Errata to "Axiomatizability of positive algebras of binary relations", AU 66:7–34

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The varieties $\mathsf{V}(\Lambda)$ generated by the representable algebras of binary relations of the similarity types $\Lambda = \{\cdot, ;, 1'\}$ and $\Lambda = \{+, \cdot, ;, 1'\}$ were stated to be finitely axiomatizable in [AM11] (Theorem 4.3 and Theorem 4.1(1), respectively), but their proofs relied on false lemmas. Finite axiomatizability of these varieties remain open.

In more detail, the third case of Definition 4.6 is ambiguous, and Lemmas 4.7 and 4.8 are not true. These lemmas are used in the proof of Theorem 4.3. As a consequence, the proof of Theorem 4.3 breaks down for the equation $1' \cdot x ; y \leq x ; (1' \cdot y ; x) ; y$. This equation is easily seen to be valid, but so far we did not manage to derive it from the axioms $Ax(\cdot, ;, 1')$ of Theorem 4.3. In fact, we conjecture that this equation does not follow from the axioms presented in [AM11]. Theorem 4.3 is used in the proof of Theorem 4.1(1).

Since we could not find other proofs for Theorem 4.3 and Theorem 4.1(1), we state them below as open problems.

Problem 1. Are the varieties generated by the representable algebras of binary relations of the similarity types $\{\cdot, ;, 1'\}$ and $\{+, \cdot, ;, 1'\}$ finitely axiomatizable?

References

[AM11] H. Andréka and Sz. Mikulás, "Axiomatizability of positive algebras of binary relations", *Algebra Universalis*, 66:7–34, 2011.

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