Groups, relation algebras, general system’s theory: in memory of Steven R. Givant

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Put together a group and a Boolean algebra and you get a relation algebra. Typically, the algebra of complexes (i.e., subsets) of a group is a relation algebra. The representation theory of relation algebras is hard, due to the fact that the whole of axiomatic mathematics can be expressed in the equational theory of relation algebras ([TG]). The talk will be about a long-term joint work with Steve Givant.

We call a relation algebra whose diagonal is the supremum of groups, in an appropriate sense, a group relation algebra. Group relation algebras are amazingly beautiful. The fact that the diagonal is a union of groups, implies that the whole algebra is put together from various quotients of these. This makes them seducingly close to being representable as relation algebras (by using the Cayley representation of groups). Indeed, it is quite surprising that there are nonrepresentable group relation algebras. In the talk we will show the smallest nonrepresentable one (it has 120 atoms), and we discuss the two major open problems concerning group relation algebras. All these will be presented through pictures.

Where does general system’s theory come in? Any algebra is a subdirect product of subdirectly irreducible algebras, and the standard method of studying an algebra is by decomposing it to its subdirectly irreducible factors and then reassemble the algebra from its factors. Subdirectly irreducible relation algebras are the same as simple ones (simple means having no nontrivial congruences). In our recent book [GA] we do the heresy of analysing simple relation algebras by cutting them into pieces, and also constructing simple relation algebras from any others. The idea of this construction is similar to the above one: we put arbitrary relation algebras on the diagonal, and then we organize their factor algebras to form a square. The method of partitioning a large network into blocks is very similar. Indeed a computer program was written for this purpose back in 1970 [NB]. This program was rather successful at that time, e.g., it could be used for inverting 400 by 400 matrices, when the availing programs could invert only about 40 by 40 matrices.

[TG] Tarski, A. and Givant, S., A formalization of set theory without variables. AMS Colloquium Publications, 1987.

[AG] Givant, S. R. and Andreka, H., Simple relation algebras. Springer, 2017.

[NB] Nemeti, I. and Bogdanffy, G., Hierarchic partition of large-scale systems and its application for power system study. Acta Tech. Acad. Sci. Hung. 71,3-4 (1971), 285-303.