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I. Main duties of the research unit in 2015

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The institute is an important center of mathematics internationally. Fellows of the institute received several Hungarian and international awards. A research professor emerita obtained the Gold Medal of the Academy this year, a research professor emeritus got an honorary doctorate, a researcher received the Officer's Cross of the Order of Merit of Hungary, a young researcher obtained the Young Researcher Award of the Academy, another one received the Géza Grünwald medal from the János Bolyai Mathematical Society. A research professor emeritus was elected to president of the European Set Theory Society, another one was awarded the Hamming medal of IEEE. Two researchers from the institute took part in the wide-ranging international project aimed at estimating the gaps between prime numbers. The Notices of the American Mathematical Society reported their spectacular result, putting pictures of the mathematicians participating in the project on the cover, including the two fellows from the MTA Rényi Institute. In 2015 another project receiving five-years of support from the European Research Council (ERC) was launched at the institute. This is the sixth research group with an ERC grant at the institute, thus the MTA Rényi Institute is one of the most successful institutions applying to the ERC from the new member states of the EU. The institute has an outstanding record at the Momentum program of the Academy as well. In 2015 the fifth Momentum group was started, this time in the field of financial mathematics.

The scientific tasks of the institute concentrate on fundamental research. However, significant efforts are devoted to some topics of applied mathematics as well. The main applied areas investigated in the institute are cryptography, the theory of large networks, and bioinformatics; mathematical statistics has also been applied in several areas (e.g., in medical research).

The institute is organized in the framework of 9 scientific departments, 4 Momentum research groups, and the newly created research group on mathematics education. The research topics of the institute are continuously adjusted to encompass the most recent developments in mathematics.

II. Outstanding research and other results in 2015

a) Outstanding research results

Low Dimensional Topology Momentum Research Group

A new property of a knot invariant found earlier has been discovered: for certain parameter values the invariant provides a lower bound for the smooth 4-dimensional unoriented genus of the knot.

The existence of contactomorphisms that are smoothly isotopic to the identity, but not via contactomorphisms has been shown. Similar techniques show that there are exotic tight contact structures on Euclidean spaces of dimension 5 and larger. Properties of a contact manifold have been identified, some of which are preserved and some of which are not preserved when one considers the obvious contact structure on the product of the manifold with the 2-torus.

The question of representability of homology classes in manifolds by subcomplexes for which only a finite number of local structures are allowed was further investigated. It was shown that in general any finite set of local structures is insufficient for realizing all homology classes in all manifolds.

The previously computed orbifold and the coarse generating series of the Euler characteristics of the Hilbert-scheme of n points on surface singularities of type $A(k)$ and $D(k)$ has been refined. A connection between the two types of generating series was established.

A formula for ECH-related invariants used in obstructing certain symplectic embeddings was proved. The combinatorial description of ECH to all toric contact manifolds was extended.

The proof that a previous identification of the geometric genus of Newton nondegenerate hypersurface singularities whose link is a rational homology sphere also identifies the Seiberg-Witten invariant associated to the canonical spin-c structure of the link has been finished.

They worked on the connection between the cobordism group of 1-codimensional prime Morin maps and the stable homotopy groups of the complex projective space. The results are extended to codimension 3 prime Morin maps of spin manifolds. They also investigated the problem of representing cohomology classes by immersions and singular maps of finite complexity.

It was shown that in the generic situation the number of Reeb chords on the fillable Legendrian submanifold is bounded from below by the stable Morse number of the filling. This is a direct improvement of the known homological Arnold type inequalities for Legendrian submanifolds. In addition, Floer theory for Lagrangian cobordisms was introduced.

Groups and Graphs Momentum Research Group

They investigated the behavior of critical parameters in Bernoulli percolation on unimodular random graphs, and examined the question of continuity of such parameters under local convergence, approaching the locality conjecture of Schramm from an alternative direction.

They investigated the torsion homology growth of lattices in Lie groups. They showed that when a higher rank simple lattice admits a generating set of elements of infinite order, such that the commuting graph on the set is connected, the torsion homology growth vanishes for arbitrary sequences of subgroups of finite index.

They continued working on the higher cost of measure preserving equivalence relations. It turned out that the higher analogues of the commuting graph condition can be used to control the higher torsion homologies.

They analyzed the eigenvectors of random regular graphs. The corresponding limiting notion is a process on the vertices of an infinite regular tree that (almost) satisfies the eigenvector equation. They conjecture that if the spectral measure of a factor of IID process is supported on a small interval, then the process is close to a Gaussian wave function. They managed to prove this in the special case when the process is a linear factor of an IID process of arbitrary distribution.

They analyzed the limiting behavior of graph parameters of large networks, both in the dense and the sparse cases. They investigated the asymptotic behavior of the chromatic measure for convergent sequences of dense graphs.

They gave an explicit lower bound for the number of conjugacy classes of a finite group in terms of a prime divisor p of its order. The same bound holds for the number of p' degree complex irreducible characters of a finite group. They also gave an asymptotic lower bound for the number of conjugacy classes of a finite group in terms of the order of the group. They proved that every nonsolvable finite group is the product of three of its proper subgroups which are conjugate to each other.

They determined the automorphisms and the continuous endomorphisms of the Einstein gyrogroup in arbitrary dimension.

Large Networks Momentum Research Group

The research group focused on three families of large graphs (with essentially different edge densities), which can be used as models of real-world networks. For random regular graphs (where the number of neighbors of a vertex is fixed), by using entropy methods, they proved that the distribution of the eigenvectors tends to the normal distribution as the number of vertices goes to infinity. This is an analogue of Heisenberg's uncertainty principle, which can be useful for understanding the structure of these kinds of random graphs. They also examined spectral and regularity properties of graphs of intermediate growth (this is the case when the number of edges, compared to the number of vertices, is between linear and quadratic). In addition to the theoretical arguments, computer simulations were also made. They achieved further development in the theory of limits of dense graphs (where the number of edges is governed by the square of the number of vertices), by clarifying questions on the uniqueness of the limit objects. They worked on the convergence speed of a particular sequence of random graphs.

The following result is also closely related to the theory of graph limits. By introducing the notion of reflection complexes and extending the earlier information theoretical approach, for various new cases they proved Sidorenko's conjecture, which states inequalities between certain subgraph densities.

Understanding basic objects (nilspaces) of higher order Fourier analysis and applying quadratic Fourier analysis to additive combinatorics were also important directions within the theoretical research of the group. In order to answer some of these questions, they introduced a concept for limits of functions defined on compact Abelian groups, and gave several equivalent formulations for it. They also managed to obtain new results related to the Wiener-Wintner theorem in ergodic theory.

Research in applied mathematics by the group was mainly concentrated on biological networks, neural networks and questions regarding the structure of the brain. They developed algorithms based on neural networks that use the new method of deep learning. These algorithms can be applied to recognition of images, and to bad quality characters and digits. Jointly with researchers working on neuroscience, they examined the similarities and differences between the functional structure of neural networks and the human brain. Furthermore, cooperating with other researchers, they succeeded in proving the Rhodes conjecture.

Financial Mathematics Momentum Research Group

The research group came into being on 1st July, 2015. The group's research activities centered around two topics: optimal investment in large financial markets and illiquid market models.

They managed to prove the existence of optimal investment strategies in the context of infinitely many assets in a classical model of microeconomics and also pointed out what can go wrong with infinitely many assets unless suitable assumptions are made on the model. These results can be applied, in particular, to bond markets where bonds of various maturity dates constitute a large market.

They proved that models with jumps behave well in the present context of illiquid markets under mild assumptions. This result contributes to a more adequate understanding of high-frequency trading.

Didactics Research Group

The members of the Didactics Research Group worked on a grant project supported by the Hungarian Academy of Sciences. In the framework of this project they tested a method to identify talented students among underprivileged students. They initiated and led an extra maths group of these talented, underprivileged students. They held maths activities for them on 11 Saturdays. Additionally they organized didactical events for teachers who teach in special maths classes, and coordinated the modernization, rewriting and update of the curriculum of these special classes.

They organized 22 weekend maths camps for talented students in 2015. Approximately 250 students visited at least one of these weekend camps.

In summer they organized two summer maths camps: MaMuT (Camp of Mathematical Amusements) and MaMuT2. Talented students from age 10 to 18 with outstanding results in Hungarian and international mathematical competitions were invited to improve their mathematical knowledge and skills.

They held 4 events in high schools in order to popularize mathematics and discovery learning of mathematics.

Many talented students were tutored in small groups by the members of the team.

They taught the basic principles of discovery learning at ELTE and Budapest Semesters in Mathematics Education.

Department of Algebra

A natural framework to develop noncommutative invariant theory is to replace the commutative polynomial algebra by relatively free algebras. These are factors of the tensor algebra modulo *T*-ideals, and any group of linear transformations of the underlying vector space acts naturally on them. It was shown that the corresponding algebra of invariants has rational Hilbert series whenever the group is reductive or is a maximal unipotent subgroup of a reductive group. A well-known basic result in this noncommutative invariant theory characterizes those varieties of associative algebras for which in any relatively free algebra the subalgebra of invariants of any finite group is finitely generated. However, it was not known whether for any such variety a bound for the degrees of the generating invariants exists that is universal in the sense that it depends only on the variety and the group, but not on the given representation. An affirmative answer to this problem was given.

The exact value of the Noether number of some small groups was determined.

The multiplicative ideal theory of algebras of polynomial invariants of finite groups has been further developed. In the case of an abelian group the canonical transfer homomorphism from the multiplicative monoid of the algebra of invariants into the monoid of zero-sum sequences over the class group of the invariant ring is related to the transfer homomorphism from the monoid of invariant monomials into the monoid of zero-sum sequences over the character group. The latter transfer homomorphism is behind the equality of the invariant theoretic Noether number and the arithmetic combinatorial Davenport constant.

The notion of the *k*-th Davenport constant of a BF-monoid was introduced, and some basic properties of the *k*-th Davenport constant of the monoid of product-one sequences over a finite group were proved.

The vanishing ideals of projective varieties associated to flow polytopes (toric quiver varieties) were studied. According to their earlier result, these ideals are always generated in degree at most three. Now a computer method was developed to study toric ideals of flow polytopes. It was shown that up to dimension 4 these toric ideals are generated in degree 2 with the exception of a single case that corresponds to the Birkhoff polytope. They compiled a list of reflexive flow polytopes up to dimension 3. These polytopes correspond to Gorenstein-Fano toric varieties.

Further results were obtained on Morita equivalence of semigroups without local unit. Using a new notion, the firm acts and semigroups, introduced in analogy to ring theory, may lead to a new foundation of this kind of Morita theory. An important advance was achieved in the categorical approach to general radical theory based on forms.

The Krull-dimension of additively idempotent semirings was studied which is defined in terms of chains of prime congruences. It was shown that when *R* is an additively idempotent semiring, the dimension of the polynomial semiring over *R* is always one bigger than the dimension of *R*.

Department of Algebraic Geometry and Differential Topology

A Riemannian manifold has the tube property if the volume of a tube of radius *r* around a curve depends only on the length of the curve and *r*, when *r* is small. Spaces of constant curvature have this property by a classical theorem of Weyl, just as symmetric spaces of rank 1 do, as was proved by Gray and Vanhecke. Now they succeeded in proving that all harmonic spaces have this property; they computed explicit formulae for the volume of tubes; and they proved that if a Riemannian space has this property, then it is a D'Atri space, Einstein, and 2-stein. They proved that among the symmetric spaces, only the 2 point homogeneous spaces have the tube property.

They studied the algebraic structure of finite groups acting on a manifold. They found a counterexample to a conjecture of Ghys, proposing that any finite group acting on a compact manifold has an abelian subgroup of bounded index, where the bound depends only on the manifold. They proved a weaker version of Ghys' conjecture about the existence of a soluble subgroup of bounded index and studied whether this statement holds also for nilpotent subgroups.

They determined the equations of some special cusp singularities.

They proved that for certain surgery 3-manifolds the Seiberg-Witten invariants are stable with respect to the universal abelian cover.

They characterized those algebraic links and surgeries that produce *L*-space 3-manifolds.

They proved that the link of a normal surface singularity is *L*-space if and only if the singularity is rational.

They investigated bounds on the concordance genus of algebraic knots. They obtained a topological obstruction to the deformation of complex plane curve singularities.

They classified the singularity types of rational unicuspidal complex projective plane curves whose singularity has two Newton pairs.

They completed their study on the cohomology of varieties over the maximal cyclotomic extension of a number field. They have stated and proved the analogue of the main cohomological result in positive characteristic, and formulated finiteness conjectures concerning the torsion of Chow groups over the maximal cyclotomic extension. They proved these conjectures in codimension 2 for a large class of varieties including complete intersections.

They proved that *p*-adic completions of certain derived de Rham algebras are universal deformation rings for a classical *p*-adic deformation problem studied by Fontaine.

They determined the Poincaré series of the local Hilbert scheme of quotient singularities of type *A* and *D*.

They studied the equivariant Poincaré series associated with links of surface singularities. They proved a special decomposition theorem which gives a polynomial generalization of the Seiberg-Witten invariant of the link. They deduced interesting relations between the Poincaré series and Hilbert series of modules over non-normal affine monoids.

They defined the generalized Poincaré series using the model of the motivic constructions from algebraic geometry and number theory. They studied the rationality of these series, and proved relations with lattice cohomology.

Department of Algebraic Logic

They solved several much-investigated problems published in 1971 and in 1985, in the field of concept algebras. Concept algebras (cylindric algebras by another name) are situated in the interface of algebra, geometry and logic. Problem 4.2 of the Henkin-Monk-Tarski monograph asks how many distinct properties can be stated about all the defined relations of a theory that uses infinitely many variables. In particular, does this number depend on the number of variables the theory uses or is this number always the continuum? They proved that this number is the cardinality of the powerset of the set of variables the theory uses, therefore it is not always the continuum.

Although this problem is interesting only in the uncountable case, they solved two further problems concerning theories using countable variables via exploiting the novel constructions used in the solution of the previous problem. Namely, they answered Problem 2.13 of the mentioned monograph which concerned a structural description of the geometric concept algebras. In the investigations, a previously unnoticed but important property of these algebras came to light. Using this, they gave a simple algorithmic enumeration of the equations valid in geometric cylindric algebras – this gives a solution to Problem 4.1 of the monograph.

They decided whether the two most natural logics for the logic of formula-schemes coincide or not. In the first logic, relations with finite but unbounded arities occur in the semantics, while in the second one relations with countable arities occur. It was known that the two logics have the same valid statements. They proved the two logics distinct by exhibiting a property all theories of the first logic have but not all theories of the second logic share. Further, they proved that both logics are compact.

Concept algebras, the subject of the above results, and the closely related definability theory of mathematical logic, are directly applicable in investigating theories occurring in physics. They make it possible to investigate whether two theories of physics that talk about the same phenomena of the physical world but use a different plethora of concepts are equivalent or not, whether one of them says more about the phenomena or not. They made investigations of this kind in relativity theory.

It is an accepted view in relativity theory that the principle of relativity is equivalent to the isotropy of space and the homogeneity of space and time. In a first order logic framework, they formalized the principle of relativity, isotropy of space and homogeneity of space and time several ways, and then they investigated their interrelationships. They proved that all these formalizations of the principle of relativity are equivalent under certain assumptions; the assumptions of isotropy and homogeneity do not always imply the principle of relativity. Further, they gave a more natural and general version of Einstein's principle of relativity that does not imply isotropy of space.

They have shown, in a general axiomatic framework for special relativity, that the symmetry axioms about space and time are equivalent only if the observers choose to use the units measuring time and space such that the observers get the same numeric value for the speed of light.

Department of Analysis

The fundamental interpolation-theoretic questions concerning unions of finitely many intervals were investigated. Upper and lower estimates for the Lebesgue function and Lebesgue constant of Lagrange interpolation on such sets were established. A unified approach to constructing optimal nodes of interpolation on unions of disjoint intervals was developed using the inverse polynomial image method. It was shown how this method works on those intervals that possess so called *T*-polynomials, and also proved that the method becomes ineffective in the absence of *T*-polynomials.

Multivariate fast decreasing polynomials on multidimensional convex domains have been studied. Even in the univariate case there is an essential difference between the decrease around inner or boundary points of the domain. It was shown how the rate of decrease of the multivariate fast decreasing polynomials depends on the smoothness of the boundary at the corresponding boundary points of the domain.

The classical Carathéodory-Fejér and Turán type extremal problems were extended to locally compact Abelian groups.

Generalizations of the classical Blaschke Rolling Ball Theorems were obtained, where contrary to the classical curvature assumption, only a discrete, much weaker condition on the change of the tangent direction of the boundary curve is assumed. The new results easily imply not only the classical ones of Blaschke, but also a later extension obtained in 1984 by Strantzen.

With respect to the Turán-Erőd type problems of lower bounds for derivatives of polynomials the decade-old results known for maximum norm were extended for the integral norms on multidimensional convex domains. Since here not only the estimation in one (maximum) point is needed, but also the size of the polynomial and its derivative must be compared essentially everywhere, this problem is harder: until now, no results except for the disk and the interval cases were obtained.

Using a modification of Delsarte's method, an improved upper bound was given for the density of planar sets avoiding the unit distance. Also, for a special class of sets with block-structure an even stronger bound was obtained in any dimension by implementing the Brunn-Minkowski inequality.

The decomposition theory of hermitian forms was used in order to obtain short-type decompositions, which is a common generalization of some well-known results, such as the Lebesgue decomposition of measures and the operator shorting. As an application an analogous result for representable functionals was proved.

Department of Discrete Mathematics

The writing-up of the proof of the conjectures on embedding of trees (150 pages) is finished. It will appear in a series of papers.

Results are proved about the uniqueness of extremal structures in the conjecture on quasi-random graphs.

An extremal problem on graph colorings is solved that yields a simple proof of an earlier theorem on half-graphs, as well.

Bounds were proved about the profile matrix of special intersecting set families.

The proof of the Helfgott-Seress diameter bound is deeply understood and is made shorter and more understandable.

An efficient algorithm is found to list-color permutation graphs.

Sharp estimates are proved on the complexity of the majority function random decision tree.

They proved the conjecture of Erdős on the number of edge disjoint triangles in 4-clique-free graphs.

They proved that a polyomino of n vertices can be decomposed into $(3n^2+4)/16$ polyominoes of at most 8 vertices.

The minimum independence number and the chromatic number of linear cycle-free 3-uniform hypergraphs are determined if the hypergraph does not contain any 5-clique.

Hypergraph applications of the regularity method were investigated. “Real” applications were studied too: the algorithmic regularity lemma (or at least some applicable version of it) was implemented. Based on this, a clustering algorithm was developed and it was successfully applied in concrete databases.

The maximum size of intersecting set families is determined if it does not contain a “butterfly.” A new proof of a theorem on k-chain-free intersecting families was found.

A sharp result was proved about the Turán number of a large hypergraph class. It might help to prove the Erdős-Sós-Kalai conjecture.

The stability version of the classical Erdős-Gallai theorem is proved. It is the first result of this type in the extremal theory of bipartite graphs.

Exciting results are proved in the mathematical modeling of the brain. The neighboring neurons are joined, and the distant neurons are directly connected more and more rarely. A theorem is proved in a random graph modeling that coincides with the observation that the brain works very fast.

They generalized two results about cover-decomposing half-planes to pseudo-halfplanes, thus giving an exact answer to questions regarding cover-decomposability of the translates of any unbounded convex set.

They found an equivalent graph coloring problem to any (sophisticated) problem in NP.

The algorithm to find the shortest grid vector was simplified.

They studied the vertex colorings of graphs such that exactly two colors appear in any triangle. It is proved that the lower chromatic number (the minimal possible number of colors) can be arbitrarily large and that the corresponding decision problem is NP-complete. It is also proved that between the possible color numbers one can have arbitrarily large gaps.

A new method was designed to determine all possible realizations of a given degree sequence.

Some new results were achieved in sampling the degree sequences of almost half-regular graphs. The newly obtained fast sampling result provides sharp estimates on the number of possible realizations of a given degree sequence.

They improved their results on the biggest cliques in 4-cycle-free graphs.

They described the spherical random polytopes if the vertices are chosen from a half sphere.

The properties of k-line-graphs were studied and they characterized the graph pairs such that their Cartesian product is a k-line-graph. It was proved that the recognition of k-line-graphs is an NP-complete problem for $k > 2$.

Similar to the graph domination game, the disjoint domination game was introduced and it was proved in two important versions that the Maker has a winning strategy.

Generalizing the previously known bin packing games to a far extent, a version has been introduced in which each current situation is evaluated using weights given in a fixed matrix. If the matrix is symmetric, then a Nash equilibrium always exists.

Department of Geometry

A graph is called 1-planar if it can be drawn in the plane such that there is at most one crossing on each edge. It is known, that a 1-planar graph with n vertices has at most $4n$-8 edges, but – unlike planar graphs – a maximal 1-planar graph can have much fewer edges. They gave a lower bound for this edge number.

A set system is called separating, if for any two elements there is a member that contains exactly one of them. They gave some estimates on the size of a separating family, under different conditions.

They proved an analogue of the Blaschke-Santaló inequality for d-dimensional convex bodies that are intersections of congruent closed balls, and they also proved the reverse isoperimetric inequality for convex discs that can be represented as the intersection of congruent closed circular discs.

They gave an upper bound for the maximum of the pairwise (non-obtuse) angles of n lines in 3-dimensional Euclidean space, thus improving an earlier result of L. Fejes Tóth.

They investigated a spherical version of the plank problem, in which one wants to cover the unit sphere of the 3-dimensional Euclidean space by n congruent spherical bands (zones) that are symmetric about the centre of the sphere. For arbitrary n, they gave a lower bound for the minimal width of the zones that can cover the sphere.

They gave the following characterization of the ball in arbitrary space of constant curvature: if K and L are convex bodies with sufficiently smooth boundary such that the intersection of any congruent copies of K and L is centrally symmetric, then K and L are congruent balls.

They constructed various twisted prisms, skewed prisms, and twisted dodecahedra that could not be triangulated, and classified those Archimedian polyhedra all of whose surface triangulations can be extended to 3D triangulations.

They extended the notion of the affine surface area to more general measures. With the aid of this, they investigated the limit shape of convex polygons determined by normally distributed random points in the plane, and studied the higher dimensional analogues. They generalized formulas for the volume of the section of l_p balls and simplices with lower dimensional subspaces, and they determined the extremal sections in special cases.

They gave a necessary and sufficient condition for an infinite sequence of x values to have the property that all polynomials of degree k can be simultaneously approximated above them.

They showed that the classic Haussler-Welzl theorem is tight with respect to both parameters.

They gave a sharp lower bound for the number of intersection points of n pairwise intersecting closed curves in the plane, no 3 of which pass through the same point.

Department of Set Theory and Topology

They started to study the so-called neighbourhood assignment versions of various topological cardinal functions. For instance, the neighbourhood assignment version $pd(X)$ of the density $d(X)$ is defined as the smallest cardinal such that for any neighbourhood assignment of X there is a set of that cardinality that meets every member of the assignment. One of their main results says that the following three statements are equivalent: (1) Every Hausdorff space X satisfies $pd(X) = d(X)$. (2) Every 0-dimensional Hausdorff space X satisfies $pd(X) = d(X)$. (3) Every limit cardinal is a strong limit. This solves two problems of Banakh and Ravsky.

They investigated the size of conjugacy classes of certain automorphism groups with respect to Christensen’s notion of Haar nullness, which is a generalization of the notion of Haar measure zero sets to non-locally compact groups. They gave a full description in several important groups, such as the automorphism group of the random graph, the ordering of the rationals, etc.

They continued their work on the regularity properties of Christensen’s notion of Haar null sets and, using a method developed earlier, they proved that certain, naive modifications of this notion behave badly--these collections of sets are not closed under unions, hence they are not good generalizations of the collection of Haar measure zero sets.

They investigated the Hausdorff (and packing) dimension of fibers of the random continuous function f in $C(K,R^d)$, where K is an uncountable compact metric space. Their main result is that the fibers and graphs of random functions have maximal Hausdorff dimension. This generalizes results of Dougherty, Bayart, Heurteaux, Antunovic, Burdzy, Peres and Ruscher.

Ordinal valued functions are called rank, and are usually used to describe the complexity of objects. Kechris and Louveau developed the theory of rank functions on Baire 1 class functions. They extended this to Baire alpha class functions, and with its application they solved a paradoxical geometric decomposition problem. In the proofs they generalized results of Kechris and Louveau from compact metric spaces to arbitrary Polish spaces.

They continued the research initiated by Tarski and Givant. The main question of this research is which fragments of the first order logic are decidable, or which have Gödel's incompleteness property? They worked out a new algebraic method and obtained easier proofs for the known results, and they proved new classes of cylindric algebras to be undecidable and to have the Gödel property. The question remains open only for one class.

New sufficient conditions have been found which imply the existence of generic automorphisms of further homogeneous structures. Connections with finite combinatorics have also been investigated.

Department of Number Theory

In 2015 they continued their research about the difference of consecutive primes that created great international interest. First Zhang, later Maynard proved the existence of infinitely many prime pairs with a bounded difference using a refinement of the method of Goldston, Pintz and Yildirim. By a further development of the method of Zhang and Maynard, they succeeded in solving several 50-60 year old problems of Erdős and Turán concerning the difference of consecutive primes. Further they showed the existence of infinitely many pairs of integers with a difference of at most 17, where one of the terms was prime and the other had an odd number of prime factors. This is another approximation to the twin prime conjecture and, before the results of Zhang and Maynard, this was not known with any constant other than 17.

They investigated problems of combinatorial and additive number theory, and related analytic questions. A paper on sets avoiding unit distance appeared. A study on the comparison of possible cardinalities of difference sets and multiple sumsets is in preparation. The so-called sum-product estimates compare, in general, large sets obtained by adding all pairs of a set of natural numbers belonging to the set to the size of the set obtained by multiplying each pair instead of adding them. Extending a conjecture of Erdős and Graham, they determined those abelian groups, which can be ordered so that they do not contain monotone three-term arithmetic progressions. Further they showed that every solvable group has a well-ordering that does not contain monotone arithmetic progressions of length six.

Automorphic forms are harmonic waves with a rich symmetry, which help us understand the whole numbers. An important task is to study the value distribution of automorphic forms. Earlier they established strong and natural bounds concerning how high a harmonic wave can get in the case when the symmetries are provided by certain 2x2 matrices over the Gaussian integers. They succeeded in extending some of these results so that the Gaussian integers can be replaced by the integers of any algebraic number field. They also studied the average fourth moment of automorphic forms.

In the Poisson-like summation formula they proved several years ago there are triple product integrals of automorphic forms as weights. Now they found an interesting relation between these weights and Rankin-Selberg L-functions. They proved an identity (a duality relation), the left-hand side of which contains Rankin-Selberg L-functions, and the right-hand side contains the mentioned triple products.

Department of Probability Theory and Statistics

The properties of random walks on certain structures were investigated. They got new results in the following subjects: strong approximations concerning local positions and local times, and occupation times. Connections to appropriate Wiener processes were investigated. These are extensions of the results about usual random walks. The Vervaat process for sums of strongly dependent random variables and their inverses were investigated. Some results about independent random variables were extended. In this case the limit process is expressed by the fractional Wiener process.

Results on error exponents of asynchronous multiple access channels have been further developed. New results were proved on minimizing entropy functionals under general convex constraints, and a new application in mathematical finance of such previous results (under linear constraints) has been given: For “multiple priors” risk models, almost worst case distributions always cluster (in Bregman distance) around an explicitly specified, perhaps incomplete distribution, even if a worst case distribution does not exist. Regarding statistical model selection via information criteria, a significant step has been made towards extending their strong consistency result for finite alphabet models to models with countable alphabets.

New estimates were proved for the tail distribution of the supremum of families of partial sums defined with the help of independent, identically distributed random variables. This is an improvement of earlier results in this field, and it may be useful in the investigation of the non-parametric maximum likelihood method.

They gave a new proof of the logarithmic Sobolev inequality in the case of strongly mixing Markov fields. This proof is simpler than the existing ones. The logarithmic Sobolev inequality was proved

(i) under Dobrushin and Shlosman's complete analiticity condition;

(ii) moreover, under Pico and Martinelli's strong mixing condition, which is a weakened version of the above condition.

They developed an efficient data representation and corresponding algorithm in bioinformatics to represent a set of multiple sequence alignments sampled in a Bayesian statistical sampling, and to construct their consensus alignment.

They showed that a Markov chain is rapidly mixing on the realizations of almost half-regular bipartite degree sequences with constraints, and also showed that the problem is self-reducible.

They developed a Gibbs-sampler on the most parsimonious labelings of an arbitrary rooted tree under the SCJ model, and showed that the sampler is irreducible. The method has been implemented and tested on real life data.

They gave necessary and sufficient conditions for the existence of a half-regular factorization of a complete bipartite graph, and they also characterized those perturbations that transform a factorization into another.

The department also did research in statistical physics. In this work, the generalization of the coupling method of standard pairs to the high dimensional case had to be worked out. They described their previous results on the dynamics of systems consisting of two falling balls. The lecture on this subject at last April's spring research school in Loughborough won first prize in the competition of young lecturers.

They proved some results on planar near-critical and dynamical Bernoulli and FK percolation models, and also wrote an appendix on the near-critical percolation window for a paper by Ahlberg and Steif. They finished their research on a conformally invariant growth process of SLE excursions.

They finished their research on noise sensitivity of critical bootstrap percolation on the square lattice and on the d-regular random graph. They proved both positive and negative results on sparse reconstruction in spin systems (percolation and Ising model).

They showed that in order to compute the distance correlation, a notion introduced about ten years ago and popular in applied statistics, one does not need n^2 operations; constant times $n \log n$ operations suffices, where n denotes the sample size. In another result they computed extremal probabilities for linear combinations of gamma random variables and applied them to stochastic algorithms.

They also worked on the solution of some practical problems with the help of applied statistics. For example, they worked on the application of the Rasch model to data on ragweed.

Applied research

The major part of the research carried out at the Rényi Institute is generated by questions raised by the inner development of mathematics. Nevertheless, together with the exploratory (theoretical) research that the institute conducts, these new results and other fundamental methods of mathematics are applied, in recent years, mostly to bioinformatics and cryptography. Together with the earlier cryptography Momentum research group (which in the meantime became a permanent research unit of the institute) the newly founded Large Networks Momentum research group also engaged in applied research in bioinformatics (use of mathematics in brain research).

The members of the institute have continued their joint collaboration with the research groups of the Institute of Experimental Medicine and started a new interdisciplinary colloquium series entitled "The application of mathematics in brain research". In the lectures held in the second part of 2015, members of the Rényi and Experimental Medicine Institutes reviewed those parts of their research where joint use of the two disciplines might lead to new results.

The main subject of the concrete joint research initiative is still the sharp wave ripples in the hippocampus. The researchers participating in this collaboration are interested in what kind of neural connections are necessary to obtain the observed dynamics. The collaboration has so far yielded a conference poster and a submitted manuscript.

The Large Networks Momentum and Limits of Structures ERC research groups' members joined this collaboration, focusing their applied research mainly on certain biological networks, neural networks, and questions related to the structure of the brain. They developed algorithms using neural networks with the help of the new, revolutionary, so-called "deep learning" method, which can be used in image processing, recognizing images of bad quality letters or digits. Together with the brain researchers, they studied the similarities and differences between the functional structure of the human brain and mathematical methods of neural networks. The group has also started a new line of investigation regarding artificial neural networks. They studied a new class of generative artificial neural network models that were inspired by concepts from probability theory. The task of a generative neural network is to learn some unknown probability distribution based on examples. This is an exceedingly general task: as an illustrating example, a strong generative neural network should be able to create forgeries after observing the works of an artist. The research group's models are obviously far from this level, but they are already competent at simpler imitation tasks such as handwriting. Besides the algorithmic advances, the project made it necessary to acquire state-of-the-art software technologies and methodologies dealing with artificial neural networks.

The institute also continued several bioinformatics collaborations with the University of Oxford, the University of South Carolina and the University of Notre Dame. The result of these collaborations was two published papers in BMC Bioinformatics, and one each in SIDMA, in PLoS ONE and in New Journal of Physics.

The Cryptology research group, which started in the framework of the first Momentum project and has been given a permanent status, continued its research based on the results of the previous years. The special issue of Studia Scientiarum Mathematicarum Hungarica containing the papers of the best talks of the previous year's Budapest Cryptography Conference has been published. The members of the research group delivered talks at several Hungarian and international conferences – CECC15, EURO2015 (the first European Conference for Operational Research and Management Science), Real World Cryptography Workshop, Cost Action Meeting, and CryptoCurrency – and joined the program committees of several other conferences, including the CECC15 and the INSCRYPT 2015. The international recognition of the group is shown by the fact that the CryptoAction Core Group international organization will held its conference “Cryptography for secure digital interaction” (https://cryptoactionsymposium.wordpress.com) in the spring of 2016 at the Rényi Institute.

Career advancement of researchers

In 2015 two researchers received the title “doctor of the Academy” and six young researchers obtained the PhD degree. At the end of the year 8 members of the Academy (7 according to the statistical number of employees), 36 doctors of MTA (stat. num. 27), and 48 researchers with PhD or CSc (stat. num. 47) worked at the institute; 28 researchers (stat. num. 25) have not yet obtained a degree. Besides the regular employees, 12 emeritus research professors (7 academicians, 5 with DSc title) take part in the scientific work of the institute. The institute puts great emphasis on involving young talents – working towards their PhD or just obtaining the degree – into the research work of the institute. In 2015 a further 6 young researchers were employed in the new or vacant positions offered by the Academy. Altogether 20 young researchers worked in the institute in 2015. The institute has an agreement with the Central European University and within this framework 24 doctoral students were supervised by members of the institute.

b) Science and society

Unfortunately, most of the research topics in pure mathematics are not suitable for discussions for the general public. On the other hand, the international success of the researchers has garnered media attention and has underlined the importance of the research conducted in the institute.

The researchers of the institute play an important role in popularizing mathematics, giving lectures for high school and university students. The institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. Members of the institute take part in fostering mathematical talents. In 2015 they organized several mathematical camps and other events for students interested in the subject. The institute plays a role in giving scientific background for the teachers of specialized mathematics classes in high schools.

III. A presentation of national and international R&D relations in 2015

National relations

Researchers of the institute teach part time at many universities both in Budapest and in other cities (Eötvös University, Budapest University of Technology, Péter Pázmány Catholic University, University of Szeged, Pannon University, etc.). They play an important role in doctoral schools and in Master programs. Seventeen fellows of the institute are core members of doctoral schools at various universities, and they supervise 60 doctoral students. Especially important is the collaboration between the institute and the Department of Mathematics and its Applications of the Central European University. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belong to the institute, including the department chair and the leader of the doctoral program. Also a large portion of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belongs to the institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model for other international programs. For the institute, the close contact with the new generation of mathematicians is of foremost importance. In this spirit 60 members of the institute (57 percent of all researchers) were active in teaching at universities in 2015; their work included supervising 2 student research projects, 18 BSc and 21 MSc dissertations.

As part of the renewal program of the Academy, the institute restarted its guest researcher program, which enables university professors and lecturers to spend one or two semesters in the institute freed from their teaching duties. As part of this program six people from the Eötvös University, two from the University of Szeged, and one each from the University of Debrecen, from the Pannon University, and from the Budapest University of Technology joined the research teams of the institute in 2015.

The weekly seminars at the institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest. This way these seminars influence the whole mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Section of Mathematics of the Hungarian Academy of Sciences (MTA) and its committees, the Hungarian Research Fund (OTKA), and the János Bolyai Mathematical Society (BJMT) can be mentioned. The president of the Section of Mathematics of MTA, the chairman and the secretary of the Mathematics Committee, the secretary of the Mathematics Doctoral Committee, one of the vice-chairmen and the secretary of the Bioinformatics Committee, the chairman of the Mathematics and Natural Sciences subcommittee of the Council of the Academy's Research Units, the chairman of the mathematics panel of OTKA, the president of the BJMT, the chairman and the secretary of the Scientific Section of BJMT, the vice-president of the Applied Mathematics Section of BJMT are all researchers of the MTA Rényi Institute.

International relations

The researchers of the institute have very extensive international relations. Among the coauthors of the members of the institute one finds mainly foreign mathematicians. Joint projects and jointly organized conferences are also typical.

In 2015 thirty people from the institute were involved in organizing international conferences, some of them even on several occasions. During the year seven conferences took place in the building of the institute. The most important events were the "Intuitive Geometry, László Fejes Tóth Centennial" conference commemorating the 100th anniversary of the birth of the former director of the institute, and the conferences "Stochastics and Interactions" and "Asymptotic Group Theory", both featuring many outstanding speakers, among them a Fields medalist. The young researchers in the institute have organized the seventh "Emléktábla Workshop" this year.

The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to cooperation with foreign partners. With the help of these programs, fruitful joint research projects, useful exchange of information, and conference participation were made possible.

Researchers of the institute took part in altogether thirteen international scientific committees. Names of the institute's researchers can be found 159 times on the list of editorial boards of various international journals. In 2015 the researchers gave altogether 311 talks at international meetings, many of these were given as an invited or plenary lecture.

In 2015 eleven researchers spent more than half a year abroad at the following institutions: University of Chicago (USA), City University of New York (USA), National Science Foundation (USA), Auburn University (USA), University of Memphis (USA), École Polytechnique Fédérale de Lausanne (Switzerland), Lancaster University (UK), Université Catholique de Louvain (Belgium), Université de Toulouse (France), and Technische Universität Graz (Austria).

Financed by the ERC and Momentum grants or from other sources, 16 foreign researchers worked in the institute for 1–6 months (altogether that makes up 61 months). They came from France, Spain, Switzerland, Italy, Israel, USA, Canada, Iran, Taiwan, and Korea. The number of foreign visitors of the institute – counting neither the conference participants nor the foreign employees – was 77 in 2015.

IV. Brief summary of national and international research proposals, winning in 2015

National grants

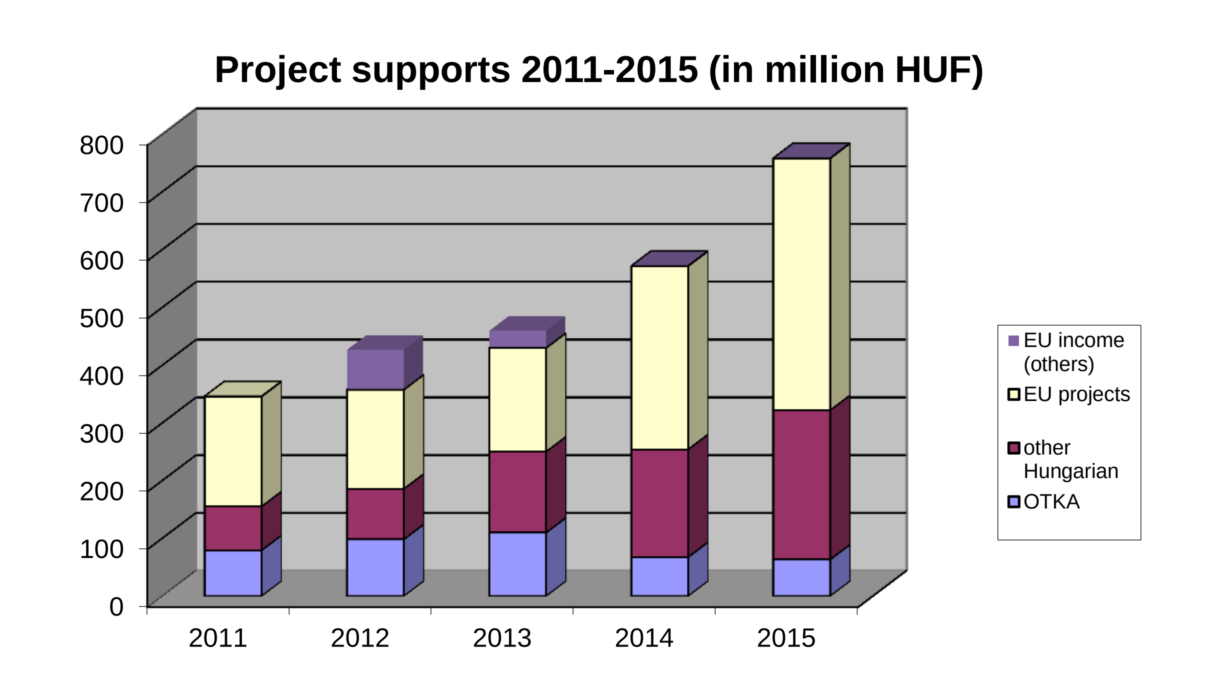
The Rényi Institute, similar to practices in previous years, successfully participated – both in terms of applications and winning projects – in 2015 in the Hungarian OTKA (Hungarian Scientific Research Fund) project proposals. In 2015 3 further research ("K", one of them with extra-large budget) OTKA and one international ARSS-OTKA (Slovenian-Hungarian cooperation's) projects won support. Further, an ERC_HU_15 (a funding tool open only for those researchers who have received an "A" mark on their EU ERC project proposal but has not been funded due to the lack of financial resources) project proposal has been submitted and declared winning, with official notification arriving only in 2016. Most of the OTKA projects winning support in 2014 started January 1st, 2015, while the projects winning support in 2015 will start at the beginning of 2016 (except the ARSS-OTKA project). With all these results the overall OTKA project support of the institute has been similar to previous years and will have the same level for 2016 as well.

International grants

The most promising and successful calls for the explanatory (theoretical) research projects of the Rényi Institute are EU European Research Council (ERC) calls and the mobility (Marie Curie) calls of the European Union. The newest ERC Consolidator Grant project of the institute, closely related to the "Groups and graphs" Momentum research group, entitled "Asymptotic invariants of discrete groups, sparse graphs and locally symmetric spaces", won in 2014, started on July 1st 2015. Another ERC Consolidator Grant project proposal was evaluated at the end of 2015, received the mark of "A", but was not funded, and so the researcher is entitled to apply for an NKFIH ERC_HU_15 grant. Two further EU mobility projects started in 2015: one, earlier winning FP7 PEOPLE IEF, only started this year, and a H2020 MSCA mobility project won and started in 2015.

Altogether, the total research grant income of the institute in 2015 was significantly higher than in all the previous years, mainly due to the income of the EU projects, but the income of the Hungarian Academy of Sciences Momentum projects has also contributed to a large extent. In 2015 there were 5 EU ERC projects, 4 EU mobility projects, 4 Hungarian Academy of Sciences Momentum projects and 22 OTKA projects running at the institute. These running Momentum, OTKA and EU funded projects together will ensure that the project-related income of the institute will not significantly decrease in 2016.

The following diagram shows the amount of project support received during the last 5 years (calculating the support of the previous years, we applied a correction: some income was moved to other years to correct the discrepancy mainly caused by the differences of the EU accounting system and the fact that their project report period is not related to the academic year).



V. List of important publications in 2015

1. Abért M, Csikvári P, Hubai T: Matching Measure, Benjamini–Schramm Convergence and the Monomer–Dimer Free Energy. J Stat Phys, 161:(1) 16-34 (2015) http://real.mtak.hu/id/eprint/33505
2. Aistleitner C, Berkes I, Seip K: GCD sums from Poisson integrals and systems of dilated functions. J Eur Math Soc. (JEMS), 17:(6) 1517-1546 (2015) http://real.mtak.hu/id/eprint/33507
3. Balka R, Buczolich Z, Elekes M: A new fractal dimension: The topological Hausdorff dimension. Adv Math, 274: 881-927 (2015) http://real.mtak.hu/id/eprint/33511
4. Bárány I, Holmsen AF, Karasev R: Topology of Geometric Joins. Discrete Comput Geom, 53:(2) 402-413 (2015) http://real.mtak.hu/id/eprint/33515
5. Böröczky KJ, Lutwak E, Yang D, Zhang G: Affine images of isotropic measures. J Differential Geom, 99:(3) 407-442 (2015) http://real.mtak.hu/id/eprint/33521
6. Carassus L, Rásonyi M, Rodrigues AM: Non-concave utility maximisation on the positive real axis in discrete time. Math Financ Econ, 9:(4) 325-349 (2015) http://real.mtak.hu/id/eprint/30233
7. Erdős PL, Kiss SZ, Miklós I, Soukup L: Approximate Counting of Graphical Realizations. PLOS ONE, 10:(7) e0131300 (2015) http://real.mtak.hu/id/eprint/33524
8. Frasca P, Garin F, Gerencsér B, Hendrick JM: One-dimensional coverage by unreliable sensors. SIAM J Control Optim, 53:(5) 3120-3140 (2015) http://real.mtak.hu/id/eprint/33542

9. Harangi V, Virág B: Independence ratio and random eigenvectors in transitive graphs. Ann Probab, 43:(5) 2810-2840 (2015) http://real.mtak.hu/id/eprint/33543

10. Janson S, T Sós V: More on quasi-random graphs, subgraph counts and graph limits. European J Combin, 46: 134-160 (2015) http://real.mtak.hu/id/eprint/21016

11. Károlyi Gy, Nagy ZL, Petrov F, Volkov V: A new approach to constant term identities and Selberg-type integrals. Adv Math, 277: 252-282 (2015) http://real.mtak.hu/id/eprint/33544

12. Kollár J, Némethi A: Holomorphic arcs on singularities. Invent Math, 200:(1) 97-147 (2015) http://real.mtak.hu/id/eprint/33546

13. Lovász L, Szegedy B: The automorphism group of a graphon. J Algebra, 421: 136-166 (2015) http://real.mtak.hu/id/eprint/33547

14. Ozsváth P, Stipsicz A, Szabó Z: Grid Homology for Knots and Links, In: Mathematical Surveys and Monographs; 208, American Mathematical Society, Providence, p. 410 (2015) (ISBN:978-1-4704-1737-6)

15. Simonyi G, Tóth Á: Dilworth rate: a generalization of Witsenhausen's zero-error rate for directed graphs. IEEE Trans Inf Theory, 61:(2) 715-726 (2015) http://real.mtak.hu/id/eprint/33548