The marriage of groups and Boolean algebras.

In memory of Steven Givant

Talk by H. Andréka and I. Németi

Renyi Institute, lecture room on the 3rd floor

December 21, 2018. 15.00—17.00

Put together a group and a Boolean algebra and you get a relation algebra. Typically, the complexes (i.e., subsets) of a group form a relation algebra. The representation theory of relation algebras is hard, due to the fact that the whole of axiomatic mathematics can be expressed in the equational theory of relation algebras ([TG]). The talk will be about a long-term joint work with Steve Givant, solution of a recent problem published by Roger Maddux, and an old vision of Steve Comer.

In more detail:

We call a relation algebra whose diagonal is the supremum of groups, in an appropriate sense, a group relation algebra. Group relation algebras are amazingly beautiful. The fact that the diagonal is a union of groups, implies that the whole algebra is put together from various quotients of these. This makes them seducingly close to being representable as relation algebras (by using the Cayley representation of groups). Indeed, it is quite surprising that there are nonrepresentable group relation algebras. In the talk we will show the smallest nonrepresentable one (it has 120 atoms). Here is where an old vision of Steve Comer enters the picture. He suggested that part of relation algebra be made via (poly)groups, and indeed our construction is more perspecuous if stripped of Boolean algebra structure and told in group theory terms only.

Let V be the variety that the completions of representable relation algebras generate. Maddux [M] shows that V contains infinitely many nonrepresentable finite relation algebras. He asks whether V is the class RA of all relation algebras. In turn, we show that our infinitely many finite nonrepresentable group relation algebras are all outside of V. This settles his problem negatively. In this proof, the Boolean algebra structure of relation algebras plays the main role, so much so that the proof can be told for any ordered discriminator varieties.

[AGN] Andreka, H., Givant, S. R., and Nemeti, I., Nonrepresentable relation algebras from group systems. arXiv 2018

[C] Comer, S., A new foundation for the theory of relations. Notre Dame J. of Formal Logic, 1983.

[M] Maddux, R. D., Subcompletions of representable relation algebras. Algebra Universalis 2018.

[TG] Tarski, A. and Givant, S., A formalization of set theory without variables. AMS Colloquium Publications, 1987.