall ph, ph_1 \in Ph)$
$$[(m \stackrel{\odot}{\to} ph \ \wedge\ k \stackrel{\odot}{\to} ph_1) \ \Rightarrow\ v_m(ph) = v_k(ph_1)],$$ p.427.

$\boxed{\textbf{Bax}^-} \stackrel{\text{def}}{=} (\textbf{Bax} \setminus \{\, \textbf{Ax5}^\mathbf{Obs}, \textbf{Ax5}^\mathbf{Ph}, \textbf{AxE}_\mathbf{00} \,\}) \cup \{\, \textbf{Ax5}_\mathbf{Obs}, \textbf{Ax5}_\mathbf{Ph}, \textbf{AxP1} \,\} = \{\, \textbf{Ax1}, \textbf{Ax2}, \textbf{Ax3}_\mathbf{0}, \textbf{Ax4}, \textbf{Ax5}_\mathbf{Obs}, \textbf{Ax5}_\mathbf{Ph}, \textbf{Ax6}_\mathbf{00}, \textbf{Ax6}_\mathbf{01}, \textbf{AxP1}, \textbf{AxE}_\mathbf{01} \,\}$ (cf. p.479), where:

AxP1 Intuitively, starting out from one point p of space-time, in every direction (forwards) there is at most one "speed of light" (i.e. photon-trace), formally:

$(\forall m \in Obs)(\forall ph_1, ph_2 \in Ph)(\forall d \in \mathsf{directions})$[1298] $\Big((ph_1$ and ph_2 are moving forwards in direction d as seen by m and $tr_m(ph_1) \cap tr_m(ph_2) \neq \emptyset) \ \Rightarrow$
$$tr_m(ph_1) = tr_m(ph_2)\Big),$$ p.472.

$\textbf{Ax5}_\mathbf{Ph}$ Intuitively, from any point p of space-time in any direction there is a photon moving forwards in that direction, cf. Fig.138 (p.477), formally:

$(\forall m \in Obs)(\forall p \in {}^nF)(\forall d \in \mathsf{directions})(\exists ph \in Ph)$
$[p \in tr_m(ph) \ \wedge\ (ph$ is moving forwards in direction d as seen by $m)]$, p.477.

[1298] Let us recall that directions are (nonzero) space-vectors, i.e. $\mathsf{directions} = {}^{n-1}F \setminus \{\bar{0}\}$, cf. p.470.

$\mathbf{Ax5_{Obs}}$ Intuitively: Let us fix an observer m, a direction d, and a point p of space-time. We will speak about things moving forwards in direction d through point p as seen by m (without mentioning all this data). Assume there is a photon moving in direction d. Then there is a photon in the same direction which is limiting in the following sense: For all speeds slower than this limiting photon, there is an observer moving with this speed, cf. Fig.139 (p.478). Formally:

$(\forall m \in Obs)(\forall p \in {}^nF)(\forall d \in \mathsf{directions})$
$\Big(\Big[(\exists ph \in Ph)(p \in tr_m(ph) \ \wedge \ (ph \text{ is moving forwards in } d \text{ as seen by } m)\Big] \ \Rightarrow$
$\Big[(\exists ph \in Ph)\Big(p \in tr_m(ph) \ \wedge \ (ph \text{ is moving forwards in } d \text{ as seen by } m) \ \wedge$
$(\forall \lambda \in F)(0 \leq \lambda < v_m(ph) \ \Rightarrow \ (\exists k \in Obs)(p \in tr_m(k) \ \wedge \ v_m(k) = \lambda \ \wedge$
$(k \text{ is moving forwards in direction } d \text{ as seen by } m))\Big)\Big]\Big)$, p.477.

$\boxed{\mathbf{Pax}} \stackrel{\text{def}}{=} \{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{Ax4}, \mathbf{Ax5_{Obs}}^{--}, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}} \,\}$ (cf. p.482) where:

§4-ben is kijavítani az intuitív szöveget! esetleg a formulaban is P síkot használni!

Ax1, Ax2, $\mathbf{Ax3_0}$, Ax4, $\mathbf{Ax6_{00}}$, $\mathbf{Ax6_{01}}$ have already been listed.

$\mathbf{Ax5_{Obs}}^{--}$ Intuitively, for each direction d there is a positive λ such that through any point there are observers moving forwards in direction d with all speeds smaller than λ. More precisely, for any observer m and for any plane P parallel with $\bar{t}$ there is $\lambda \in {}^+F$ such that for any straight line ℓ in P, with $ang^2(\ell) < \lambda$, ℓ is the trace of an observer (as seen by m, of course). In other words:

$(\forall m \in Obs)(\forall d \in \mathsf{directions})(\forall p \in {}^nF)(\exists \lambda \in {}^+F)$
$(\forall q \in {}^nF)\Big[space(p) - space(q) = \delta \cdot d \text{ for some } \delta \in F \ \Rightarrow \ (\forall 0 \leq \varepsilon < \lambda)$
$(\exists k \in Obs)(k \text{ moves forwards in direction } d \text{ with speed } \varepsilon \text{ and } q \in tr_m(k))\Big]$,
p.481.

* * *

Assume **Ax1**, **Ax2**, $\mathbf{Ax3_0}$, **AxP1**. Let $m \in Obs$. Then

$$c_m : {}^nF \times \mathsf{directions} \xrightarrow{\circ} F \cup \{\infty\}$$

is a partial function such that $c_m(p,d)$ is defined iff m sees a photon at point p moving forwards in direction d, <u>and</u> $c_m(p,d)$ is the speed of this photon,[1299] cf. pp. 473, 535.

[1299] There is only one such speed because of **AxP1**.

Let Th be one of our theories such that $Th \models \{\,\mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{AxP1}\,\}$. Then

$\boxed{Th^{\oplus}} \stackrel{\text{def}}{=} Th + c_m(p,d) < \infty$, p.643.

Next, we turn to listing the Reichenbachian versions of our theories. For this we recall some notation.

Assume $\mathbf{Bax}^-$. By Thm.4.3.17 (p.488), the speed $c_m(p,d)$ does not depend on p. This motivates the following:

$$\mathsf{c}_m(d) \stackrel{\text{def}}{=} c_m(\bar{0}, d),$$

cf. p.488. Intuitively, $\mathsf{c}_m(d)$ is the (square of the) speed of light in direction d as seen by observer m.

speed of light vagy square of the speed of light?

<u>Notation:</u> Let $m \in Obs$ and $d \in \mathsf{directions}$. Then

$$T_m(d) \stackrel{\text{def}}{=} \begin{cases} 1/\sqrt{\mathsf{c}_m(d)} & \text{if } 0 \neq \mathsf{c}_m(d) < \infty, \\ \infty & \text{if } \mathsf{c}_m(d) = 0, \\ 0 & \text{if } \mathsf{c}_m(d) = \infty, \end{cases}$$

cf. p.555. $T_m(d)$ is the reciprocal of the "speed of light", i.e. it is the time needed for a photon to cover the unit distance in direction d (as seen by observer m).

$\boxed{\mathbf{Reich_0(Bax)}} \stackrel{\text{def}}{=} \mathbf{Bax}^- + \mathbf{R(AxE_{00})}$ (cf. p.562), where

$\mathbf{R(AxE_{00})}$ $\mathbf{Ax}(\sqrt{\ })$ and
$(\forall d, d_1 \in \mathsf{directions})[T_m(d) + T_m(-d) = T_m(d_1) + T_m(-d_1)]$, p.557.

$\boxed{\mathbf{Reich_0(Flxbasax)}} \stackrel{\text{def}}{=} \mathbf{Bax}^- + \mathbf{R(AxE_{02})}$ (cf. p.562), where

$\mathbf{R(AxE_{02})}$ $(\forall m, k \in Obs)(\forall d, d_1 \in \mathsf{directions})$
$T_m(d) + T_m(-d) = T_k(d_1) + T_k(-d_1)$, and $\mathbf{Ax}(\sqrt{\ })$, p.557.

$\boxed{\mathbf{Reich_0(Newbasax)}} \stackrel{\text{def}}{=} \mathbf{Bax}^- + \mathbf{R(AxE)}$ (cf. p.562), where

$\mathbf{R(AxE)}$ $\mathbf{Ax}(\sqrt{\ })$ and
$(\forall m \in Obs)(\forall d \in \mathsf{directions}) T_m(d) + T_m(-d) = 2$, p.557.

$\boxed{\mathbf{Reich_0(Basax)}} \stackrel{\text{def}}{=} \mathbf{Reich_0(Newbasax)} + \mathbf{Ax6}$, p.562.

Let $Th \in \{\,\mathbf{Bax}, \mathbf{Flxbasax}, \mathbf{Newbasax}, \mathbf{Basax}\,\}$. Then

$\boxed{\mathbf{Reich}(Th)} \stackrel{\text{def}}{=} \mathbf{Reich_0}(Th) + \mathbf{R}_\triangle(E)$ (cf. p.576), where

$\mathbf{R}_\Delta(E)$ $\quad (\forall m \in Obs)(\exists r \in F)(\forall d_1, d_2, d_3 \in \mathsf{directions})\Big[d_1 + d_2 + d_3 = \bar{0} \;\Rightarrow$

$$\frac{|d_1| \cdot T_m(d_1) + |d_2| \cdot T_m(d_2) + |d_3| \cdot T_m(d_3)}{|d_1| + |d_2| + |d_3|} = r\Big], \text{ p.574.}$$

(2) Axioms concerning the direction of flow of time

The binary relation $\uparrow \subseteq Obs \times Obs$ is defined as follows.

$$m \uparrow k \;\stackrel{\text{def}}{\Longleftrightarrow}\; (\mathsf{f}_{km}(1_t)_t - \mathsf{f}_{km}(\bar{0})_t > 0), \text{ p.296.}$$

Intuitively, $m \uparrow k$ means that m sees k's clock running forwards. Further, if $m, k \in Obs$ then $m\ STL\ k$ means that m sees k moving slower than light (cf. Def.4.2.6 on p.460).

Ax(↑) $\quad (\forall m, m' \in Obs)\ (\ tr_m(m') = \bar{t} \quad \Rightarrow \quad m \uparrow m'\ )$, p.296.

Ax(↑↑) $(\forall m, k \in Obs) m \uparrow k$, p.426.

Ax(↑↑$_0$) $(\forall m, k \in Obs)\,(m \stackrel{\odot}{\to} k \;\to\; m \uparrow k)$, p.840.

Ax(↑↑$_{00}$) $(\forall m, k \in Obs)\,(m\ STL\ k \;\to\; m \uparrow k)$, p.840.

(3) Auxiliary axioms

Recall that

$$Triv = \{\, f \;:\; f \text{ is an isometry of } {}^n\mathbf{F} \text{ and } f(1_t) - f(\bar{0}) = 1_t \,\},$$

cf. p.135.

Ax(*Triv*) $\quad (\forall m \in Obs)(\forall f \in Triv)(\exists k \in Obs)\ \mathsf{f}_{mk} = f$, p.135.

Ax(*Triv$_t$*) $\quad (\forall m \in Obs)(\forall f \in Triv)\ \Big(f[\bar{t}] = \bar{t} \quad \Rightarrow \quad (\exists k \in Obs)\ \mathsf{f}_{mk} = f\Big)$, p.135.

Ax(*Triv$_t$*)$^-$ Assume we are given an observer m and a *Triv* transformation f that leaves the time-axis fixed. Then m has a brother, call it k, such that m thinks that (i) the coordinate axes of k are the f-images of the original coordinate axes $\bar{x}_i$, and (ii) the clock of k runs forwards, formally:

$(\forall m \in Obs)(\forall f \in Triv)\,[\, f[\bar{t}] = \bar{t} \quad \Rightarrow$
$\qquad (\exists k \in Obs)(\forall i \in n)(\,\mathsf{f}_{km}[\bar{x}_i] = f[\bar{x}_i] \quad \wedge \quad m \uparrow k\,)\,]$, p.812.

Ax(||) $(\forall m,k \in Obs)\Big(tr_m(k) \parallel \bar{t} \quad \Rightarrow \quad (\mathsf{f}_{mk} \text{ is an isometry})\Big)$, p.136.

Ax(||)$^-$ $(\forall m,k \in Obs \cap Ib)$
$[\, tr_m(k) \;=\; \bar{t} \quad \Rightarrow \quad (\mathsf{f}_{mk} \;=\; h \circ I, \quad \text{for some expansion } h \text{ and isometry } I)\,]$,
p.828.

Ax($\sqrt{\ }$) $(\forall 0 < x \in F)(\exists y \in F)\; y^2 = x$, p.91.

Ax(rc) (Axiom schema for real-closed fields)

Ax($\sqrt{\ }$) $+ \{\, \phi_{2n+1} \;:\; n \in \omega \,\}$, where

$$(\phi_n) \qquad \forall x_0 \ldots \forall x_n \exists y\, [x_n \neq 0 \;\rightarrow\; (x_0 + x_1 \cdot y + \ldots + x_n \cdot y^n = 0)],$$

p.301.

Ax(diswind) (Axiom of disjoint windows)

$(\forall m,k \in Obs \cap Ib)\; [(m \overset{\odot}{\rightarrow} ph \;\wedge\; k \overset{\odot}{\rightarrow} ph) \;\Rightarrow\; m \overset{\odot}{\rightarrow} k]$, p.812.

(4) Axioms concerning measuring distances

Ax(eqtime) Observers with common life-line agree on time-like distances, i.e.
$(\forall m, m' \in Obs)$
$$\Big(tr_m(m') = \bar{t} \quad \Rightarrow \quad (\forall p,q \in \bar{t}\,)\; |p-q| = |\mathsf{f}_{mm'}(p) - \mathsf{f}_{mm'}(q)|\Big),$$ p.127.

Ax(eqspace) Observers agree on spatial distances, i.e.
$(\forall m,k \in Obs)(\forall p,q \in {}^nF)$
$$\Big(\,(p_t = q_t \;\wedge\; \mathsf{f}_{mk}(p)_{\,t} = \mathsf{f}_{mk}(q)_{\,t}) \quad \Rightarrow \quad |p-q| = |\mathsf{f}_{mk}(p) - \mathsf{f}_{mk}(q)|\,\Big),$$
p.136.

Ax(eqm) Inertial observers agree on distances, i.e.
$(\forall m,k \in Obs \cap Ib)(\forall i,j \in n)(\forall p,q \in \bar{x}_i)(\forall p',q' \in \bar{x}_j)$
$$\Big([w_m(p) = w_k(p') \;\wedge\; w_m(q) = w_k(q')] \;\Rightarrow\; |p-q| = |p'-q'|\Big),$$ p.796.

(5) Axiom systems $\mathbf{Pax^+}$, $\mathbf{Pax^{++}}$, $\mathbf{Pax^+_+}$, $\mathbf{Pax^{++}_+}$, **Wax**, $\mathbf{Wax^+}$

Ax(Bw) $(\forall m, k \in Obs)[m \overset{\odot}{\to} k \;\Rightarrow\; (\mathsf{f}_{mk}$ is betweenness preserving$)\,]$, p.1028.

Ax♡ $\quad B = Obs \cup Ph$, p.296.

Ax(∞ph) $(\forall m \in Obs)(\forall ph, ph' \in Ph)\Big(\,[\,\bar{0} \in tr_m(ph) \cap tr_m(ph') \;\wedge\; (\,ph$ and ph' move in the same direction as seen by $m\,) \wedge v_m(ph) = \infty\,] \;\to\; v_m(ph') = \infty\Big)$, p.1028.

sorrend?

Intuitively, no observer can emit simultaneously in the same direction two photons one with infinite speed and the other one with finite speed.

Ax(ext) $\quad (\forall m, k \in Obs)\,[w_m = w_k \;\Rightarrow\; m = k] \quad \wedge$
$\quad (\forall b, b_1 \in B \setminus Obs)(\forall m \in Obs)\,[tr_m(b) = tr_m(b_1) \;\Rightarrow\; b = b_1]$,
p.298.

Ax(Ph) $(\forall m \in Obs)(\forall p \in {}^nF)(\exists ph_1, ph_2 \in Ph)\, tr_m(ph_1) \cap tr_m(ph_2) = \{p\}$, p.1073.

$\boxed{\mathbf{Pax^+}} \overset{\text{def}}{=} \mathbf{Pax} + \mathbf{AxE_{01}} + \mathbf{Ax(Bw)} + \mathbf{Ax(\infty \mathit{ph})} +$
$\Big([\,\mathbf{Ax(eqtime)} \;\wedge\; (\forall m, k \in Obs)(\forall 0 < i \in \omega)\, tr_m(k) \neq \bar{x}_i\,] \quad \vee \quad \mathbf{Ax(eqm)}\Big)$,
p.1029.

$\boxed{\mathbf{Pax^{++}}} \overset{\text{def}}{=} \mathbf{Pax^+} + \mathbf{Ax(eqm)} + \mathbf{Ax(ext)} + \mathbf{Ax}\heartsuit$, p.1081.

$\boxed{\mathbf{Pax^+_+}} \overset{\text{def}}{=} \mathbf{Pax^+} + \mathbf{Ax(diswind)}$, p.1086.

$\boxed{\mathbf{Pax^{++}_+}} \overset{\text{def}}{=} \mathbf{Pax^{++}} + \mathbf{Ax(diswind)}$, p.1093.

$\boxed{\mathbf{Wax}} \overset{\text{def}}{=} \{\,\mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}, \mathbf{Ax4}, \mathbf{Ax6}, \mathbf{Ax(Bw)}, \mathbf{Ax(\mathit{Ph})}\,\}$, p.1073.

$\boxed{\mathbf{Wax^+}} \overset{\text{def}}{=} \mathbf{Wax} + \mathbf{Ax(ext)} + \mathbf{Ax}\heartsuit + \mathbf{Ax(\infty \mathit{ph})} + (\forall m, k)(\mathsf{f}_{mk} \in \mathit{Aftr})$, p.1081.

(6) Symmetry axioms

Ax(symm$_0$) $(\forall m,k\in Obs)(\exists m',k'\in Obs)$
$$\Big(tr_m(m')=tr_k(k')=\bar t \quad\wedge\quad \mathsf{f}_{mk}=\mathsf{f}_{k'm'}\Big),$$ p.124.

Ax(symm) is defined to be **Ax(symm$_0$)** + **Ax(eqtime)**, p.127.

Ax(syt$_0$) $(\forall m,k\in Obs)\Big(tr_m(k)\neq\emptyset \quad\Rightarrow$
$(\forall p\in\bar t\,)\ |\mathsf{f}_{mk}(p)_t-\mathsf{f}_{mk}(\bar 0)_t|=|\mathsf{f}_{km}(p)_t-\mathsf{f}_{km}(\bar 0)_t|\Big)$, p.134.

Ax(syt$_{00}$) $(\forall m,k\in Obs)\ [\mathsf{f}_{mk}(\bar 0)=\bar 0\ \Rightarrow\ |\mathsf{f}_{mk}(1_t)_t|=|\mathsf{f}_{km}(1_t)_t|]$, p.138.

Ax(syt)* $\mathsf{f}_{mk}(\bar 0)=\bar 0\ \Rightarrow\ \mathsf{f}_{mk}(1_t)_t=\mathsf{f}_{km}(1_t)_t$, p.721.

Ax(syx)* (m,k are in pre-standard configuration[1300]) $\Rightarrow$ $|\mathsf{f}_{mk}(1_x)_x| = |\mathsf{f}_{km}(1_x)_x|$, p.725.

Ax(speedtime) $(\forall m,k,m',k'\in Obs)\ \Big(v_m(k)=v_{m'}(k')\quad\Rightarrow$
$(\forall p\in\bar t\,)\ |\mathsf{f}_{mk}(p)_t-\mathsf{f}_{mk}(\bar 0)_t|=|\mathsf{f}_{m'k'}(p)_t-\mathsf{f}_{m'k'}(\bar 0)_t|\Big)$, p.137.

Ax□1 $(\forall m,k,m'\in Obs)(\exists k'\in Obs)\mathsf{f}_{mk}=\mathsf{f}_{m'k'}$, p.350.

Ax□2 $(\forall m,k,m',k'\in Obs)(tr_m(k)=tr_{m'}(k')\to$
there is an isometry N of ${}^n\mathbf{F}$ such that $N[\bar t\,]\parallel\bar t$ and $\mathsf{f}_{mk}=\mathsf{f}_{m'k'}\circ N)$, p.350.

Ax△1 $(\forall m,k\in Obs)(\exists k'\in Obs)(tr_m(k)=tr_m(k')\ \wedge\ \mathsf{f}_{mk'}=\mathsf{f}_{k'm})$, p.351.

Ax△2 $(\forall m,k\in Obs)$ (there is an isometry N of ${}^n\mathbf{F}$ such that
$N[\bar t\,]\parallel\bar t$ and $\mathsf{f}_{mk}=N\circ\mathsf{f}_{km}\circ N)$, p.351.

Ax(ω)^0 is defined to be the <u>disjunction</u> of the following symmetry axioms: **Ax(syt$_0$)**, **Ax(symm)**, **Ax(speedtime)**, **Ax△1+Ax(eqtime)**, **Ax△2**, **Ax□1+Ax(eqtime)**, **Ax□2**, p.844.

Ax(ω)^{00} is defined to be the <u>disjunction</u> of the following symmetry axioms: **Ax(ω)^0**, **Ax(eqspace)**, **Ax(eqm)+Ax($Triv_t$)$^-$**, p.844.

Ax(ω)$^\sharp$ is defined to be **Ax(ω)^0+Ax($Triv_t$)$^-$+Ax($\sqrt{\ }$)**, p.844.

[1300] m and k are said to be in *pre-standard configuration* iff $\mathsf{f}_{mk}(\bar 0)=\bar 0$ and $\mathsf{f}_{mk}[\mathsf{Plane}(\bar t,\bar x)]=\mathsf{Plane}(\bar t,\bar x)$. Cf. Def.4.6.5 (p.602) and Fig.201 (p.603).

Ax$(\omega)^{\sharp\sharp}$ is defined to be **Ax**$(\omega)^{00}$ + **Ax**$(\mathit{Triv}_t)^-$ + **Ax**$(\sqrt{\ })$, p.844.

Ax(symm)† is defined to be **Ax(symm)** + **Ax**(Triv) + **Ax**$(\|)$, p.151.

Ax(ω) **Ax□1** $\wedge$ **Ax□2** $\wedge$ **Ax△1** $\wedge$ **Ax△2**, p.351.

Ax(ω^-) **Ax□1** $\vee$ **Ax□2** $\vee$ **Ax△1** $\vee$ **Ax△2**, p.351.

(7) Symmetry axioms adequate for Reichenbachian theories

$\mathbf{R}^+$**(Ax eqsp)** Intuitively, the thickness of spaceships do not change in the direction orthogonal to movement (cf. pp. 608–614), formally:

Assume $m, k \in \mathit{Obs}$ such that $m \overset{\odot}{\to} k$. Assume P, Q are parallel planes of ${}^nF$ such that they are parallel with both $\bar{t}$ and $tr_m(k)$. Then

$$\begin{aligned} \mathsf{Eudist}(P,Q) &= \mathsf{Eudist}(\mathsf{f}_{mk}[P], \mathsf{f}_{mk}[Q]), \text{ p.614, where} \\ \mathsf{Eudist}(H,H_1) &\stackrel{\text{def}}{=} \inf\{\, \|p-q\| \;:\; p \in H \text{ and } q \in H_1 \,\}\,. \end{aligned}$$

R(Ax eqsp) Intuitively, the thickness of spaceships do not change in the direction orthogonal to movement (cf. pp. 608–614), formally:

Assume m and k are in pre-standard configuration[1301]. Let P be a (2-dimensional) plane parallel with $\mathsf{Plane}(\bar{t},\bar{x})$. Then the distance between P and $\mathsf{Plane}(\bar{t},\bar{x})$ is the same as the distance between $\mathsf{f}_{mk}[P]$ and $\mathsf{f}_{mk}[\mathsf{Plane}(\bar{t},\bar{x})]$. Formally,

$$\begin{aligned} \mathsf{Eudist}(P,\mathsf{Plane}(\bar{t},\bar{x})) &= \mathsf{Eudist}(\mathsf{f}_{mk}[P], \mathsf{f}_{mk}[\mathsf{Plane}(\bar{t},\bar{x})]), \text{ p.611, where} \\ \mathsf{Eudist}(H,H_1) &\stackrel{\text{def}}{=} \inf\{\, \|p-q\| \;:\; p \in H \text{ and } q \in H_1 \,\}\,, \text{ cf. p.609.} \end{aligned}$$

See Fig.205 on p.611.

R(Ax syt$_0$) Intuitively m and k *literally* see, via photons, each other's clocks slowing down with the same rate, see Fig.207 (p.616), formally:

$(\forall m,k \in \mathit{Obs})[\mathsf{f}_{mk}(\bar{0}) = \bar{0} \;\Rightarrow$
$\qquad (\forall p \in \bar{t}\,)|\mathit{view}_m(\mathsf{f}_{km}(p))| = |\mathit{view}_k(\mathsf{f}_{mk}(p))|]$ (cf. p.615),

where $\mathit{view}_m \stackrel{\text{def}}{=} \{\, \langle p,q\rangle \in {}^nF \times \bar{t} \;:\; p_t \le q_t \text{ and } (\exists ph \in \mathit{Ph})\; p,q \in tr_m(ph) \,\}$, cf. Fig.206 (p.615).

R(sym) is defined to be **R(Ax eqsp)**+ **R(Ax syt$_0$)**, p.616.

[1301] Cf. footnote 1300 on p.1260 for the notion of a pre-standard configuration.

(8) Twin paradox

Let $m, k \in Obs$. Then $m\ STL\ k$ means that m sees k moving slower than light, cf. Def.4.2.6 on p.460 for details.

Ax(TwP) $(\forall m, k_1, k_2 \in Obs)(\forall p, q, r \in {}^nF)$
$\Big([\, m\ STL\ k_1 \quad \wedge \quad m\ STL\ k_2 \quad \wedge \quad p_t < q_t < r_t \quad \wedge$
$\{p\} = tr_m(m) \cap tr_m(k_1) \ \wedge\ \{q\} = tr_m(k_1) \cap tr_m(k_2) \ \wedge\ \{r\} = tr_m(m) \cap tr_m(k_2)\,] \ \Rightarrow$
$|p_t - r_t| > |\mathsf{f}_{mk_1}(p)_t - \mathsf{f}_{mk_1}(q)_t| + |\mathsf{f}_{mk_2}(q)_t - \mathsf{f}_{mk_2}(r)_t| \ \Big)$, p.460
(cf. Fig.43 on p.141).

The *existential version* **Ax(∃TwP)** of the twin paradox is defined as follows. First, let us notice that **Ax(TwP)** is a formula of the pattern

$$(\forall m \ldots)(\forall p \ldots)\Big([\ldots] \ \Rightarrow |\ldots| > |\ldots|\Big).$$

Let ψ_1, ψ_2 be formulas such that **Ax(TwP)** is the formula

$$(\forall m, k_1, k_2 \in Obs)(\forall p, q, r \in {}^nF)(\psi_1 \Rightarrow \psi_2).$$

§4.2 **Ax(∃TwP)** def-ját kijavítani: $\forall p, q, r$ helyett $\exists p, q, r$ kellene hogy legyen.

Now we define **Ax(∃TwP)** as follows.

Ax(∃TwP) $(\exists m, k_1, k_2 \in Obs)(\exists p, q, r \in {}^nF)[\psi_1 \ \wedge\ \psi_2]$, p.461.

(9) Axiom systems Specrel, Flxspecrel, BaCo, Compl, NewtK⁻, NewtK

$\boxed{\textbf{Specrel}} \overset{\text{def}}{=}$ **Basax** + **Ax(symm)**†, p.151.

$\boxed{\textbf{Flxspecrel}} \overset{\text{def}}{=}$ **Bax** + **Ax6** + **Ax(symm)**† + **AxE$_{02}$**, p.428.

$\boxed{\textbf{Compl}} \overset{\text{def}}{=}$ { **Ax(symm)**, **Ax♡**, **Ax(↑)**, **Ax5$^+$**, **Ax(ext)**, **Ax(**$Triv_t$**)** } (cf. p.298), where

Ax5$^+$ $\quad \ell \in \mathsf{SlowEucl} \quad \Rightarrow \quad (\exists k \in Obs)\ (\ell = tr_m(k) \quad \wedge \quad m \uparrow k)$, p.297.

$\boxed{\textbf{BaCo}} \overset{\text{def}}{=}$ **Basax** + **Compl**, p.298.

$\boxed{\textbf{NewtK}^-} \overset{\text{def}}{=}$ **Bax** + **Ax6** + **Ax(symm)**† + $(\forall m \in Obs) c_m = \infty$ (cf. p.426), where c_m is the speed of light for observer m, assuming **Bax**.

$\boxed{\textbf{NewtK}} \overset{\text{def}}{=}$ **NewtK$^-$** + **Ax(↑↑)** + **Ax□1**, p.426.

(10) Geometrical axioms and axiom systems

Axioms $\mathbf{A_0}$–$\mathbf{A_4}$ and $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{Pa}$ below apply to geometries with reducts $\langle Mn;\, Bw\rangle$ or $\langle Mn;\, coll\rangle$. In the case of "$\langle Mn;\, Bw\rangle$" *coll* is a defined relation, cf. p.818. The new universe (or sort) *lines* is (explicitly) defined over $\langle Mn;\, coll\rangle$ on p.1037. For $H \subseteq Mn$, $Plane'(H)$ is intuitively the "n-long closure of H under *coll*" (cf. Def.6.2.15 on p.819), where throughout n is the dimension of our geometry and $n \geq 2$.

$\mathbf{A_0}$ $(\forall a, b, c \in Mn)[\, coll(a, b, c) \leftrightarrow (\exists \ell \in lines)\, a, b, c \in \ell\,]$, p.1038.

$\mathbf{A_1}$ $(\forall a, b \in Mn)(\, a \neq b \;\rightarrow\; (\exists! \ell \in lines)\, a, b \in \ell\,)$, p.1038.

$\mathbf{A_2}$ Intuitively, if H is a less than $n+2$ element subset of Mn then the "n-long closure" $Plane'(H)$ of H under *coll* will be closed under *coll*, hence the plane $Plane(H)$ generated by H coincides with $Plane'(H)$ (cf. Def.6.2.15, p.819), formally:

$$(\forall H \subseteq Mn)$$
$$\Big(\big(\, |H| \leq n{+}1 \;\wedge\; a, b \in Plane'(H) \;\wedge\; coll(a, b, c)\,\big) \;\rightarrow\; c \in Plane'(H)\Big), \text{ p.1039.}$$

$\mathbf{A_3}$ Intuitively, if $i \leq n$ and H is an $i{+}1$ element independent subset[1302] of Mn then there is exactly one i-dimensional plane[1303] that contains H, formally:

$$(\forall H, H' \subseteq Mn)\Big(\big(\, |H| = |H'| \leq n{+}1 \;\wedge\; \text{(both } H \text{ and } H' \text{ are independent)} \;\wedge$$
$$H \subseteq Plane'(H')\,\big) \;\rightarrow\; Plane'(H) = Plane'(H')\Big), \text{ p.1039.}$$

$\mathbf{A_4}$ Mn is an n dimensional plane, p.1039.

In connection with axioms $\mathbf{P_1}$, $\mathbf{P_2}$ below we note that the relation of parallelism $\parallel$ on *lines* is defined the usual way in Def.6.6.19 on p.1039.

$\mathbf{P_1}$ (Euclid's axiom)

$$(\forall \ell \in lines)(\forall a \in Mn)(\exists! \ell' \in lines)(a \in \ell' \;\wedge\; \ell \parallel \ell'), \text{ p.1040.}$$

$\mathbf{P_2}$ $(\ell \parallel \ell' \;\wedge\; \ell' \parallel \ell'') \;\rightarrow\; \ell \parallel \ell''$, p.1040.

[1302] Let $H \subseteq Mn$. H is called *independent* iff $(\forall e \in H)\, e \notin Plane'(H \setminus \{e\})$, cf. p.1039.

[1303] Let $P \subseteq Mn$. P is called an i-dimensional iff there is an $i+1$ element independent subset H of Mn such that $Plane'(H) = P$, where for the notion of an independent subset cf. footnote 1302. Cf. Def.6.6.18(ii) on p.1039.

$\boxed{\mathbf{ag}}$ $\stackrel{\text{def}}{=}$ $\{\mathbf{A_0}, \mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}, \mathbf{A_4}, \mathbf{P_1}, \mathbf{P_2}\}$, p.1040. **ag** is the axiom system for _affine geometries_.

For $a, b, c, d \in Mn$, the abbreviation $\langle a, b\rangle \parallel \langle c, d\rangle$ means that $a \neq b$, $c \neq d$, and there are $\ell, \ell' \in lines$ such that $a, b \in \ell$, $c, d \in \ell'$ and $\ell \parallel \ell'$, cf. p.1042.

Pa (Pappus-Pascal property)

$(\forall \ell, \ell' \in lines)(\forall a, b, c \in \ell \setminus \ell')(\forall a', b', c' \in \ell' \setminus \ell)$

$$[\,(\,\langle a, b'\rangle \parallel \langle a', b\rangle \;\;\wedge\;\; \langle a, c'\rangle \parallel \langle a', c\rangle\,) \quad\rightarrow\quad \langle b, c'\rangle \parallel \langle b', c\rangle\,],$$

see Fig.319, p.1042.

$\boxed{\mathbf{pag}}$ $\stackrel{\text{def}}{=}$ $\mathbf{ag} + \mathbf{Pa}$, p.1042. **pag** is the axiom system for _Pappian affine geometries_.

Axioms $\mathbf{B_1}$–$\mathbf{B_3}$ below apply to geometries with reducts $\langle Mn;\, Bw\rangle$. (*coll* is a defined relation.)

$\mathbf{B_1}$ $Bw(a, b, c) \;\rightarrow\; (\,a \neq b \neq c \neq a \;\wedge\; Bw(c, b, a) \;\wedge\; \neg Bw(b, a, c)\,)$, p.1043.

$\mathbf{B_2}$ $a \neq b \;\rightarrow\; (\exists c) Bw(a, b, c)$, p.1043.

$\mathbf{B_3}$ (Pasch's Law)

Intuitively, if a line ℓ lies in the plane determined by a triangle abc, and passes between a and b but not through c, then ℓ passes between a and c, or between b and c, formally:

$(\,\neg coll(a, b, c) \;\wedge\; \ell \subseteq Plane'(\{a, b, c\}) \;\wedge\; (\exists d \in \ell) Bw(a, d, b)\,) \;\rightarrow$
$(\exists e \in \ell)(Bw(a, e, c) \;\vee\; Bw(b, e, c))$, p.1043 (cf. Fig.320 on p.1044).

$\boxed{\mathbf{opag}}$ $\stackrel{\text{def}}{=}$ $\mathbf{pag} + \{\mathbf{B_1}, \mathbf{B_2}, \mathbf{B_3}\}$, p.1044. **opag** is the axiom system for _ordered Pappian affine geometries_.

Axioms $\mathbf{L_1}, \ldots, \mathbf{L_{10}}$ below apply to geometries with reducts

$$\langle Mn, L;\; L^T, L^{Ph}, L^S, \in, \prec, Bw, \perp_r, eq\rangle.$$

Further, *coll* is a defined relation and it is defined over $\langle Mn;\, Bw\rangle$, cf. p.818, and *lines* is the new sort firts-order defined from *coll*.

$\mathbf{L_1}$ $L \subseteq lines$, p.1071.

$\mathbf{L_2}$ $(\forall a \in Mn)(\exists \ell, \ell' \in L^{Ph})\, \ell \cap \ell' = \{a\}$, p.1071.

$\boxed{\textbf{lopag}} \stackrel{\text{def}}{=} \textbf{opag} + \mathbf{L_1} + \mathbf{L_2}$, p.1071.

$\mathbf{L_3}$ $(\,[\, a \prec b \;\wedge\; (\, Bw(a,b,c) \vee Bw(a,c,b)\,)\,] \;\rightarrow\; a \prec c\,) \;\wedge$
$(\,[\, a \prec b \;\wedge\; (\, Bw(c,a,b) \vee Bw(a,c,b)\,)\,] \;\rightarrow\; c \prec b\,)$, p.1076.

$\mathbf{L_4}$ Intuitively, eq is (very) symmetric, formally:
$\langle a,b\rangle \; eq \; \langle c,d\rangle \quad \rightarrow \quad (\,\langle c,d\rangle \; eq \; \langle a,b\rangle \;\wedge\; \langle b,a\rangle \; eq \; \langle c,d\rangle \;\wedge\; \langle a,a\rangle \; eq \; \langle c,c\rangle\,)$,
p.1076.

$\mathbf{L_5}$ eq is transitive, i.e.
$(\,\langle a,b\rangle \; eq \; \langle c,d\rangle \;\wedge\; \langle c,d\rangle \; eq \; \langle e,f\rangle\,) \quad \rightarrow \quad \langle a,b\rangle \; eq \; \langle e,f\rangle$, p.1076.

$\mathbf{L_6}$ (For the intuitive meaning of this axiom see Fig.326 on p.1076.)
$(\forall \ell, \ell' \in L)(\forall o, e, e', a, a' \in Mn)\Big(\,[\, \ell \cap \ell' = \{o\} \;\wedge\; e, a \in \ell \;\wedge\; e', a' \in \ell' \;\wedge$
$\langle e, e'\rangle \parallel \langle a, a'\rangle \;\wedge\; \langle o, e\rangle \; eq \; \langle o, e'\rangle\,] \quad \rightarrow \quad \langle o, a\rangle \; eq \; \langle o, a'\rangle\,\Big)$, p.1076.

$\mathbf{L_7}$ (For the intuitive meaning of this axiom see Fig.327 on p.1077.)
$(\forall \ell \in L^T \cup L^S)(\forall a,b,c,d,e,f \in Mn)\,[\,(\, a,b,c,d \in \ell \;\wedge$
$\langle a,b\rangle \parallel \langle e,f\rangle \parallel \langle c,d\rangle \;\wedge\; \langle a,e\rangle \parallel \langle b,f\rangle \;\wedge\; \langle c,e\rangle \parallel \langle d,f\rangle\,) \;\rightarrow\; \langle a,b\rangle \; eq \; \langle c,d\rangle\,]$,
p.1076.

$\mathbf{L_8}$ $\perp_r$ is symmetric, i.e.
$(\forall \ell, \ell' \in L)\,(\ell \perp_r \ell' \;\rightarrow\; \ell' \perp_r \ell)$, p.1076.

$\mathbf{L_9}$ $\perp_r$ is closed under parallelism, i.e.
$(\forall \ell, \ell_1, \ell_2 \in L)\,[\,(\, \ell \perp_r \ell_1 \;\wedge\; \ell_1 \parallel \ell_2\,) \quad \rightarrow \quad \ell \perp_r \ell_2\,]$, p.1076.

$\mathbf{L_{10}}$ $\perp_r$ is closed under taking limits, p.1077.

$\boxed{\textbf{lopag}^+} \stackrel{\text{def}}{=} \textbf{lopag} + \mathbf{L_3} + \mathbf{L_4} + \mathbf{L_5} + \mathbf{L_6} + \mathbf{L_7} + \mathbf{L_8} + \mathbf{L_9} + \mathbf{L_{10}}$, p.1081

(11) "Speed of light free" axiom systems for relativity (axioms and axiom systems used in Chapter 5)

$\boxed{\textbf{Relnoph}_\mathbf{0}} \stackrel{\text{def}}{=}$ $(\mathbf{Ax1}\text{–}\mathbf{Ax4})$[1304] $+ \mathbf{Ax6} + \mathbf{Ax\Box 1} + \mathbf{Ax\triangle 1} + \mathbf{Ax}(\sqrt{\ }) + \mathbf{Ax}(\mathit{Triv}) + \mathbf{Ax}(\|)$, p.705.

Ax(5nop) $\forall m, k\ (\forall \lambda \in {}^+F)[\lambda < v_m(k) \ \Rightarrow\ \exists k'\ (v_m(k') = \lambda)]$, p.706.

The intuitive idea of **Ax(5nop)** is that if a certain speed is realized by some observer then the *smaller speeds* are also realized by some observers.

$\boxed{\textbf{Relnoph}} \stackrel{\text{def}}{=}$ $\textbf{Relnoph}_\mathbf{0}$ + **Ax(5nop)**, p.707.

Ax(group$^+$) $(\forall m, k, m', k' \in \mathit{Obs})(\exists k'' \in \mathit{Obs})\mathsf{f}_{mk} \circ \mathsf{f}_{m'k'} = \mathsf{f}_{mk''}$, p.410.

Ax(syt)* $\mathsf{f}_{mk}(\bar{0}) = \bar{0} \ \Rightarrow\ \mathsf{f}_{mk}(1_t)_t = \mathsf{f}_{km}(1_t)_t$, p.721.

Ax(syx)* (m, k are in pre-standard configuration)[1305] $\Rightarrow$ $|\mathsf{f}_{mk}(1_x)_x| = |\mathsf{f}_{km}(1_x)_x|$, p.725.

$(*1)$–$(*3)$ below are also potential axioms[1306] (or principles) connected with **Relnoph**.

$(*1)$ The sum of finitely many small positive velocities is not infinite, formally: Let $j \in \omega$. Let $m_0, \dots, m_j \in \mathit{Obs}$. Assume m_i, m_{i+1} are in strict standard configuration[1307] and $v_{m_i}(m_{i+1}) < 1$, for all $i < j$. Then $v_{m_0}(m_j) \neq \infty$, p.710.

$(*2)$ The sum of finitely many small positive velocities is nonnegative, formally: Let $j \in \omega$. Let $m_0, \dots, m_j \in \mathit{Obs}$. Assume m_i, m_{i+1} are in strict standard configuration and $v_{m_i}(m_{i+1}) < 1$, for all $i < j$. Then m_0 sees m_j moving forwards in direction 1_x, p.710.

[1304] **Ax1**, **Ax2**, **Ax3**, **Ax4**.

[1305] Cf. footnote 1300 on p.1260 for the notion of a pre-standard configuration.

[1306] $(*1)$–$(*3)$ are schemas of formulas.

[1307] Assume $m, k \in \mathit{Obs}$. Then m and k are defined to be in *strict standard configuration* if they are in standard configuration, m sees k moving forwards in direction 1_x, k sees m moving backwards in direction 1_x, further $[v_m(k) = 0 \ \Rightarrow\ \mathsf{f}_{km}(1_t)_t \cdot \mathsf{f}_{km}(1_x)_x > 0]$ and $[v_m(k) = \infty \ \Rightarrow\ (\mathsf{f}_{km}(1_t)_x > 0 \ \&\ \mathsf{f}_{km}(1_x)_t < 0)]$. Cf. Def.5.0.42 (p.709) and Remark 5.0.43 (p.709).

(∗3) The sum of finitely many small positive velocities is nonzero, formally: Let $0 < j \in \omega$. Let $m_0, \ldots, m_j \in Obs$. Assume m_i, m_{i+1} are in strict standard configuration and $0 < v_{m_i}(m_{i+1}) < 1$, for all $i < j$. Then $v_{m_0}(m_j) \neq 0$, p.710.

"**Flxspecrel**", the notation $\mathfrak{M} \models$ **"Flxspecrel"** was introduced on p.708.

Ax(natu) $(*1) \vee (*2) \vee (*3)$,[1308] p.752

Ax(natu)$^+$ $(\forall m, k, k' \in Obs)\sqrt{v_m(k')} \leq \sqrt{v_m(k)} + \sqrt{v_k(k')}$, p.753.

Ax(∃body) $\forall m, k\ (\forall \ell \in \mathsf{Eucl})[(v_m(k) > 0\ \wedge\ \mathsf{f}_{mk}[\ell] = \ell)\ \Rightarrow\ (\exists b \in B) tr_m(b) = \ell]$, p.757.

Ax(5nop)$^-$ $\forall m\ (\exists c \in {}^+F)(\forall \lambda \in {}^+F)[\lambda < c\ \Rightarrow\ (\exists k) v_m(k) = \lambda]$, p.761.

$\boxed{\textbf{Relnoph}^-}$ is obtained from **Relnoph** by replacing **Ax(5nop)** with **Ax(5nop)$^-$**, p.761.

$\boxed{\textbf{Bax}^-_{\textbf{nobs}}} \stackrel{\text{def}}{=} \textbf{Bax}^- \setminus \{\textbf{Ax5}_{\textbf{Obs}}\} + \textbf{Ax(5nop)}^-$, p.762.

$\boxed{\textbf{Relnoph}^{--}} \stackrel{\text{def}}{=} \textbf{Relnoph}^- \setminus \{\textbf{Ax}\triangle\textbf{1}\} + \textbf{Ax(symm)}$, p.764.

egyezetetni §4.3-ben levő definicióval

(12) The cone-smooth versions of our theories

Assume **Ax1**, **Ax2**, **Ax3$_0$**, **AxP1**. Let $m \in Obs$. Then

$$c_m : {}^nF \times \mathsf{directions} \xrightarrow{\circ} F \cup \{\infty\}$$

is a partial function such that $c_m(p, d)$ is defined iff m sees a photon at point p moving forwards in direction d, <u>and</u> $c_m(p, d)$ is the speed of this photon,[1309] cf. pp. 473, 535. Further, for any $m \in Obs$ and $d \in \mathsf{directions}$

$$\mathsf{c}_m(d) \stackrel{\text{def}}{=} c_m(\bar{0}, d),$$

[1308] **Ax(natu)** is a schema of formulas.

[1309] There is only one such speed because of **AxP1**.

cf. p.488. Hence, c_m : $\mathsf{directions} \overset{\circ}{\longrightarrow} F \cup \{\infty\}$ is a partial function. (**Ax(consm)** below will imply that it is not partial. The notion of strongly continuous functions, partial derivatives etc. used in **Ax(consm)** below are formulated in our frame language on pp. 536, 518.) Now,

Ax(consm) $\stackrel{\text{def}}{=}$ **Ax(cnsm$_0$)** + **Ax(cnsm$_1$)** + **Ax(cnsm$_2$)** (cf. p.518), where

Ax(cnsm$_0$) c_m is a strongly continuous function defined on $\mathsf{directions}$, p.519.

Ax(cnsm$_1$) For all $0 < i < n$, the partial derivative $(\partial_i \mathsf{c}_m) : \mathsf{directions} \longrightarrow F$ is everywhere defined on the domain $\mathsf{directions}$, p.519.

Ax(cnsm$_2$) For all $0 < i < n$, $\partial_i \mathsf{c}_m$ is strongly continuous on the domain $\mathsf{directions}$, p.519.

Let Th be one of our theories such that $Th \models \{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{AxP1} \,\}$. Then $\boxed{Th_\partial} \stackrel{\text{def}}{=} Th + \mathbf{Ax(consm)}$ is called the *cone-smooth version* of the theory Th, cf. p.521.

(13) Axioms and axiom systems not listed in this list

Ax$_{\text{OF}}$, p.30.
Ax$_{\text{G}}$, p.31. (**Ax**$_{\text{OF}}$ and **Ax**$_{\text{G}}$ are assumed throughout the present work.)
Ax1', p.45.
$\mathbf{AxE_1}$, p.223.
$\mathbf{AxE_2}$, p.223.
$\mathbf{Ax2_1}$, p.223.
$\mathbf{Ax5_1}$, p.223.
$\boxed{\mathbf{Relphax}}$, p.223.
Ax□1', p.354.
Ax△2*, p.359.
Ax△3, p.406.
Ax(isotropy), p.399.
Ax(isotropy'), p.400.
Ax(isotropy$^-$), p.401.
homogeneity **Ax(homogenity)**, p.405.
Ax(group$^-$), p.410.
Ax(group), p.410.
$\boxed{\mathbf{Flxspecrel}^+}$, p.425.

Credits

Figures 258, 261, 333 are created from works by M. C. Escher, which appeared in D. R. Hofstadter: "Gödel, Escher, Bach", Hungarian translation, Typotex Kiadó, 1998. Figure 355 (representing Gödel's rotating universe) and Figure 290 are created from figures in S. W. Hawking and G. F. R. Ellis [126]. Figure 281 (view from near a black hole) is created from a figure in K. S. Thorne [258].

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