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I. Main duties of the research unit in 2016

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The institute is an important center of mathematics internationally. Fellows of the institute received several Hungarian and international awards in 2016. The Hungarian Academy of Sciences elected one researcher of the institute to full membership of the Academy, another one was elected to corresponding membership. A researcher received the Széchenyi Prize, another one obtained the Erdős Prize from the Mathematics Section of the Hungarian Academy of Sciences. A young researcher obtained the Young Researcher Award of the Academy, another one received the Géza Grünwald medal from the János Bolyai Mathematical Society. A research professor emeritus obtained the lifetime achievement award for Rutgers University. During 2016 five research groups were supported by grants from ERC. The institute is also very successful in the Momentum program of the Academy, they launched the sixth Momentum research group in 2016, this time on the topic of random spectra.

Researchers of the institute published 171 papers during the year. The most important results have appeared in the most significant international mathematical periodicals (Annals of Mathematics, Journal of the American Mathematical Society, Duke Mathematical Journal, Journal of the European Mathematical Society, Annals of Probability, etc.).

The scientific tasks of the institute concentrate on fundamental research. However, significant efforts were devoted to some topics of applied mathematics as well. The main applied areas investigated in the institute are cryptography, the theory of large networks (within this area their new direction is “deep learning” – a fundamental paradigm in artificial intelligence), as well as bioinformatics, but mathematical statistics has also been applied in several related areas (e.g., in medical research).

The institute is organized in the framework of 9 scientific departments, 5 Momentum research groups, and the recently created research group on mathematics education. The research topics of the institute are continuously adjusted to the most recent developments in mathematics.

II. Outstanding research and other results in 2016

a) Outstanding research results

Low Dimensional Topology Momentum Research Group

The minimal number of singular fibers in Lefschetz fibrations over the torus has been examined. Surprisingly, these studies have close connections to the topological constructions of fake complex projective planes. The examination of fillability properties of contact structures on Seifert fibered 3-manifolds was continued. The first 10 chapters of the planned book on basics of Heegaard-Floer homology have been completed; in this work known results have been reconsidered, new proofs were found for some basic results and some of the earlier results have been put in different context. Members of the group took part in writing the lecture notes on Freedman's theorem based on the semester-long seminar held in Bonn in 2013.

A formula for the generating function of the Euler characteristics of the Hilbert schemes of points on cyclic quotient singularities of type (p,1) has been obtained. This result provides a new method to calculate the multivariable generating function of the diagonally coloured Young diagrams. Questions about the knot concordance group and the three-dimensional homology sphere groups were studied, in particular properties of homomorphisms between these groups. In addition, results about embeddings of 3-manifolds into certain spin 4-manifolds have been obtained. Further results about concordance invariants for links have been found through link grid homology. Lagrangian cobordisms between Legendrian submanifolds were also studied.

Homotopical refinements of embedded contact homologies and Gromov invariants for 4-manifolds have been developed. As a result of this research, a twisted version of Taubes' invariants has been defined, and it has been related to the twisted Alexander polynomials of 3-manifolds. A combinatorial reformulation of the embedded contact homology of toric contact manifolds has been given. A corollary of this result improves symplectic embedding results in dimension 4. Tight exotic contact structures on R^{2n+1} were studied, and fillability properties of the Bourgeois contact structures were investigated.

They simplified and enhanced the proof of the connection between the cobordism group of 1 codimensional prime Morin maps and the stable homotopy groups of the complex projective space. They generalized the results to higher codimensional prime Morin maps.

They expressed the inducing maps in the so-called key fibration construction of the classifying spaces of singular maps more explicitly.

Groups and Graphs Momentum Research Group

They showed that there exists a way to partition R^3 into connected "nice" pieces, so that the adjacency graph determined by the pieces is a 3-regular infinite tree, and the partition is isometry-invariant furthermore, there is no way to distinguish the partition classes by any invariantly defined property. It was a (seemingly well supported) conjecture that such a construction cannot exist. They investigated embeddings of transitive and unimodular random graphs into Euclidean spaces. In particular, they proved that unimodular random hyperfinite graphs can be invariantly embedded into R^3.

It is known that for a factor of IID process on the infinite regular tree the correlation of two distant vertices is small. They managed to extend this result by proving that the so-called "1-ended tail" of any factor of IID process is trivial and that for a finite-state process a convex part of the tree provides little information about a distant vertex.

They determined the limit distribution of the co-rank of the adjacency matrix of a random d-regular bipartite graph modulo p. This limit distribution turns out to be the same as in the dense random bipartite graph case. The work on the non-bipartite case is in progress.

They extended Kun's construction of hyperfinite complexes: Their goal is to carry out the strategy of Freedman and Hastings in order to attack the quantum PCP conjecture.

They showed that for a fixed semisimple Lie group, any sequence of locally symmetric spaces that are Ramanujan and have volume tending to infinity must Benjamini-Schramm converge to the symmetric space of the Lie group. According to Serre's conjecture, all arithmetic locally symmetric spaces are Ramanujan. They showed that for a local-globally convergent graph sequence, the cost of the limiting graphing equals the combinatorial cost of the sequence. This gives new simplified proofs for some known results, and also the new theorem that for an amenable group, the rank gradient vanishes for any Farber sequence – this has been known only for Farber chains.

A common generalization of dense graph convergence and Benjamini-Schramm convergence was introduced. Basic properties of the resulting graph limit theory were described, and many interesting examples of convergent graph sequences and estimable graph parameters were given. Tentative limit objects were introduced. Limit objects for the sequence of hypercubes, resp. finite projective planes were constructed. In a weaker sense, limit objects were constructed for any convergent graph sequence of large essential girth.

They proved a conjecture of Pyber from 1993 that determines, up to a constant factor, the minimal base size of a primitive permutation group.

Large Networks Momentum Research Group

In 2016, the group continued exploring the limits of graphs (which arise as natural combinatorial structures). This research topic is motivated by questions about large real world networks, and its goal is to obtain results on finite graphs by combining tools of analysis (considering continuous objects) and probability theory. In particular, the research group generalized the theory of dense graph limits to the sparse case (with subquadratic number of edges). They proved that the limit of a convergent graph sequence can be represented by a Borel measure on the square of the Cantor set. This is motivated by the fact that real-world networks (social networks, connections of neurons, etc.) are usually sparse. In addition, they found methods describing the fractal-like behaviour of the graphs. Another important result (which is also related to random matrices) is about random regular graphs. They proved that the distribution of the elements of the eigenvectors of these graphs is close to the Gaussian distribution. They examined and compared certain dense random graphs, which grow and evolve in time, and have the preferential attachment property: vertices with larger degree have larger chance to get new edges.

They continued the elaboration of higher order Fourier analysis. They extended the lecture notes written on this topic, and proved new results as well. This theory has an important role in the examination of arithmetic sequences among primes. Furthermore, they examined the possible applications of a special case of this theory, the quadratic case. They developed softwares that can be used to predict time series of financial or scientific data.

Research of the recently intensively studied and applied algorithms based on artificial intelligence and deep learning methods had important role in the work of the group. These methods are more and more often used in applications for everyday use, industry or information technology. For example, they are contained in software for image recognition, data compression or data analysis. The main goal of the group is to find stronger, clearer mathematical bases of these successful deep learning methods, and to build a connection between theoretical research and the applied research approach of the industry (Google, Facebook, etc.). In 2016, they focused on generative neural network models (applied for statistical problems, approximation of probability distributions), and on residual networks, which they used for classification of natural images. They performed computer experiments to study different neural network topologies. They investigated the effect of the network depth and wideness in the learning process, and developed neural networks with stochastic depth and autoencoder based models.

Financial Mathematics Momentum Research Group

Their main line of research focuses on stochastic optimal control and its applications to investment problems.

In financial markets trading volume of assets largely impacts their prices, this is called illiquidity. Mathematical modelling has been recently catching up to adequately describe this phenomenon. They managed to establish the existence of optimal portfolio strategies in illiquid markets for agents with a very general preference structure, allowing risk-averse and risk-seeking attitudes which are both observable in investors' behaviour. Such results contribute to a better understanding of actual market phenomena.

Statistical analysis of asset prices shows that past prices may strongly influence present ones. They studied such "memory effects" and found an approximation formula which expresses how the performance of optimal portfolios is influenced by the "length" of the memory.

They started working on adaptive algorithms which follow the dynamics of the underlying stochastic systems without actually identifying it. Such algorithms are thus optimal candidates for high-frequency trading in financial markets. They managed to obtain new results on the convergence of such adaptive schemes for updating functions that show discontinuities: this is often the case in applications but it has scarcely received a theoretical treatment. This research was realized in the framework of a "small research group" project of the Alan Turing Institute (ATI), London, in collaboration with two colleagues from Edinburgh University.

They also extended results on optimal investment to cases where the agent possesses insider information. Formulas for the optimal portfolio performance were obtained.

Random Spectra Momentum Research Group

There are a few important probability distributions that appear in nature whenever the effects of multiple events are compounded. The most well-known of these is the normal distribution. Another important one is the Tracy-Widom distribution that appears in random permutations, random matrices as well as aggregation processes. They have shown that this distribution also appears in the world of random Schrödinger operators. This brings an entire research area into the realm of the Tracy-Widom distribution.

In another joint work they have also shown some important results for interchange processes. They showed that if n ordered particles are reversed randomly swapping nearest neighbours and this happens in a relatively short time, then despite the randomness the particles tend to follow some prescribed process. In particular, quite surprisingly, they move along sine curves.

Didactics Research Group

The members of the Didactics Group successfully applied for a grant in the framework of the Content Pedagogy Research Program of the Hungarian Academy of Sciences in 2016. They started to work on this in September.

They organized didactical events for teachers of special maths classes, and coordinated the modernization and update of their curriculum.

They organized 21 weekend maths camps for talented students in 2016. Approximately 250 students participated at least one of these weekend camps. In the summer they organized two summer maths camps: MaMuT (Camp of Mathematical Amusements) and MaMuT2.

Talented students from age 10 to 18 with outstanding results in Hungarian and international mathematical competitions were invited to improve their mathematical knowledge and skills.

They organized several events in high schools in order to popularize mathematics and discovery learning of mathematics. Many talented students were fostered by the members of the group, in the form of private tutoring or small groups too. They led an extra maths group of talented, underprivileged students. They held maths activities for them on 11 different Saturdays, each of them lasted 4 hours.

They taught the basic principles of discovery learning in mathematics at Eötvös Loránd University and within Budapest Semesters in Mathematics Education program.

Department of Algebra

A link between additive combinatorics and invariant theory is provided by the fact that the Noether number of a finite abelian group (the maximal possible degree of an indecomposable polynomial invariant) equals its Davenport constant (the maximal length of a zero-sum sequence over the group). The study of separating systems (a generalization of generating systems) became a popular topic in invariant theory in the past fifteen years. Analogously to the above mentioned basic fact, the exact degree bound for separating systems of a finite abelian group has been characterized purely in terms of the theory of zero-sum sequences. This result was applied to show that for almost any finite abelian group the universal degree bound for a separating system is smaller than the degree bound for systems of generators of the algebras of polynomial invariants.

The Noether numbers of the groups of order less than 32 have been determined, and the small and large Davenport constants of these groups were also computed with the aid of computer. The results confirm a conjecture of Geroldinger and Grynkiewicz asserting that the Noether number is grater than the small Davenport constant. On the other hand the first example of a group whose Noether number is greater than the large Davenport constant was found. A general upper bound on the Noether number of p-groups was also given.

Nagata gave the first example of a group representation such that the corresponding algebra of invariants is not finitely generated. The Hilbert series of Steinberg's variant of this famous algebra was computed. It turned out that although this algebra is not finitely generated, the Hilbert series is a nice rational function. This result was achieved in a Bulgarian-Hungarian mobility project of the Academy.

For investigating Morita equivalence of semigroups, the notion of firm semigroups has been introduced in analogy to rings. Firm semigroups form a rather large subclass of factorizable semigroups. Significant results have been obtained on Morita equivalence of firm semigroups. It has been proved that any equivalence functor between the categories of firm acts over two firm semigroups is a tensor product functor, and the categories of firm right acts over two firm semigroups are equivalent if and only if there is a Morita context with bijective mappings between the two semigroups. This theorem has been known so far only for semigroups with local units, and firm semigroups form a much larger class.

The investigations of Peirce rings were continued, by describing their direct decomposition which is similar to Wedderburn principal decomposition of certain finite dimensional algebras. A description of the automorphism groups of so-called strict n-Peirce rings was given in terms of those of their structural components.

It was shown that the notion of Krull dimension introduced for commutative semirings in earlier work coincides with the usual notion of dimension for tropical varieties. In particular it was proved that the quotient semiring of the bend relations defining a tropical variety has Krull dimension one larger than that of the original algebraic variety.

Toric ideals of matroid polytopes and generalized polymatroids were studied, and degree bounds for their generators were established in some special cases.

Department of Algebraic Geometry and Differential Topology

They formulated and proved an important conjecture regarding uni-cuspidal projective plane curves. They computed the Seiberg-Witten invariant of the universal abelian cover of certain graph 3-manifolds (in this direction this is the first such computation).

They proved A. Durfee's famous conjecture from 1978 (regarding the negativity of the signature of the Milnor fiber) for 2-dimensional hypersurface singularities. Furthermore, for the signature of the complete intersections they proved rather strict estimates based on improved combinatorial inequalities.

They computed the generating functions associated with Hilbert schemes of simple singularities.

They determined the boundary of the Milnor fiber of certain non-isolated hypersurface singularities as graph-manifolds.

They achieved important results regarding the etale homotopy groups of arbitrary algebraic groups. Using this they provided an explicit formula for the second homotopy groups of homogeneous spaces.

They have shown that the Conjecture of Ghys (any finite subgroup of the diffeomorphism group of a compact manifold has an abelian subgroup with bounded index) is not true in its original form; however, the existence of solvable subgroups is guaranteed. Furthermore, they proved a more general and much stronger statement: the subgroups of the homeomorphism group have nilpotent subgroups with bounded index.

Department of Algebraic Logic

They continued investigating the interrelationships between three principles, the special principle of relativity, isotropy and homogeneity of space-time, formalized in the precise, unambiguous language of mathematical logic. These are three natural, but significantly different formalizations of Einstein's special principle of relativity. They proved, among others, that isotropy of space implies homogeneity of space. Furthermore, homogeneity of time implies homogeneity of space. If there are clocks that get out of synchronism, then homogeneity of space implies homogeneity of time. Under some assumptions, homogeneity of time and isotropy of space imply the special principle of relativity; and if we assume that there are clocks that get out of synchronism, isotropy of space implies principle of relativity.

They have shown that if we distinguish a set of observers stationary with respect to each other in special relativity theory, then the theory becomes definitionally equivalent to classical kinematics. This is an application of the methods of mathematical logic to physics, a step in a larger project, in which they investigate physics in the form of a network of first-order logic theories and the emphasis is on the passages between theories.

They have developed a framework allowing time-travel where it is possible to investigate, independently of the dynamics used, if there are self-consistent solutions or not to an arbitrary initial condition.

They gave a proof system for the logic of formula schemes of first-order predicate calculus, and proved that it is strongly complete. This solves two open problems published more than 30 years ago in Tarski's group.

They showed that two axioms can be omitted from Tarski's ten axioms for relation algebras, on the expense of slightly modifying two other axioms. They proved that both the original axiom system and the modified one consist of independent axioms. Tarski's ten original axioms have been used in the theory of relation algebras ever since Tarski introduced them in 1941. The modified axiom system is easier to check and is more intuitive.

They proved that complete additivity of functions on partially ordered sets is preserved via ultraproducts. This implies that complete additivity of functions is a first-order expressible property, though it looks like an inherently second-order one.

Department of Analysis

Application of the Fourier analytic version of Delsarte's method was used to obtain an improved upper bound on the cardinality of sets avoiding quadratic or cubic residues in cyclic groups. A certain version of Delsarte's bound over non-commutative groups was established. It was applied to the problem of mutually unbiased bases.

The convergence-divergence problems of barycentric interpolation were investigated. This discrete linear interpolatory operator is similar to Lagrange interpolation, although it is not a polynomial but a rational function.

The investigation of the decomposition theory of Hermitian forms was continued. In this respect a unified approach to verifying several different decomposition theorems in analysis was found. Furthermore, it was also shown that the Lebesgue type decomposition of representable positive functionals plays a crucial role in the investigation of some extremal or order theoretic questions.

They addressed the well-known conjecture according to which the uniform norm of multivariate algebraic polynomials on an arbitrary bounded convex domain can be replaced in an asymptotically optimal way by discrete norm taken on a finite subset. This conjecture was settled in the two dimensional case.

The Bernstein-Erdős conjecture on optimal nodes of interpolation was investigated for certain Haar systems.

Department of Discrete Mathematics

It was proved that restricting non-adaptive search to a large family of permissible questions can be done with little loss of efficiency. To separate n elements from each other one trivially needs log n sets. Their nice result shows that with just one more set this can be done using only members of a prescribed family as long as the family contains more than half of the possible sets.

The "two part" Erdős-Ko-Rado theorem was proved 20 years ago. Now they proved a far reaching generalization of this theorem for the case when the sets considered have given number of elements in the parts. They determined the exact Turán number for a large class of hypergraphs. These investigations might help in the proof of the Erdős-Sós-Kalai conjecture.

They proved the stability version of the classical Erdős-Gallai theorem, a first result concerning the extremal numbers of bipartite graphs. They introduced a natural generalization of extremal problems defined by forbidden subposets: instead of the cardinality of the family one studies how many copies of another poset it can contain.

They proved the conjecture of Faudree on the minimum of the maximum degree in path-pairable graphs and answered further open questions in the more general topic of terminal-pairability. The 25 year old conjecture of Erdős on the number of edge disjoint triangles in 4-clique-free graphs was proved. They reproved the famous Pósa-Seymour conjecture for large graphs by means of two new lemmas that have other applications too. The "largeness" required in the theorem has a much smaller order of magnitude than the earlier one. They solved a problem concerning the biggest bipartite 4-cycle free subgraph of 6-cycle free graphs.

They proved the list version of the Bollobás-Eldridge packing conjecture for graphs with few edges. They proved result on the intersecting Ryser's conjecture, for example for the t-intersecting case and also proved a sharp bound for the following problem: How large fraction of the vertices of a k-edge-coloured complete graph can be covered by $k-1$ monochromatic components of distinct colours? Exact or approximate values were established for the Grundy domination number of grid-like and toroidal graphs (Cartesian-, lexicographic-, direct- and strong products of paths and/or cycles). Considering all possible clique covers of the edge set in a graph G, the local clique covering number is the minimum of the maximum number of cliques incident with one vertex. Several results were obtained in connection with a Nordhaus-Gaddum type conjecture on the local clique covering number. They gave upper and lower bounds on the maximum number of cycles that a graph (a directed graph) can contain provided the number of different cycle lengths allowed is finite.

Related to the MMS conjecture, analogously to the vertex sign balance they introduced the edge sign balance parameter and investigated its properties for graphs and (mainly 3 uniform) hypergraphs. Various hypergraph versions of the Erdős-Gallai theorem on paths were proved. They studied the covering of complete r-partite k-edge-coloured hypergraphs by monochromatic components. In their main theorem they determined the smallest m, such that every r-uniform $(r–l)$-partite k-edge-coloured hypergraph can be covered by m monochromatic components if the colouring is spanning. They characterised sufficiently large hypertrees that have minimal number of edges, and generalized the lower bound on the number of edges of hypertrees with disconnected line graph.

They improved the conjectured optimal pebbling number construction in square grids.

They proved an elementary result in permutation group theory that made it possible to prove the famous graph isomorphism without using the Classification theorem of Finite Simple Groups.

The poly-Bernoulli numbers and their relatives were studied by combinatorial methods. They gave new combinatorial interpretations of these numbers. With the help of these they gave transparent explanations of previously known analytical properties of them.

They achieved important progress in the area about colouring homothets of a square, introducing a new tool useful in attacking such problems. They presented this result on the prestigious SoCG conference in Boston.

Generalizing an earlier random graph model they managed to represent the phenomenon brain researchers assume that two different types – excitatory and inhibitory – neurons influence the functioning of the brain.

As applications of the Regularity Lemma, they studied classical Ramsey type problems: Of note is the result on the multicolour Ramsey number of a path; this is the first improvement on this classical problem in a long time.

They achieved significant progress in combinatorics of finite metric spaces, proved the non-metrizability of the Fano plane.

It has been shown that one can define the notion of vertex even in the case of non-convex polytopes and prove several of their properties.

Tverberg's famous theorem was extended to coefficients with negative values.

Department of Geometry

They gave a quadratic time algorithm to estimate the rectilinear crossing number of dense graphs, up to an arbitrarily small percentage of error. To this end, they have developed geometric and combinatorial partition results that can be regarded as far-reaching strengthenings of Szemerédi's regularity lemma.

They proved an old conjecture, according to which for any small ε, if n is large enough, then there exists a partition of the unit square into n smaller squares whose side lengths differ only by at most epsilon per cent.

They gave upper bounds on the chromatic numbers of disjointness graphs of segments in 3-space and of several other geometric objects, in terms of their clique number.

They studied the following problem: determine the smallest number $w(n)$ with the property that the unit sphere can be covered by n congruent copies of a zone of width w, symmetric on the equator. They essentially improved the previously known best lower bound on $w(n)$.

They investigated the limit shapes of convex polytopes generated by random geometric point sets. They continued research about the Steinitz problem, and strengthened the bounds for ordering of vector sequences with small partial sums. They improved the bounds in quantitative Helly type theorems.

In 1911, Toeplitz made a conjecture asserting that every closed Jordan curve in the plane contains four points forming the corners of a square. That variation of Toeplitz problem was considered, where a polygon P is strongly inscribed in a Jordan curve C, if all of its vertices are on C and the interior of P is a subset of the region enclosed by C. Among others, they characterized triangles T, for which every Jordan curve has a strongly inscribed triangle similar to T.

The Hajós Lemma refers to a very nice elementary geometric problem of minimizing the area of those polygons which contain a fixed unit circle and inscribed in the concentric circle with radius $r > 1$. The dual problem where one wants to maximize the area of those polygons which are contained in the larger circle and are circumscribed to the unit circle is interesting on its own. It turned out that while the Hajós lemma is the key to the solution of a circle packing problem, the dual lemma leads to the solution to a circle covering problem.

They managed to solve the planar case of the Lp Minkowski problem if p is between 0 and 1. In addition, partial results have been achieved in any dimension if p is negative.

They characterized the maximum area of convex polygon(s) circumscribed about a fixed convex polygon and gave an efficient algorithm to find it. This problem has many applications, e.g. in statistics.

Department of Set Theory and Topology

There have been numerous sporadic results concerning the cardinal invariants of Hausdorff measures (which are fundamental in geometric measure theory). Their group managed to give an essentially full description of the inequalities between these invariants, and to answer three corresponding questions by D.H. Fremlin, J. Zapletal, and M. Laczkovich-P. Humke.

Automorphism groups of countable structures are interesting from the point of view of set theory as well as group theory. Conjugate automorphisms can be treated as isomorphic, so it is natural to ask if there exists a typical automorphism, in other words, if there are large conjugacy classes with respect to various notions of smallness. Their group investigated this problem using Christensen's notion of Haar null sets. For numerous automorphism and homeomorphism groups they managed to give a full description of the non-null conjugacy classes.

They continued to study the question under what set theoretic assumptions what kind of topological spaces may exist such that their pinning down number differs from their density. In this they closely collaborated with Jan van Mill who worked in the institute in the framework of the Visiting Fellowship Scheme of MTA. They succeeded to prove that all the earlier examples may be assumed to be either (locally) connected or topological groups, without any further assumptions. However, the existence of a connected Tychonov example already does require further assumptions. Even further assumptions will provide examples that are topological vector spaces over the real field.

They proved that every (infinite) homogeneous compactum that is the union of countably many dense or finitely many arbitrary countably tight subspaces has cardinality continuum.

They showed that in the class of monotonically normal spaces the weakly linearly Lindelöf property implies the (in general much stronger) Lindelöf property. (A space is weakly linearly Lindelöf if every system of open sets of uncountable regular size has a complete accumulation point.)

They proved that every almost discretely Lindelöf and first countable regular space has cardinality at most continuum. (A space is almost discretely Lindelöf if any discrete subset in it can be covered by a Lindelöf subspace.)

Based on methods of algebraic logic, a new, cylindric-like semantics of independence logics has been introduced which is different from those appeared earlier in the related literature. The locally finite dimensional elements of the variety generated by these algebras have been axiomatized.

A game introduced by Väänänen has been investigated, which gives an analogue of perfectness for generalized Baire spaces. It was shown that the existence of a weakly compact cardinal implies the consistency of the determinacy of this game for arbitrary subsets of the generalized Baire space as well as a perfect set theorem for the size of independent sets of certain definable graphs on the generalized Baire space. Before, the consistency of both statements was known to follow from the existence of a measurable cardinal. Games generalizing the notion of Cantor-Bendixson ranks for generalized Baire spaces have also been investigated and a related variant of the above mentioned perfect set theorem was shown to follow already from ZFC.

Department of Number Theory

One of the significant results of number theory is the theorem of Maynard and Tao according to which for any natural number *m* there exists a configuration of *m* numbers below a fixed bound, such that there are infinitely many translated configurations where all the numbers are primes. Another outstanding result is the Green-Tao theorem which states that the primes contain arbitrarily long arithmetic progressions. A common generalization of these two results was reached by the Department of Number Theory in 2016. Another result of them asserts that for almost all arithmetic progressions one has a finite, bounded configuration of *m* numbers such that it has infinitely many translations within the same arithmetic progression containing only primes. The diameter of this bloc can be bounded from above by a constant depending only on *m*, multiplied by the difference of the arithmetic progression and the first such bloc appears below the cube of the difference.

Automorphic forms are harmonic waves with a rich symmetry, which help us to understand the whole numbers. An important task is to study the value distribution of automorphic forms. They established strong and natural bounds concerning how high a harmonic wave can get in the case when the symmetries are provided by 2x2 matrices over an arbitrary algebraic number field. They also studied the error term in the hyperbolic circle problem. They estimated the sup-norm of GL(2) Hecke-Maaß cusp forms over number fields, both in the eigenvalue and in the level aspect. Their results are stronger than the generic (purely analytically derivable) bounds, and they are better and better when the underlying field has a larger and larger totally real subfield (in particular, for totally real fields, they managed to prove the bound which had been earlier known over the rational field).

They finalized their results about generalized Montréal functors. They succeeded in generalizing the results of A.C. Cojocaru and Á. Tóth concerning elliptic curves over function fields to the low characteristic case. Moreover, they disproved a conjecture which is related to it.

The classical Gaussian circle problem asks for estimating the number of lattice points inside a circle of radius *R* around the origin of a coordinate system in the Euclidean plane. As *R* tends to infinity, this number can be estimated by the area of the circle, and the problem is the estimation of the error term. The hyperbolic circle problem is the analogue of this problem on the hyperbolic plane: let *G* be a discrete group of isometries of the hyperbolic plane and *z* a point on the plane, one wants to estimate the number of points inside the circle of radius *R* around *z* which are equivalent to *z* under *G*. The main term is the area of the circle. It is well-known that for given *z* and *G* the error term can be estimated by the power of the main term with exponent 2/3, and the conjecture is that 2/3 can be lowered here to any number greater than 1/2. But 2/3 is still the best result. Recently Risager and Petridis improved 2/3 to 7/12 in a certain average sense (if we average the error term over *z*), but only for arithmetic groups *G*. In 2016 they succeeded in improving the exponent 2/3 to 5/8 in this average sense also for general groups *G*. So this is a more general result, but it is not as strong numerically as the result of Risager and Petridis in the special case they considered.

Working out a new result in additive combinatorics, they have developed a fast algorithm to determine the range of a polynomial over a nilpotent ring.

Department of Probability Theory and Statistics

They worked out a new method to investigate whether the coordinates of several dimensional random vectors are completely independent. They are testing the goodness of this method with the help of MTA cloud. According to the results they obtained until now the proposed test method is better than the previous ones. They also started to work on the analysis of vector pairs independence.

Some studies were related to the following problems of probability theory, mathematical statistics and analysis: dilated and lacunary series, random trigonometric series, discrepancy theory, autoregressive processes. Some modern methods of probability theory, number theory and analysis played an important role in these investigations.

They investigated distances between independent random walkers in dependence of the number of steps carried out. They studied the question that if we consider several independent random walks which do not meet in a point, then how large is the distance between them in the function of the time. They also considered some natural related problems on different graphs, e.g. the random walk on a so-called comb. Besides, they got some results on the asymptotic properties of local and occupation times of a random walk on a so-called spider structure.

Fundamental research in Information Theory has been conducted, concerning exponential error bounds for multiuser and/or asynchronous systems, as well as extremum problems for generalized information measures. Results of the latter kind have been applied to Mathematical Finance.

They continued both national and international collaborations: They achieved new results on factor of iid. processes on large random graphs. They showed that the process on distant parts of the graph necessarily has to be nearly independent. Together with Belgian collaborators they analysed the policy iteration algorithm for dynamic programming to give new bounds on its complexity. By exploiting the possibilities of a workshop they initiated a cooperation to better understand the mixing behaviour of non-reversible (asymmetric) Markov chains, including cutoff properties. They joined a group working on stochastic approximation algorithms.

They tried to improve the earlier estimates on the logarithmic Sobolev constant of the Gibbs sampler for probability measures satisfying Dobrushin's uniqueness condition (on finite product spaces). It seems reasonable to conjecture that if the L_2 norm of Dobrushin's interdependence matrix is $1 - \delta$, then the logarithmic Sobolev constant should be of order $1/\delta$, in contrast to the existing bound of order $1/\delta^2$. For the time being the attempts to improve the existing bound were not successful. However, it could be shown that for the distributions generated by the Gibbs sampler in consecutive moments t and $t + 1$, there exists a coupling that follows most naturally from the development of the process, and in many measure concentration problems it works equally well as the widely used coupling which is not so natural. Perhaps this will lead to an improved logarithmic Sobolev constant.

Some of the researchers dealt with the realizations of graphical degree sequences with constraints and degree sequence packing. They showed that it is NP-complete to decide if there is a graph with prescribed degree sequence and number of second neighbours. They also showed that the problem is already NP-complete on bipartite graphs. On the other hand, by another result of them, it is in P to decide whether a graph exists with prescribed degree sequence and number of edges between the two classes of a prescribed bipartition. They also proved that the swap operations are not irreducible on the set of realizations satisfying the above mentioned properties, however, swaps and double swaps are irreducible. They also showed with the help of a graph decomposition technique that the swap Markov chain is rapidly mixing on a new class of degree sequences. This class is exponentially larger than the previously known class of degree sequences for which the rapid mixing was proved.

They proved that if a pair of tree degree sequences does not have common leaves, then they also have edge disjoint caterpillar realizations. They also showed that the edge disjoint realizations of two tree degree sequences without common leaves are in FPRAS and FPAUS. A further theorem of theirs is that it is NP-complete to decide if a tree degree sequence and an arbitrary degree sequence have edge-disjoint realizations.

They studied some phenomena in statistical physics, e.g. the surprising phenomenon that non-Markovian spreading processes on critical random graphs can be drastically sped up by adding a single edge. The study of sparse reconstruction in spin systems (percolation and Ising model) also belongs to this circle of problems. Another research subject which is closely related also to the theory of complex analysis is the construction of conformally invariant growth process of SLE excursions which was motivated by the theory of Diffusion Limited Aggregation and Conformal Loop Ensembles.

They also dealt with practical application of mathematical statistics. They worked out a new method for determination of the statistical significance of GIANT COSMIC RING, and gave a good approximation of the spread of different allergen pollens in Europe on the basis of the properties of the negative binomial distribution.

Applied research

The research carried out at the Rényi Institute has focused on exploratory (theoretical) research. On the applied research projects the research groups of the previous years have continued their work in cryptograph, bioinformatics and other mathematical methods applied in life sciences, e.g. in neural networks.

The Cryptography research group of the Rényi Institute has played an important role in 2016 in the organization of the *CryptoAction Symposium* held in Budapest from the 6th till 8th of April. The event was cosponsored by COST (European Cooperation in Science and Technology) and hosted more than 40 lectures and 200 participants. One member of the group was invited to give a series of lectures about secret sharing to *Encounters of Mathematicians* conference held at the Universidad National (Columbia). The group invited and hosted a researcher from Prague with whom they worked together on quantum secret sharing questions.

The members of the institute participated in numerous bioinformatics projects. They continued their work with researchers of the University of South Carolina and University of Notre Dame. This joint research resulted in one paper on genome realignment and three ones on network research, all being peer reviewed. They have started three new research topics, the one on discrete tomography already produced a research manuscript, and the two others are on the dynamics of networks (joint work with the colleagues from the universities of Szeged, Pécs and Ljubljana) and on the research of ageing. A consortium consisting of the Rényi Institute and Research Centre for Natural Sciences of the Hungarian Academy of Sciences, together with the Swiss Institute of Bioinformatics prepared and submitted the Teaming for Bioinformatics Focus in Hungary (BioinfHU) proposal in 2016 to the Teaming Phase 1 call of the EU Spreading Excellence and Widening Participation WIDESPREAD-04-2017 program.

Members of the institute participated in the organization of the Dagstuhl seminar “Pattern avoidance and genome sorting” in 2016. The seminar turned out to be highly successful.

The applied research (on neural networks) of the members of the Large Networks Momentum Research Group and the Limits of Structures ERC group in 2016 focused on establishing the new machine learning paradigm "Deep Learning" which gained enormous interest in recent years. They aim to unfold the surprising but organic relation with the theoretical research results around the Regularity Lemma.

This new research profile attempts to connect the theoretical research directions traditionally pursued at the institute to the applied research activities characteristic to the big industrial research centers like Google or Facebook. In 2016 the focus of the research was at the generative neural networks and the so called residual neural networks.

The main task of the generative neural network is to discover unknown random variables based on given samples. This is a rather general task, as shown by the following example: a properly configured generative neural network might be able to recognize a fake art piece after recognizing the original works of an artist. The models so far created by the group are not yet able to achieve his task but are already competitive in case of simpler tasks like recognizing or imitating handwriting or drawing realistic portrait.

The residual neural network is a very successful new paradigm to create "Deep Learning" models which interpret the solution of a machine learning task as a process starting with a loose solution refining it step by step. This model promises close connections to the different branch of theoretical mathematical results.

Unlike the theoretical research in mathematics, the study of deep learning requires an exceeding amount hardware and software resources. Therefore, the institute invested extra resources in order to build up the required hardware and software infrastructure and to acquire the necessary technical know-how.

So far the following (preliminary) results have been obtained by the research group:

Building a generative neuron network model based on the optimization of the empirical Wasserstein distance.

Building a state-of-the-art performing recurrent image recognizing neural network based on teaching residual neural network with stochastic depth using relaxed weight-sharing strategy.

A state-of-the-art artificial intelligence result on the standard CIFAR-100 benchmark task using deep networks with stochastic depth.

Developing a semantic compressing method based on neural networks. Developing a 2-dimensional auto encoder model based on Gauss RBF.

Career advancement of researchers

In 2016 one researcher was elected to the membership of the Academy, six young researchers received the PhD degree, three members obtained habilitation. At the end of the year 10 members of the Academy, 34 doctors of HAS, 55 researchers with PhD or CSc worked at the institute, 25 researchers have not yet obtained a degree. Besides the regular employees 13 emeritus research professors (7 academicians, 6 with DSc title) take part in the scientific work of the institute. The institute puts great emphasis on involving young talents – working towards their PhD or just obtaining the degree – into the research work of the institute. In 2016 further 5 young researchers were employed in the new or vacant positions offered by the Academy. Altogether 20 young researchers worked in the institute in 2016. The institute has an agreement with the Central European University (CEU). In this framework 26 doctoral students were supervised by members of the institute.

b) Science and society

Unfortunately, most of the research topics in pure mathematics are not suitable for discussions for the general public. On the other hand, the international success of the researchers has underlined the importance of the research conducted in the institute even in the media.

The researchers of the institute play an important role in popularizing mathematics, giving lectures for high school and university students. The institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. At the 2016 event, however, most of the audience belonged to older generations. Members of the institute take part in fostering mathematical talents. In 2016 they have organized several mathematical camps and other events for students interested in the subject. The institute plays a role in giving scientific background for the teachers of specialized mathematics classes in high schools.

III. A presentation of national and international R&D relations in 2016

National relations

Researchers of the institute teach part time at many universities both in Budapest and in other cities (Eötvös University, Budapest University of Technology, Péter Pázmány Catholic University, University of Szeged, Pannon University, etc.). They play an important role in doctoral schools and in Master programs. 13 fellows of the institute are core members of doctoral schools in various universities, they supervise 56 doctoral students. Especially important is the collaboration between the institute and the Department of Mathematics and its Applications of the Central European University. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belong to the institute, including the department chair and the leader of the doctoral program. Also a large part of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belongs to the institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model for other international programs. For the institute the close contact with the new generation of mathematicians is of foremost importance. In this spirit 62 members of the institute (57 percent of all researchers) were active in teaching at universities in 2016; that included supervising 4 student research projects, 9 BSc and 22 MSc theses.

As part of the renewal program of the Academy, the institute restarted its guest researcher program, which enables university professors and lecturers to spend one or two semesters in the institute freed from their teaching duties. As part of this program three people from Eötvös University, and one each from the University of Szeged and from the Pannon University joined the research teams of the institute in 2016.

The weekly seminars in the institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest as well. This way these seminars influence the whole mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Section of Mathematics of the Hungarian Academy of Sciences (MTA) and its committees, the Hungarian Research Fund (NKFIH), and the János Bolyai Mathematical Society (BJMT) can be mentioned. The president of the Section of Mathematics of MTA, the chairman and the secretary of the Mathematics Committee, the secretary of the Mathematics Doctoral Committee, one of the vice-chairmen and the secretary of the Bioinformatics Committee, the president of the BJMT, the chairman and the secretary of the Scientific Section of BJMT, the vice-chairman of the Applied Mathematics Section of BJMT are all researchers of the MTA Rényi Institute.

International relations

The researchers of the institute have very extensive international relations. Among the coauthors of the members of the institute one finds mainly foreign mathematicians. Joint projects and jointly organized conferences are also typical.

In 2016 twenty-eight people from the institute were involved in organizing international conferences, some of them even on several occasions. Because of the construction work going on in the institute's building the conferences had to take place at other venues.

The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to the cooperation with foreign partners. With the help of these programs fruitful joint research projects, useful exchange of information, and conference participations were made possible.

Researchers of the institute took part in altogether eleven international scientific committees. Names of the institute's researchers can be found 164 times on the list of editorial board of various international journals. In 2016 the researchers gave altogether 280 talks at international meetings, many of these were given as an invited or plenary lecture.

In 2016 eight researchers spent more than half a year abroad at the following institutions: University of Chicago (USA), City University of New York (USA), National Science Foundation (USA), Auburn University (USA), École Polytechnique Fédérale de Lausanne (Switzerland), Lancaster University (UK), Technische Universität Graz (Austria).

Financed by the ERC and Momentum grants or from other sources 12 foreign researchers worked in the institute for 1–7 months (altogether that makes up 35 months). They came from France, Switzerland, Israel, USA, Iran, Taiwan, China, Belgium, Sweden, and Russia. The number of foreign visitors of the institute – not counting the conference participants, neither the foreign employees – was 84 in 2016.

IV. Brief summary of national and international research proposals, winning in 2016

National grants

The Rényi Institute, similar to the practices of the previous years, successfully participated in the national NKFIH researcher-initiated project proposals. In 2016 further 4 research and 1 SNN_16 (Slovenian-Hungarian cooperation) projects won support. Beyond these one further project proposal has been submitted (a call for those researchers whose EU ERC proposal received an "A" evaluation but was finally not supported due to the lack of financial resources), which – together with an earlier one submitted in 2015 – officially was awarded and started in 2016. Further 2 NKFIH_PD individual postdoc proposals submitted by young colleagues of the institute received support. Due to these numerous winning projects – some with high support intensity – and to the fact that some earlier in 2015 awarded projects officially started early 2016, the overall NKFIH researcher-initiated project support of the institute has almost tripled in 2016 compared to the earlier years. This is partly due to the overall (national) increase of these funds, but in a greater extend to the non-reproducible, one time projects (the two ERC_HU projects and three international – Austrian-Hungarian and

Slovenian-Hungarian – cooperation projects give the majority of the increase in financial support).

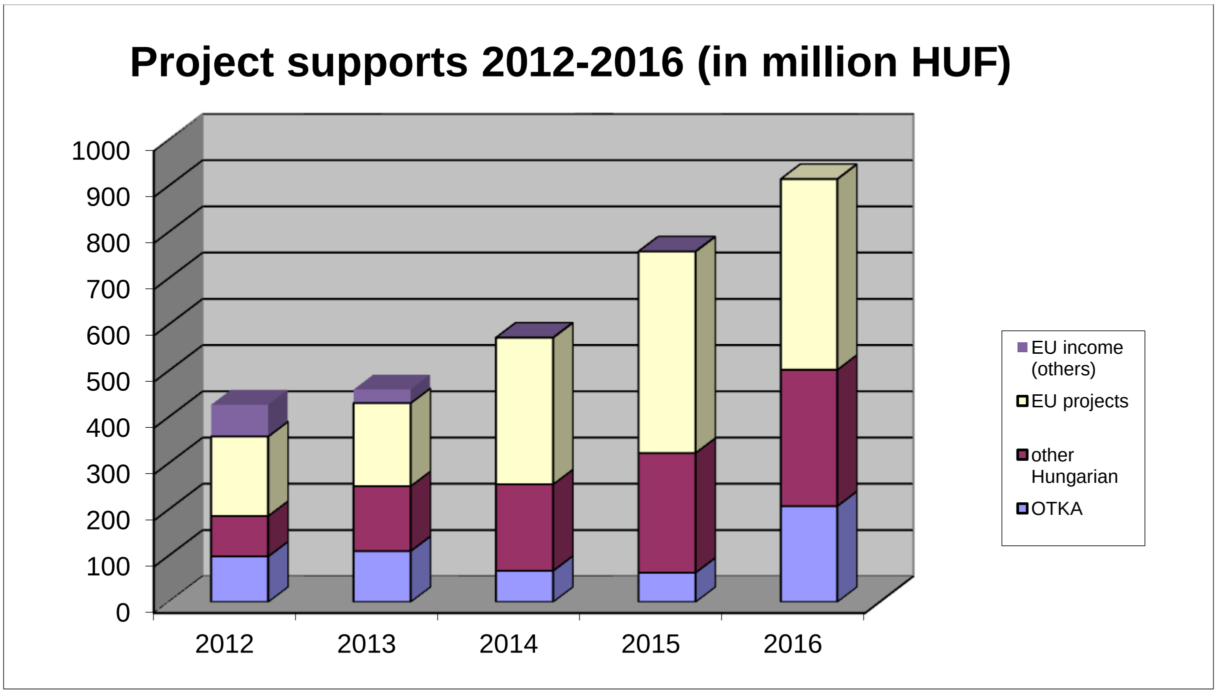
In the project calls of the Hungarian Academy of Sciences one young researcher from abroad received a Premium Post Doctorate Research grant carrying out the project at the institute, the didactics research group won support on the course pedagogy research program of the Academy and one colleague returning from abroad was awarded a Momentum project. These projects altogether resulted in a financial support level of Academy and other national (non-NKFIH) projects similar to high level of the previous years.

International grants

The most promising and successful international calls for the explanatory (theoretical) research projects of the Rényi Institute are EU European Research Council (ERC) calls and the mobility (Marie Curie) calls of the European Union. Among the ERC project proposals submitted in 2015 one advanced and consolidator proposal received "A" evaluations in 2016 and so they were eligible for the NKFIH ERC_HU support. The Advanced proposal has been resubmitted and received evaluation "A" again, but the final decision about the support has not been made yet. One H2020 MSCA mobility project won and will start early 2017, and one further is on the waiting list. Besides this other consortial project proposals have been submitted and/or are being prepared.

Altogether the total research grant income of the institute in 2016 has again exceeded the similar income of the earlier years. While earlier the increase was mainly due to the increasing income from the EU projects, in 2016 this income and the research grant income from the Academy and other resources did not change, the increase was mainly due to the NKFIH researcher-initiated projects.

The following diagram shows the amount of project support received during the last 5 years (some minor corrections were applied between consecutive years to correct discrepancies mainly due to special accounting rules of the EU projects).



V. List of important publications in 2016

1. Abért M, GlasnerY, Virág B: The measurable Kesten theorem. Ann Probab, 44:(3) 1601-1646 (2016) http://real.mtak.hu/44165/
2. Bárány I, Matoušek J, Pór A: Curves in R^d intersecting every hyperplane at most $d + 1$ times. J Eur Math Soc, 18:(11) 2469-2482 (2016) http://real.mtak.hu/44195/
3. Blomer V, Harcos G, Milićević D: Bounds for eigenforms on arithmetic hyperbolic 3-manifolds. Duke Math J, 165:(4) 625-659 (2016) http://real.mtak.hu/44369/
4. Böröczky KJ, Henk M: Cone-volume measure of general centered convex bodies. Adv Math, 286: 703-721 (2016) http://real.mtak.hu/44378/
5. Breuer T, Csiszár I: Measuring distribution model risk. Math Finance, 26:(2) 395-411 (2016) http://real.mtak.hu/44216/
6. Carassus L, Rásonyi M: Maximization of nonconcave utility functions in discrete-time financial market models. Math Oper Res, 41:(1) 146-173 (2016) http://real.mtak.hu/44160/
7. Füredi Z, Kostochka A, Verstraëte J: Stability in the Erdős-Gallai theorems on cycles and paths. J Comb Theory B, 121: 197-228 (2016) http://real.mtak.hu/44181/
8. Harari D, Szamuely T: Local-global questions for tori over *p*-adic function fields. J Algebraic Geom, 25:(3) 571-605 (2016) http://real.mtak.hu/44233/
9. Bodnár J, Némethi A: Lattice cohomology and rational cuspidal curves. Math Res Lett, 23:(2) 339-375 (2016) http://real.mtak.hu/44213/
10. Juhász I, Soukup L, Szentmiklóssy Z: Pinning down versus density. Isr J Math, 215:(2) 583-605 (2016) http://real.mtak.hu/44236/
11. Keleti T, Matolcsi M, de Oliveira Filho FM, Ruzsa IZ: Better bounds for planar sets avoiding unit distances. Discrete Comput Geom, 55:(3) 642-661 (2016) http://real.mtak.hu/44372/
12. Kun G, Szegedy M: A new line of attack on the dichotomy conjecture. Eur J Combin, 52: 338-367 (2016) http://real.mtak.hu/44212/
13. Major P: Sharp tail distribution estimates for the supremum of a class of sums of i.i.d. random variables. Stoch Proc Appl, 126:(1) 100-117 (2016) http://real.mtak.hu/32402/
14. Maróti A: A lower bound for the number of conjugacy classes of a finite group. Adv Math, 290: 1062-1078 (2016) http://real.mtak.hu/44150/
15. Némethi A, Sigurdsson B: The geometric genus of hypersurface singularities. J Eur Math Soc, 18:(4) 828-851 (2016) http://real.mtak.hu/44386/
16. Pyber L, Szabó E: Growth in finite simple groups of Lie type. J Am Math Soc, 29: 95-146 (2016) http://real.mtak.hu/44517/
17. Szemerédi E: Structural approach to subset sum problems. Found Comput Math, 16:(6) 1737-1749 (2016) http://real.mtak.hu/44175/