# Abstracts of Building Bridges II

A conference to celebrate $70^{th}$ birthday of
László Lovász

Budapest, July $2^{nd}$ - $6^{th}$, 2018

# Invited Talks

### Lovász, vectors, graphs and codes

**Noga Alon**
Tel Aviv and Princeton, Israel and USA

- What is the maximum possible Euclidean norm of a sum of $n$ unit vectors so that any three of them contain an orthogonal pair ?
- What is the maximum possible value of the Lovász theta function of an $n$ vertex graph with independence number 2 ?
- What is the minimum possible size of the maxcut of a triangle-free graph with $m$ edges ?

These are three questions raised by Lovász. Somewhat surprisingly all can be settled using a construction of a family of pseudo-random Cayley graphs that are useful in tackling several additional extremal problems in Graph Theory and Information Theory.

I will discuss these results, mentioning related open problems and focusing on a recent application of the graphs in the study of zero-rate list decodable codes, obtained jointly with Bukh and Polyanskiy.

### A palace made of gems: The Laci Lovász treasure chest

**László Babai**
Chicago, USA

We review a selection of wondrous ideas that have, for half a century, mesmerized the speaker and the world.

### On the graph limit approach to the eigenvectors of random graphs

**Ágnes Backhausz**
Budapest, Hungary

The goal of the talk is to show how the theory of graph limits can be used for certain problems concerning the spectral theory of random graphs (or random matrices). In particular, we are interested in the empirical distribution (the distribution of a uniformly chosen entry) of the eigenvectors of a random $d$-regular graph. Since these random graphs locally look like trees, and they converge in the Benjamini–Schramm (local weak) sense to the infinite $d$-regular tree, we could formulate an analogous problem in the limit. By using counting arguments and inequalities for entropies, we could show that eigenvector processes that can be approximated with eigenvectors of random $d$-regular graphs are Gaussian. Getting back to the finite case, this implies that the empirical distribution tends to the Gaussian distribution in an appropriate sense. In the talk we summarize this result and some of the background from graph limit theory. Joint work with Balázs Szegedy.

## Continuous matroids revisited

**Anders Björner**
Stockholm, Sweden

In this talk I will review work done in the mid-1980s in collaboration with László Lovász.

We give conditions for embeddings of finite geometric lattices that allow passing to the limit, the limit being a lattice with a continuous rank-measure-dimension function taking values in the unit interval. Borel-Lebesgue measure algebras and hyperfinite vonNeumann geometries are examples of this kind. From a combinatorial point of view, they can be thought of as continuous analogues of free matroids and linear matroids. We show that also partition lattices and field extensions have such continuous analogs, corresponding to the classes of graphic and algebraic matroids, respectively.

## The Sharp Threshold for Making Squares

**Béla Bollobás**
Cambridge and Memphis, U.K. and U.S.A.

Motivated by the fastest known algorithms for factoring large integers, in 1994 Pomerance posed the following problem. *Select integers* $a_1, a_2, \ldots$ *at random from the interval* $[1, x]$*, until the product of some (nonempty) subsequence is a perfect square. Find a good estimate for the expected stopping time* $t(x)$ *of this process.* In 1996, Pomerance proved that, for an explicitly constructed function $J_0(x)$, with probability tending to 1 we have $J_0(x)^{1+o(1)} \leq t(x) \leq J_0(x)$, and conjectured that the transition is sharp.

In 2012, Croot, Granville, Pemantle and Tetali significantly improved these bounds, bringing them within a constant factor of each other, and conjectured that their upper bound is sharp. In a recent paper, Paul Balister, Robert Morris and I have proved this conjecture. The problems arising are closely connected to smooth numbers, cores of hypergraphs, and random xorsat, about which there are numerous deep results. In my talk, I shall give a brief overview of these results together with some of the ideas used from number theory, combinatorics and stochastic processes in the proof of the sharp threshold.

## Graphexes and Multigraphexes: A Deeper Look

**Christian Borgs**
Cambridge, MA,, USA

In a recent series of papers, it was shown that the completion of the set of sparse graphs under sampling convergence (a notion of left convergence for sparse graphs) is the space of graphexes, where a graphex consists of a graphon over a sigma-finite measure space, a star density function from this space into $\mathbb{R}_+$, and real parameter called the dust density. In this talk, I further develop the theory of graphexes, including the definition of an analogue of the cut metric, the question of uniqueness, and an application to sparse configuration models. This work is joint work with J.T. Chayes, H. Cohn, S. Dhara, L.M. Lovasz, and S. Sen.

### Large deviations for random graphs

**Sourav Chatterjee**
Stanford, CA, USA

I will give a brief overview of large deviations for random graphs, and the role of graph limit theory in the development of this area.

### Graphons and Graphexes as Limits of Sparse Graphs

**Jennifer Chayes**
Cambridge, MA,, USA

Bounded graphons over a probability space play an important role in the theory of graph limits and modeling of dense graphs. After reviewing the dense theory, as well the extension to sparse graphs using unbounded graphons and convergence in the rescaled cut metric, I discuss a recent alternative theory. This new theory is based on the notion of sampling convergence, which is the appropriate generalization of subgraph convergence to sparse graphs. The natural limit objects in this theory are graphons over sigma-finite measure spaces, and, more generally, their extension to graphexes, which form the completion of the space of sparse graphs under sampling convergence. This is joint work with C. Borgs, H. Cohn, N. Holden and V. Veitch.

### Sidorenko-type inequality for determinants

**Péter Csikvári**
Budapest, Hungary
joint with Balázs Szegedy

We study a class of determinant inequalities that are closely related to Sidorenko's famous conjecture (also conjectured by Erdős and Simonovits in a different form). Our main result can also be interpreted as an entropy inequality for Gaussian Markov random fields. Connection with graph homomorphisms, Ihara zeta function and the number of spanning trees is also discussed. Joint work with Balázs Szegedy.

### Local algorithms on random graphs

**Endre Csóka**
Budapest, Hungary

Local algorithms on graphs mean constant-time randomized distributed algorithms. The most common objectives of these algorithms are to construct large cuts or independent sets or other locally defined structures on large graphs. It turns out that our best algorithms for most of these problems can be approximated by local algorithms. We give an overview about positive and negative results about what can be constructed by them, including some very recent algorithms and entropic bounds. All these questions have strong connections to sparse graph limit theory and statistical physics.

### Approximate counting; a conversation with Laci

**Persi Diaconis**
Stanford, CA, USA

There is a useful class of backtracking algorithms also known as sequential importance sampling that allow approximations to intractable counting problems (eg, self-avoiding walks on an $n \times n$ grid). Laci and I have started a conversation about how to use these algorithms as a proof technique.There are applications to Siderenko's conjecture,estimating partition functions and counting graph homomorphisims. I'll explain the ideas and hope to include the audience in the conversation.

### Polynomial method and graph bootstrap percolation

**Hamed Hatami**
Montreal, Canada
joint with Lianna Hambardzumyan and Yingjie Qian

We introduce a simple method for proving lower bounds for the size of the smallest percolating set in a certain graph bootstrap process. We apply this method to determine the sizes of the smallest percolating sets in multidimensional tori and multidimensional grids (in particular hypercubes). The former answers a question of Morrison and Noel, and the latter provides an alternative and simpler proof for one of their main results.

### Globally rigid graphs and frameworks

**Tibor Jordán**
Budapest, Hungary

A $d$-dimensional geometric graph (also called bar-and-joint framework) is said to be globally rigid if its edge lengths uniquely determine all pairwise distances between pairs of vertices. Global rigidity is known to be a generic property for every fixed dimension, which means that it depends only on the graph if the vertex coordinates are algebraically independent.

We shall summarize the key results, the main applications, as well as the recent developments of this branch of rigidity theory, focusing on the combinatorial aspects.

### Demolishing bridges and building new ones

**Gil Kalai**
Jerusalem, Israel

Quantum computers and "Quantum supremacy" serve as a wonderful bridge between quantum physics and computation that ignites the imagination of many people in academy, buisness, and politics. A quantum computer is a hypothetical physical device that exploits quantum phenomena such as interference and entanglement in order to enhance computing power. In the 1990s, Peter Shor discovered that quantum computers would make it possible to perform certain computational tasks hundreds of orders of magnitude faster than ordinary computers and, in particular, would break most of today's encryption methods.

Are quantum computers possible? I will describe my argument against quantum computers which also explains why the near term goals of many research groups and commercial companies of demonstrating "quantum supremacy" and building stable qubits will fail. The argument relies on a 2014 paper with Guy Kindler. Softly and carefully dismantling the quantum supremacy bridge may lead to new solid connections between quantum physics and computation and we will concentrate on the relevance of Benjamini-Kalai-Schramm's notion of noise stability.

### Singular Value Decomposition in Algorithms: Proofs to Practice

**Ravi Kannan**
Bangalore, India

Singular Value Decomposition (SVD) has long been studied by Numerical Analysts. More recently, it has found a number of applications in Machine Learning and Optimization. The talk will give a from-first-principles survey of the applications - to Clustering, Maximum Cut and related problems, Gaussian Mixtures, Topic Modeling, Non-negative Matrix factorization - for which we have proves of correctness, polynomial running time and error bounds. These provable algorithms also translate well to practice as we will briefly describe.

### Coloured and directed designs

**Peter Keevash**
Oxford, UK

We will give some illustrative applications of our recent result on decompositions of labelled complexes, such as the existence of resolvable hypergraph designs and large sets of hypergraph designs. We will also discuss some new results on decompositions of hypergraphs with coloured or directed edges. For example, we give fairly general conditions for decomposing an edge-coloured graph into rainbow triangles, and for decomposing an r-digraph into tight q-cycles.

### Some challenges in high-dimensional combinatorics

**Nathan Linial**
Jerusalem, Israel

The study of high-dimensional combinatorial objects leads to many fascinating new open problems. Here are a few examples: Brown Erdős and Sós showed that an $n$-vertex 3-uniform hypergraph with $cn^{5/2}$ hyperedges contains a triangulation of a 2-sphere, and the bound is sharp up to the value of $c$. Is the same statement true also for a torus? For other 2d surfaces? A simple probabilistic argument shows that the exponential of 5/2 cannot be improved. Major advances in the area of combinatorial designs (Keevash followed by Kuehn, Osthus , Lu and Glock) suggest that we should start investigating combinatorial designs in the well-established spirit of local graph theory. The existence of short cycles and discrepancy properties of Steiner Triple Systems, Latin squares and more are particularly suggestive. The ongoing search for locally testable codes and other challenges coming from computer science raise the interest in higher-dimensional notions and constructions of expanders. While we already have a reasonably good understanding of the difference between acyclicity and collapsibility in random simplicial complexes, the analogous questions for hypertrees remain widely open. How can we generate efficiently and uniformly random one-factorizations, Latin squares and other HD combinatorial structures? How can we generate many $3d$ permutations?

### Structural limits, lifts and clustering

**Jaroslav Nešetřil**
Prague, Czech Republic
joint with P. Ossona de Mendez

In the lecture we survey the recent development related to asymptotic properties of graphs (and structres) studied from the point of view of their structural convergence "at and near the limit". This provides an unifying view of various types of convergence and solves some open problems in the area related to graphings and modelings.

### Semantic Limits of Combinatorial Objects

**Alexander Razborov**
Chicago and Moscow, USA and Russia
joint with Leonardo Coregliano

The theory of limits of discrete combinatorial objects has been thriving for over a decade. There are two known approaches to it, one is geometric and semantic ("graph limits") and another is algebraic and syntactic ("flag algebras"). The language of graph limits is more intuitive and expressive while flag algebras are more versatile when it comes to generalizations to combinatorial objects other than graphs.

In this talk we present a general framework intending to combine useful features of both theories and compare it with previous attempts of this kind. As its underlying language, our theory uses the same simple concepts from first-order logic and model theory as flag algebras, and it capitalizes on the notion of an open interpretation that often allows to transfer methods and results from one situation to another. Many previous examples (hypergraphons, digraphons, permutons etc.) can be treated in this language quite uniformly.

## On $\vartheta$ and $\Theta$

**Alexander Schrijver**
Amsterdam, The Netherlands
joint with Sven Polak

An intriguing discovery of Jeroen Zuiddam is that for any graph $G$:

$$\Theta(G) = \min_{f\in\Delta} f(G). \tag{1}$$

Here $\Theta(G)$ is the Shannon capacity of $G$. Moreover, $\Delta$ is the set of all functions $f : \{\text{graphs}\} \to \mathbb{R}$ satisfying $f(K_1) = 1$ and for all graphs $G, H$:

> $f(G \sqcup H) = f(G) + f(H)$, $f(G \boxtimes H) = f(G)f(H)$, and if there exists a homomorphism $\overline{G} \to \overline{H}$ then $f(G) \leq f(H)$.

Zuiddam derived (1) from Strassen's semiring theorem, which uses Zorn's lemma. It gives rise to several questions, like to construct for any graph $G$ a function $f$ attaining the minimum in (1). The only functions in $\Delta$ explicitly known seem Laci Lovász's $\vartheta$, the fractional clique cover number $\overline{\chi}^*$, and Blasiak's fractional Haemers bounds. (The function $\Theta$ does not belong to $\Delta$.)

Another question is to understand the limit behaviour of the space of functions $\{\phi_G : \Delta \to \mathbb{R} \mid G \text{ graph}\}$, where $\phi_G(f) := f(G)$ for $f \in \Delta$. It is a subspace of the complete metric space $C(\Delta, \mathbb{R})_\infty$. (Note that $\Delta$ is a compact topological space in the Tychonoff product $\mathbb{R}^{\{\text{graphs}\}}$.)

We will discuss such questions and some results, and moreover consider extensions to hypergraphs.

## Needles in Exponential Haystacks

**Joel Spencer**
New York, USA

For Paul Erdős, the goal was to prove the existence of a mathematical object. In this century we further search for an efficient algorithm to find the object. We discuss the celebrated Local Lemma of Lovász and this speaker's "Six Standard Deviation" discrepency result. Their original proofs were existential. Modern arguments give efficient algorithms – and new proofs!

## Limits and exchangeability in the additive setting

**Balázs Szegedy**
Budapest, Hungary

The theory of dense graph limits, developed by Lovász and various co-authors, provides a powerful unified view point on many different subjects including Szemerédi's regularity, exchangeability and dense extremal graph theory. Motivated by the works of Szemerédi, Fürstenberg, Gowers and many others it is very natural to aim for a similar limit approach to additive combinatorics. This general program has lead to the development of nilspace theory and to periodic inverse theorems for the Gowers norms on compact abelian groups. In this talk we present a recent direction in this subject. We solve various exchangeability type problems related to additive combinatorics. As a major application we determine the structure of the characteristic factors of the Host-Kra seminorms for nilpotent actions. In particular, by solving an open problem, we generalise a celebrated result of Host and Kra to finitely generated nilpotent groups. As a second application we obtain limit objects for functions on abalian groups and more generally on nilmanifolds. The third application is a generalisation of the inverse theorem for the Gowers norms to arbitrary nilmanifolds. Joint work with Pablo Candela.

### Embedding graphs into larger graphs, results, methods and problems

**Endre Szemerédi**
Budapest, Hungary

Extremal Graph Theory is one of the fastest developing areas in Graph Theory. Fascinating and deep problems of Extremal Graph Theory largely contributed to the development of several deep methods of Discrete Mathematics. One of the aims of this lecture is to describe some of these methods, and to describe the strong interaction of Extremal Graph Theory and of the other parts of Discrete Mathematics or, more generally, of other branches of mathematics.

We shall describe several methods, and their applications, mainly in Combinatorics, Geometry, and Combinatorial Number Theory. The methods we shall described are the Stability method, Regularity Lemma, Blowup Lemma, Semi-random method, Absorbing method, application of Finite Geometries, and many others, mainly in Extremal Graph Theory.

### Directions and blocking sets

**Tamás Szőnyi**
Budapest, Hungary

In the sixties László Rédei wrote a book on *Lacunary Polynomials over Finite Fields*. One of the problems in the book (Section 36) is about the number of directions determined by a set of $q$ points in $\mathrm{AG}(2, q)$. If the points are not collinear and $q$ is a prime, then the number of directions is at least $(p+3)/2$. The proof seemed to use the machinery introduced in the book. Later Lovász and Schrijver gave a very nice short proof of the above bound and also described the sets attaining it. In the nineties, the corresponding problem for general $q$ was in the centre of interest and Blokhuis, Ball, Brouwer, Storme, Szőnyi essentially described sets determining less than $(q + 3)/2$ directions: they are translates of a vector space over a subfield of $\mathrm{GF}(q)$ (embedded in $\mathrm{AG}(2, q)$. Later Ball gave a very nice, short(er) proof treating the missing cases. Another very attractive result in the prime order case is due to Gács: if the set in $\mathrm{AG}(2, p)$ is neither a line nor the example described by Lovász and Schrijver, then it determines more than $2(p - 1)/3$ directions. In the talk we shall survey results about this problem, together with possible modifications, variants, generalizations.

A natural variant of the problem is related to blocking sets. The set of $q$ points together with the set of determined directions (as infinite points) has the property that it meets every line of the projective closure of $\mathrm{AG}(2, q)$. Such sets are called *blocking sets*. The analogue of Rédei's theorem for projective planes $\mathrm{PG}(2, p)$ of prime order $p$ is the famous theorem of Blokhuis, stating that a blocking set in these planes either contains a line or has at least $3(p + 1)/2$ points. This was conjectured in the sixties by Jane di Paola. In the second part of the talk generalizations and variants of the blocking set problem will be considered. In particular, possible sizes of minimal blocking sets will be discussed. We will also see bounds for multiple blocking sets, sets with few lines not meeting them, higher dimensional analogues, etc.

### Recent progress on Liouville sign patterns

**Terence Tao**
Los Angeles, USA

The Liouville function $\lambda(n)$ is defined to equal $+1$ when $n$ the product of an even number of primes, and $-1$ if it is the product of an odd number of primes. It is conjectured that for each $k$, the $k$-tuples $(\lambda(n), \ldots, \lambda(n+k-1))$ attain each sign pattern in $\{-1, \}^k$ with positive density. Thanks to recent advances, this is now known for $k = 1, 2, 3, 4$; we survey this and other related developments in this talk.

### Small-loss bounds for online learning with partial information

**Eva Tardos**
Ithaca, USA
joint with Thodoris Lykouris and Karthik Sridharan

We consider the problem of adversarial (non-stochastic) online learning with partial information feedback, where at each round, a decision maker selects an action from a finite set of alternatives. We develop a black-box approach for such problems where the learner observes as feedback only losses of a subset of the actions that includes the selected action. When losses of actions are non-negative, under the graph-based feedback model introduced by Mannor and Shamir, we offer algorithms that attain the so called "small-loss" $o(\alpha L^\star)$ regret bounds with high probability, where $\alpha$ is the independence number of the graph, and $L^\star$ is the loss of the best action.

### Extremal theory of vertex or edge ordered graphs

**Gábor Tardos**
Budapest, Hungary

Turán-type extremal graph theory asks the maximum number of edges in an $n$-vertex simple graph without a subgraph isomorphic to a specified forbidden pattern. In this talk we survey initial steps toward extending this theory to graphs with a specified linear order on the set of its vertices or edges. Instead of forbidding any subgraph isomorphic to a forbidden pattern, we only forbid it if the vertices (or edges) come in a certain specified order. This leads to a large class of new extremal problems, many of them open. Here is my favorite: Assume a vertex-ordered tree $T$ has a proper 2-coloring with the two color classes being intervals in the ordering. Is it true that the corresponding extremal function (the maximal number of edges in an $n$-vertex simple vertex-ordered graph without $T$ as a subgraph in this order) at most $n$ times polylog of $n$? This is known for many, but not all trees. In the case of edge ordered graphs the exact order of magnitude is not known for some very simple forbidden patterns including exactly one edge ordering of the four edge cycle and of the four edge path.

### A constant-factor approximation algorithm for the asymmetric traveling salesman problem

**László Végh**
London, United Kingdom
joint with Ola Svensson and Jakub Tarnawski

We give a constant-factor approximation algorithm for the asymmetric traveling salesman problem. Our approximation guarantee is analyzed with respect to the standard LP relaxation, and thus our result confirms the conjectured constant integrality gap of that relaxation.Our techniques build upon the constant-factor approximation algorithm for the special case of node-weighted metrics. Specifically, we give a generic reduction to structured instances that resemble but are more general than those arising from node-weighted metrics. For those instances, we then solve Local-Connectivity ATSP, a problem known to be equivalent (in terms of constant-factor approximation) to the asymmetric traveling salesman problem. This is joint work with Ola Svensson and Jakub Tarnawski.

### With or Without KLS

**Santosh Vempala**
Atlanta, USA

The Kannan-Lovász-Simonovits conjecture says that the Cheeger constant of any logconcave density is achieved to within a universal, dimension-independent constant factor by a hyperplane-induced subset. We survey several consequences of the conjecture (in geometry, probability, information theory and algorithms). A positive resolution of the conjecture suggests an $O^*(n^3)$ algorithm for computing the volume of a well-rounded convex body in $\mathbf{R}^n$. We present such a cubic algorithm; its analysis bypasses the full conjecture and uses only a special case when it provably holds (with Ben Cousins, 2015). We then present recent progress on the conjecture itself, resulting in the current best bound of $O(n^{1/4})$, as well as a tight bound for the log-Sobolev constant of isotropic logconcave densities (with Yin Tat Lee, 2017; 2018).

### Real roots of Random functions

**Van H. Vu**
New Haven, CT, USA

Locating the real roots of a real function (say, a algebraic or trigonometric polynomial) is a basic problem in mathematics. In the case the function is random, the study of this question, starting by Littlewood and Offord in the 1940s, has led to the theory of random functions.

In this talk, we present an introduction and several key results of this theory, concluding with recent results of Tao, Nguyen, Do and the speaker concerning some old questions.

### Mathematics and Computation (through the lens of one problem and one algorithm)

**Avi Wigderson**
Princeton, USA

Mathematics and computation have gone hand in hand for millennia. Many of the greatest mathematicians we great algorithm designers as well, including Euclid, Newton, Gauss and Hilbert. And with the arrival of the theory of computation, and then computational complexity, these connections have become far broader, deeper and stronger.

This lecture will illustrate these connections by focusing on a single computational problem, *Singularity of Symbolic Matrices*, and a single algorithmic technique for it, *Alternate Minimization.* As it happens, recent attempts to understand these have uncovered a surprisingly rich web of connections between diverse areas of mathematics and computer science, all of which contributing and benefitting from this interaction. In math these include non-commutative algebra, invariant theory, quantum information theory and analysis. In computer science they include optimization, algebraic complexity and pseudorandomness.

I will motivate the problem and the algorithm, and discuss at high level how a combination of the areas above drive its complexity analysis. This algorithm and its extensions efficiently solve new non-convex programs, and give weak membership oracles to new classes of linear programs with exponentially many facets.

Based on several recent works, with Zeyuan Allen-Zhu, Peter Buergisser, Cole Franks, Ankit Garg, Leonid Gurvits, Pavel Hrubes, Yuanzhi Li, Rafael Oliveira, and Michael Walter.

# Contributed Posters

### Coin tossing with a die

**Simon Apers**
Ghent, Belgium
joint with Florian Adriaens

Consider a coin toss experiment, where we toss $n$ times a fair coin and we sum the outcomes ($+1$ for heads, $-1$ for tails). Can we simulate this experiment with a fair die (sides $\pm 1, \pm 2, \ldots, \pm k$), while significantly lowering the number of tosses?

Surprisingly, we show that tossing only a constant number of times a die of $O(\sqrt{n})$ sides allows to simulate the coin experiment, up to constant error. More specifically, we prove that the outcome probability distribution of tossing $m$ times a die of $\sqrt{3n/m}$ sides is $1/\sqrt{m}$-close in total variation distance to the outcome of the coin experiment.

Finally, we highlight the origin of this question in research on quantum algorithms, and we show how the observation relates to the use of second-order Markov chains to speed up mixing, as described in for instance [Diaconis, Miclo (2013)].

[1] Diaconis, P., and Miclo, L. "On the spectral analysis of second-order Markov chains." Annales de la Faculté des Sciences de Toulouse. Mathématiques. Série 6 22.3 (2013): 573-621.

### On path-path Ramsey minimal graphs

**Hilda Assiyatun**
Institut Teknologi Bandung, Bandung, Indonesia
joint with D. Rahmadani, E.T. Baskoro

For any two given graphs $G$ and $H$, the notation $F \to (G, H)$ means that any red-blue coloring of all edges of $F$ creates either a red subgraph isomorphic to $G$ or a blue subgraph isomorphic to $H$. A graph $F$ is a Ramsey $(G, H)-$minimal graph if $F \to (G, H)$ but $F - e \nrightarrow (G, H)$, for every $e \in E(F)$. The class of all Ramsey $(G, H)-$minimal graphs is denoted by $\mathcal{R}(G, H)$. It is known that $\mathcal{R}(P_m, P_n)$, for $n \geq m \geq 3$ is infinite. In this talk we will present some latest results on $\mathcal{R}(P_m, P_n)$, for $n \geq m \geq 3$.

### Illustrating the Co-authorship Network of László Lovász

**Béla Barabás**
Budapest, Hungary
joint with Ottilia Fülöp, Roland Molonta

Research collaboration is a fundamental mechanism that unites all kinds of knowledge and different areas of expertise into common new original ideas and significant results. Considering the lists of publications of László Lovász from Google Scholar and from the Hungarian bibliographic database MTMT retrieved on 21 December 2017 we found 602 and 373 scientific publications (articles, books, book chapters, conference papers, patents, etc.), respectively. In the present poster we illustrate and analyze the network determined by all co-authors of Lovász , considering only common papers from the above-mentioned lists.

### Unicyclic Ramsey $(mK_2, P_4)$-minimal graphs

**Edy Tri Baskoro**
Institut Teknologi Bandung (ITB), Bandung, Indonesia

Let $F, G$ and $H$ be simple graphs. We say that $F \to (G, H)$ if in any red-blue coloring of all edges of $F$ there is always a red subgraph isomorphic to $G$ or a blue subgraph isomorphic to $H$. Such a graph $F$, then, is called a *Ramsey* $(G, H)$ *graph.* A Ramsey $(G, H)$ graph $F$ is *minimal* if for any edge $e \in E(F)$, the subgraph $F - e$ is no longer a Ramsey $(G, H)$ graph. The problem of characterising all Ramsey $(G, H)$-minimal graphs for certain graphs $G$ and $H$ is classified as one of difficult problems in graph theory. Even, it is for small graphs $G$ and $H$. In this paper, we characterize all unicyclic Ramsey $(G, H)$-minimal graphs if $G$ is a matching and $H$ is a path $P_4$.

### On the greedy algorithm for Steiner forest

**Yixin Cao**
Hong Kong, China
joint with Yixin Cao

Given an edge-weighted graph $G$ and a set of terminal pairs, the Steiner forest problem asks for a cheapest forest such that each of the terminal pairs are in the same tree. A 2-approximation, based on primal-dual, has been known for nearly three decades (Agrawal, Klein, and Ravi, STOC 1991). Unlike Steiner tree, however, this remains the best known result. On the other hand, an algorithm using "purely combinatorial" techniques and achieving the same ratio has been sought for a while. One candidate was the following greedy algorithm. A terminal is active until it is connected its pair via picked edges. The algorithm repititively takes two active terminals with the minimum distance (disregarding whether they are a pair) and pick a shortest path to connect them. It stops when all terminals a re inactive, and the subgraph on the pick edges is a solution. Gupta and Kumar (STOC 2015) proved that this algorithm has a ratio 96, and conjectured that its ratio is actually 2. We show that this is not true, by giving an infinite set of instances on which the ratio is arbitrarily close to 8/3. We provide also strong evidence that 8/3 is the correct ratio.

### Tiling edge-coloured graphs with few monochromatic tiles

**Jan Corsten**
London, United Kingdom
joint with Sebastián Bustamante, Nóra Frankl, Alexey Pokrovskiy, Jozef Skokan

A *tiling* $\mathcal{T}$ of a graph $G$ is a set of subgraphs of $G$ whose vertex-sets partition $V(G)$. If a colouring (of the edges) of $G$ is given, a tiling $\mathcal{T}$ is called monochromatic if every $H \in \mathcal{T}$ is monochromatic (not necessarily in the same colour). In 1991, Erdős, Gyárfás and Pyber proved that, for every $r, n \in \mathbb{N}$, every $r$-coloured $K_n$ contains a monochromatic tiling consisting of $O(r^2 \log r)$ cycles. In particular, the number of cycles needed to guarantee a monochromatic cycle-tiling of $K_n$ does not depend on $n$. We are interested in finding other examples, where it is possible to tile complete graphs with few (i.e. a number bounded in $n$) monochromatic tiles.

In 2013, Gyárfás and Sárközy generalised this result to hypergraphs by showing that the complete $k$-uniform hypergraph can be tiled with few monochromatic loose cycles. Confirming a conjecture of Gyárfás, we prove the corresponding result for tight cycles. We further show that in fact every graph with bounded independence number can be tiled with few monochromatic tight cycles (similar results have been proved for loose cycles by Gyárfás and Sárközy).

As an easy corollary of our result we obtain that complete graphs can be tiled with few monochromatic $k$-th powers of cycles, which solves a problem of Elekes, D. Soukup, L. Soukup and Szentmiklóssy. A conjecture of Grinshpun and Sárközy states that something even stronger should hold. Given a family $\mathcal{F}$ of graphs with bounded degree, containing exactly one graph with $i$ vertices for each $i \in \mathbb{N}$, complete graphs can be tiled with few tiles from $\mathcal{F}$. Grinshpun and Sárközy proved the conjecture for two colours, but it remains open for any $r \geq 3$. We prove an infinite analogue of the conjecture (for every $r$). This extends a result of Elekes, D. Soukup, L. Soukup and Szentmiklóssy, who proved the corresponding result for powers of cycles.

### Almost orthogonal vectors

**Christopher Cox**
Pittsburgh, USA
joint with Boris Bukh

How can $d+k$ vectors in $\mathbb{R}^d$ be arranged so that they are as close to orthogonal as possible? In particular, define $\theta(d,k) := \min_X \max_{x \neq y \in X} |\langle x, y\rangle|$ where the minimum is taken over all collections of $d+k$ unit vectors $X \subseteq \mathbb{R}^d$; we focus on the case where $k$ is fixed and $d \to \infty$. In establishing bounds on $\theta(d,k)$, we find an intimate connection to the existence of large systems of equiangular lines in $\mathbb{R}^k$. Using this connection, we are able to pin down $\theta(d,k)$ precisely whenever $k \in \{1, 2, 3, 7, 23\}$ and establish reasonable asymptotics for general $k$. This connection is formed by providing tight upper bounds on $\mathbb{E}_{x,y\sim\mu}|\langle x, y\rangle|$ whenever $\mu$ is an isotropic probability mass on $\mathbb{R}^k$, which may be of independent interest.

Star chromatic index of the Cartesian product of graphs

**Marzieh Vahid Dastjerdi**
Isfahan, Iran
joint with Behnaz Omoomi

In 2008, Liu and Deng in [An upper bound for the star chromatic index of graphs with $\Delta \geq 7$. Journal of Lanzhou University, 44(2):98-99, 2008] introduced the concept of star edge coloring that is defined as follows. A *star edge coloring* of $G$ is a proper edge coloring of $G$ such that each path and cycle of length four in $G$ uses at least three colors. The smallest integer $k$ for which $G$ admits a $k$-star edge coloring is called the *star chromatic index* of $G$ and is denoted by $\chi'_s(G)$.

For a star edge coloring of a graph $G$, and for every vertex $v \in V(G)$, let $A_f(v)$ be the set of colors of edges incident to $v$. Two star edge colorings $f_1$ and $f_2$ of $G$ are called *star compatible* if for every vertex $v$, $A_{f_1}(v) \cap A_{f_2}(v) = \emptyset$. We say that graph $G$ is $(k,t)$*-star colorable* if $G$ has $t$ pairwise star compatible colorings $f_i : E(G) \to \{0, 1, \ldots, k-1\}$, $1 \leq i \leq t$.

In this paper using the concept of star compatibility, we find upper bounds on the star chromatic index of the Cartesian product of graphs and determine the exact velue of the star chromatic index of some inportant famileis of graphs. The *Cartesian product* of two graphs $G$ and $H$, denoted by $G \square H$, is a graph with vertex set $V(G) \times V(H)$, and $(a,x)(b,y) \in E(G \square H)$ if either $ab \in E(G)$ and $x = y$, or $xy \in E(H)$ and $a = b$.

The main results of this paper are as follows:

## Main Results

(i) If $G$ and $H$ are two graphs such that $G$ is $(k_G, t_G)$-star colorable and $t_G \geq \chi(H)$, then
$$\chi'_s(G \square H) \leq k_G + \chi'_s(H).$$

(ii) For two paths $P_m$ and $P_n$ with $m, n \geq 2$, we have

$$\chi'_s(P_m \square P_n) = \begin{cases} 3 & \text{if } m = n = 2, \\ 4 & \text{if } (m = 2, n \geq 3) \text{ or } (m \geq 3, n = 2), \\ 5 & \text{if } (m \in \{3,4\}, n = 3) \text{ or } (m = 3, n \in \{3,4\}), \\ 6 & \text{otherwise.} \end{cases}$$

(iii) For path $P_m$ and cycle $C_n$, we have

$$\chi'_s(P_m \square C_n) = \begin{cases} 4 & \text{if } m = 2, n = 4, \\ 5 & \text{if } m = 2, n \geq 5, \\ 6 & \text{if } (m \geq k-1, n = 0 \pmod{k}, k \in \{3,4\}) \text{ or } (m \in \{3,4\}, n = 2 \pmod{4}), \\ \leq 7 & \text{otherwise.} \end{cases}$$

(iv) If $m$ and $n$ are even integers, then $\chi'_s(C_m \square C_n) \leq 7$.
(v) If $m$ or $n$ is an odd integer, then $\chi'_s(C_m \square C_n) \leq 8$.

## Large Monochromatic Components and Long Monochromatic Cycles in Sparse Random Hypergraphs

**Sean English**
Kalamazoo, United States of America
joint with Patrick Bennett, Louis DeBiasio and Andrzej Dudek

It is known, due to Gyárfás and Füredi, that for any $r$-coloring of the edges of $K_n$, there is a monochromatic component of order $(1/(r-1)+o(1))n$. Recently, Bal and DeBiasio, and independently Dudek and Prałat showed that the Erdős-Rényi random graph $\mathcal{G}(n,p)$ behaves very similarly with respect to the size of the largest monochromatic component. More precisely, it was shown that a.a.s. for any $r$-coloring of the edges of $\mathcal{G}(n,p)$ and arbitrarily small constant $\alpha > 0$, there is a monochromatic component of order $(1/(r-1)-\alpha)n$, provided that $pn \to \infty$. As before, this result is clearly best possible.

We present a generalization of this result to hypergraphs. Specifically we show that in the $k$-uniform random hypergraph, $\mathcal{H}^{(k)}(n,p)$ a.a.s. for any $k$-coloring of the edges, there is a monochromatic component of order $(1-\alpha)n$ and for any $k+1$ coloring, there is a monochromatic component of order $(1-\alpha)\frac{k}{k+1}n$, as long as $pn^{k-1} \to \infty$.

It is also known due to Gyárfás, Sárközy and Szemerédi that the Ramsey number for loose cycles on $n$ vertices in $k$-uniform hypergraphs is asymptotically $\frac{2k-1}{2k-2}n$. We will present a generalization of this which shows that even if the host graph is $H^{(k)}_{n,p}$, this result still holds a.a.s. provided that $pn^{k-1} \to \infty$.

## Slow mixing of Glauber dynamics for the six-vertex model in the ferroelectric phase

**Matthew Fahrbach**
Atlanta, United States of America
joint with Dana Randall

We analyze the effect of boundary conditions on the mixing time of Glauber dynamics for the six-vertex model in the ferroelectric phase and on the critical line separating the ferroelectric and disordered phases. Specifically, we show that there exist boundary conditions in these regimes such that local Markov chains require exponential time to converge to equilibrium. This is the first rigorous result about the mixing time of Glauber dynamics for the six-vertex model when the stationary distribution is nonuniform.

Originally introduced to investigate the entropy and hydrogen bond structure of ice, the six-vertex model is a multivariate model of ferroelectric solids. It is defined on a square lattice where each vertex is connected by an edge to its four nearest neighbors. A state in the model is an orientation of the edges such that each vertex has two edges pointing inwards. There are six possible types of edge configurations around a vertex, and each type is assigned its own Boltzmann weight. The phase diagram of the six-vertex model, which can be derived from the Yang–Baxter equation, demonstrates that slight changes in the Boltzmann weights can cause the system to have remarkably different thermodynamic properties. A second compelling feature of this model is its sensitivity to boundary conditions. In this work we explore algorithmic challenges that arise from boundary conditions in various phases.

The six-vertex model with domain wall boundary conditions has been extensively studied in statistical physics and plays an essential role in our analysis of the Glauber dynamics. For the domain wall boundary conditions, the partition function has a determinantal formula with well-understood asymptotic bounds and a surprisingly deep connection to the enumeration of alternating sign matrices in combinatorics. Using insight and recent developments for the partition function of the six-vertex model with domain wall boundary conditions, we analyze a new boundary condition consisting of smaller, disjoint domain wall boundary conditions. This construction allows us to show that the conductance of local Markov chains can be exponentially small in all of the ferroelectric phase and on the critical line separating the ferroelectric phase from the disordered phase. This Markov chain was previously only known to be rapidly mixing in the disordered phase, for any boundary conditions, when the Boltzmann distribution is uniform.

### Necessary and sufficient conditions for sub-exponential rate of growth of the number of multiplicative structures

**Boris Granovsky**
Haifa, Israel

We prove the necessity of the main sufficient condition of Meinardus for subexponential growth of the number of decomposable structures (such as e.g. partitions), having a multiplicative generating function of a general form and establish new necessary and sufficient condition for local limit theorem in Khintchine-Meinardus method for asymptotic enumeration. The exposition is based on the recent paper by the author, published in The Ramanujan journal.

### A degree sequence Komlós theorem

**Joseph Hyde**
Birmingham, United Kingdom
joint with Hong Liu, Andrew Treglown

Given graphs $G$ and $H$, we define an *$H$-tiling in $G$* to be a collection of vertex-disjoint copies of $H$ in $G$. Let $\varepsilon > 0$. We call an $H$-tiling *perfect* if it covers all of the vertices in $G$ and *$\varepsilon$-almost perfect* if it covers all but at most an $\varepsilon$-proportion of the vertices in $G$. An important theorem of Komlós provides the minimum degree of $G$ which ensures an $\varepsilon$-almost perfect $H$-tiling in $G$. We present a degree sequence strengthening of this result. This is joint work with Hong Liu and Andrew Treglown.

Spanning Tree Congestion and Computation of Generalized Győri-Lovász Partition

**Davis Issac**
Saarbruecken, Germany
joint with L. Sunil Chandran, Yun Kuen Cheung

We study a natural problem in graph sparsification, the Spanning Tree Congestion (STC) problem.

Informally, the STC problem seeks a spanning tree with no tree-edge *routing* too many of the original edges. The root of this problem dates back to at least 30 years ago, motivated by applications in network design, parallel computing and circuit design. Variants of the problem have also seen algorithmic applications as a preprocessing step of several important graph algorithms.

For any general connected graph with $n$ vertices and $m$ edges, we show that its STC is at most $\mathcal{O}(\sqrt{mn})$, which is asymptotically optimal since we also demonstrate graphs with STC at least $\Omega(\sqrt{mn})$. We present a polynomial-time algorithm which computes a spanning tree with congestion $\mathcal{O}(\sqrt{mn} \cdot \log n)$. We also present another algorithm for computing a spanning tree with congestion $\mathcal{O}(\sqrt{mn})$; this algorithm runs in sub-exponential time when $m = \omega(n \log^2 n)$.

For achieving the above results, an important intermediate theorem is *generalized Győri-Lovász theorem*, for which Chen et al. [JACM 2007] gave a non-constructive proof. We give the first elementary and constructive proof by providing a local search algorithm with running time $\mathcal{O}^*(4^n)$, which is a key ingredient of the above-mentioned sub-exponential time algorithm. We discuss a few consequences of the theorem concerning graph partitioning, which might be of independent interest.

We also show that for any graph which satisfies certain *expanding properties*, its STC is at most $\mathcal{O}(n)$, and a corresponding spanning tree can be computed in polynomial time. We then use this to show that a random graph has STC $\Theta(n)$ with high probability.

The Ulam-Hammersley problem for heapable sequences

**Gabriel Istrate**
Timişoara, Romaniia
joint with (joint work with Cosmin Bonchiş, János Balogh, Diana Diniş, Ioan Todincă)

A set of integers is, informally, called *heapable* (Byers et al., ANALCO'2011) if its elements can be inserted into a binary tree (not necessarily complete) that respects the heap property. The definition naturally extends to partial orders.

We investigate the partitioning of partial orders into a minimal number of heapable subsets. We prove a characterization result reminiscent of the proof of Dilworth's theorem, which yields as a byproduct a flow-based algorithm for computing such a minimal decomposition. On the other hand, for interval partial orders a longest heapable subsequence can be computed in polynomial time. for trapezoid partial orders we prove that a minimal decomposition can be computed by a simple greedy-type algorithm.

The talk will also discuss a couple of open problems related to the analog of the Ulam-Hammersley problem for decompositions of sets and sequences of elements of a partial order into heapable subsets.

### Grove probabilities and the double-dimer model

**Helen Jenne**
Eugene, United States

Given a finite edge-weighted planar graph and a set of vertices on its outer face called nodes, a grove is a spanning forest in which each component tree contains at least one node. Kenyon and Wilson computed the probabilities of different possible node connections in a grove when the probability of a grove is proportional to the product of its edge weights. Given a planar partition $\sigma$, let $\dddot{\Pr}(\sigma)$ denote an appropriately normalized probability that a random grove partitions the nodes according to $\sigma$. Kenyon and Wilson showed that $\dddot{\Pr}(\sigma)$ is an integer-coefficient polynomial in the entries of the response matrix (or Dirichlet-to-Neumann matrix) on the nodes. Similarly, in the double-dimer model, connection probabilities of boundary nodes are integer-coefficient polynomials of boundary measurements.

I will show that given a planar partition $\sigma$ of $1, 2, \ldots, n$, a certain specialization of the polynomial $\dddot{\Pr}(\sigma)$ is an integer linear combination of polynomials $\dddot{\Pr}(\tau)$, where each $\tau$ is a partition of $\{1, 2, \ldots, n-2\}$. I will also present a conjecture that a similar result holds for double-dimer configurations. A corollary to this conjecture is that an identity similar to Kuo's condensation identity for perfect matchings of a bipartite graph holds for double-dimer configurations.

### Intervals in the Hales–Jewett theorem

**Nina Kamčev**
Zürich, Switzerland
joint with David Conlon

The Hales–Jewett theorem is one of the central results in Ramsey theory. Quoting Graham, Rothschild and Spencer, it "strips van der Waerden's theorem of its unessential elements and reveals the heart of Ramsey theory. It provides a focal point from which many results can be derived and acts as a cornerstone for much of the more advanced work."

Stating the theorem requires some notation. Given natural numbers $m$ and $n$, let $[m]^n$ be the collection of all $n$-letter words, where each letter is taken from the alphabet $[m] = \{1, 2, ..., m\}$. Given a word $w$ from $[m]^n$, a subset $S$ of $[n]$ and an element $i$ of $[m]$, let $w(S, i)$ be the word obtained from $w$ by replacing the $j$th letter with $i$ for all $j$ in $S$. A *combinatorial line* in $[m]^n$ with *wildcard set* $S \neq \emptyset$ is then a subset of the form $\{w(S, 1), w(S, 2), \ldots, w(S, m)\}$. Hales and Jewett have shown that for any $m$ and $r$, there exists a natural number $n$ such that any $r$-colouring of the elements of $[m]^n$ contains a monochromatic combinatorial line.

It is easy to see that any $r$-colouring of $[2]^r$ contains a monochromatic line whose wildcard set is an interval in $[r]$. Moreover, for an alphabet of $m = 3$ and any $r$, Shelah's single-induction proof grants a monochromatic combinatorial line in $[3]^n$ where the wildcard set is the union of at most $r$ intervals.

We show that there are situations where one can do no better, suggesting that Shelah's proof strategy is, at least in some sense, necessary. Namely, for any $n$ and any odd $r > 1$, there is an $r$-colouring of $[3]^n$ containing no monochromatic combinatorial line whose wildcard set is the union of fewer than $r$ intervals.

Surprisingly, Imre Leader and Eero Räty have shown that our result does not extend to $r = 2$, that is, in any two-colouring of $[3]^n$, there exists a combinatorial line whose wildcard set is an interval. Whether the minimal number of intervals for even $r \geq 4$ is $r - 1$ or $r$ remains an open question.

## Sufficient connectivity conditions for rigidity of symmetric frameworks

**Viktória E. Kaszanitzky**
Budapest, Hungary
joint with Bernd Schulze

A $d$-dimensional *(bar-joint) framework* is a pair $(G, p)$, where $G = (V, E)$ is a finite simple graph and $p : V \to \mathbb{R}^d$ is a map. A $d$-dimensional framework is called *rigid* if, the vertices cannot be moved continuously in $\mathbb{R}^d$ to obtain another non-congruent framework while keeping the lengths of all edges fixed.

A framework $(G, p)$ is called *generic* if the coordinates of the image of $p$ are algebraically independent over $\mathbb{Q}$. Laman's landmark result from 1970 gives a combinatorial characterisation of generic rigid frameworks in $\mathbb{R}^2$ [2]. In [3] Lovász and Yemini established sufficient graph connectivity conditions for the rigidity of generic frameworks in $\mathbb{R}^2$. Their result was recently improved by Jackson and Jordán in [1]. Analogous results for higher dimensions have not yet been found.

Rigidity theory has applications in various areas of science and technology. Since many structures in these areas of application exhibit non-trivial symmetries, the study of how symmetry impacts the rigidity and flexibility of frameworks has become a highly active research area in recent years.

There are two basic approaches to this problem. First, one may ask whether a framework is 'forced-symmetric rigid', that is it can not be deformed without breaking the original symmetry of the structure. One may also ask if a symmetric framework is 'incidentally symmetric', i.e., whether it does not have *any* non-trivial deformations.

We extend the sufficient graph connectivity conditions for generic rigidity established in [1,3] to both forced-symmetric and incidentally symmetric frameworks. For all of these results, we also provide examples that show that our conditions are best possible.

[1] B. Jackson and T. Jordán, A sufficient connectivity condition for generic rigidity in the plane, *Discrete Applied Mathematics* **157**(8) (2009), 1965–1968.

[2] G. Laman, On graphs and rigidity of plane skeletal structures, *J. Engineering Math.* **4** (1970), 331–340.

[3] L. Lovász and Y. Yemini, On generic rigidity in the plane, *SIAM J. Algebraic Discrete Methods* **3**(1) (1982), 91–98.

### The Isomorphism Problem for Monomial Digraphs

**Alex Kodess**
Kingston, RI, United States

Let $p$ be a prime, let $e$ be a positive integer, $q = p^e$, and let $\mathbb{F}_q$ denote the finite field of $q$ elements. Let $m, n$, $1 \leq m, n \leq q-1$, be integers. The monomial digraph $D = D(q; m, n)$ is defined as follows: the vertex set of $D$ is $\mathbb{F}_q^2$, and $((x_1, x_2), (y_1, y_2))$ is an arc in $D$ if $x_2 + y_2 = x_1^m y_1^n$. We study the question of isomorphism of monomial digraphs $D(q; m_1, n_1)$ and $D(q; m_2, n_2)$. We conjecture that $D(q; m_1, n_1) \cong D(q; m_2, n_2)$ if and only if $(m_2, n_2) = k(m_1, n_1)$ for some integer $k$ coprime with $(q-1)$. While the sufficiency of this condition is known, its necessity remains an open question. We present a number of partial results that support the conjecture.

### Linear algebraic analogues of the graph isomorphism problem and the Erdős-Rényi model

**Yinan Li**
Sydney and Amsterdam, Australia and Netherlands
joint with Youming Qiao

A classical difficult isomorphism testing problem is to test isomorphism of $p$-groups of class 2 and exponent $p$ in time polynomial in the group order. It is known that this problem can be reduced to solving the alternating matrix space isometry problem over a finite field in time polynomial in the underlying vector space size. We propose a venue of attack for the latter problem by viewing it as a linear algebraic analogue of the graph isomorphism problem. This viewpoint leads us to explore the possibility of transferring techniques for graph isomorphism to this long-believed bottleneck case of group isomorphism.

In 1970's, Babai, Erdős, and Selkow presented the first average-case efficient graph isomorphism testing algorithm (SIAM J Computing, 1980). Inspired by that algorithm, we devise an average-case efficient algorithm for the alternating matrix space isometry problem over a key range of parameters, in a random model of alternating matrix spaces in vein of the Erdős-Rényi model of random graphs. For this, we develop a linear algebraic analogue of the classical individualization technique, a technique belonging to a set of combinatorial techniques that has been critical for the progress on the worst-case time complexity for graph isomorphism, but was missing in the group isomorphism context. As a consequence of the main algorithm, we establish a weaker linear algebraic analogue of Erdős and Rényi's classical result that most graphs have the trivial automorphism group. We also show that Luks' dynamic programming technique for graph isomorphism (STOC 1999) can be adapted to slightly improve the worst-case time complexity of the alternating matrix space isometry problem in a certain range of parameters.

Most notable progress on the worst-case time complexity of graph isomorphism, including Babai's recent breakthrough (STOC 2016) and Babai and Luks' previous record (STOC 1983), has relied on both group theoretic and combinatorial techniques. By developing a linear algebraic analogue of the individualization technique and demonstrating its usefulness in the average-case setting, the main result opens up the possibility of adapting that strategy for graph isomorphism to this hard instance of group isomorphism. The linear algebraic Erdős-Rényi model is of independent interest and may deserve further study. In particular, we indicate a connection with enumerating $p$-groups of class 2 and exponent $p$.

### The quantum homomorphism order is universal


**Yangjing Long**
Wuhan, China
joint with Zhicong Lin, Hehui Wu


Graph multihomomorphisms were introduced by Lovász in the proof of Kneser Conjecture. We give an explicit expression which transfer the number of multihomomorphisms to the number of homomorphisms. We also proved that $\hom(P_n, G) = \hom(S_n, G)$ if and only if any connected components of $G$ are regular graphs.


### The quantum homomorphism order is universal



**Yangjing Long**
Wuhan, China


Quantum homomorphisms on undirected graphs was introduced by Marinska and D.E. Roberson. This can be naturally generalized on the setting of directed graphs. Very little is known about the existence of quantum homomorphisms. We mainly consider the partial orders induced by the existence of quantum homomorphisms and proved that the partial orders on undirected graphs is universal.

### On weakly distinguishing graph polynomials


**Johann Makowsky**
Haifa, Israel
joint with Vsevolod Rakita


Let $P$ be a graph polynomial. A graph $G$ is *$P$-unique* if every graph $H$ with $P(G;X) = P(H;X)$ is isomorphic to $G$. A graph $H$ *is a $P$-mate* of $G$ if $P(G;X) = P(H;X)$ but $H$ is not isomorphic to $G$. In [M. Noy, 2003] $P$-unique graphs are studied for the Tutte polynomial $T(G;X,Y)$, the chromatic polynomial $\chi(G;X)$, the matching polynomial $m(G;X)$ and the characteristic polynomial $char(P;X)$.

A graph polynomial $P$ is *almost complete* if almost all graphs $G$ are $P$-unique, and it is *weakly distinguishing* if almost all graphs $G$ have a $P$-mate. In [B. Bollobás and L. Pebody and O. Riordan, 2000] it is conjectured that almost all graphs are $\chi$-unique, and hence $T$-unique, in other words, both $\chi(G;X)$ and $T(G;X,Y)$ are almost complete. There are plenty of trivial graph polynomials which are weakly distinguishing, like $X^{|V(G)|}$ or $X^{|E(G)|}$. However, one might expect that the prominent graph polynomials from the literature are not weakly distinguishing. However, here we show that various non-trivial graph polynomials are still weakly distinguishing.

For a vertex $v \in V(G)$ of a graph $G$ let $d_G(v)$ denote the degree of $v$ in $G$. The degree polynomial $Dg(G;X)$ of a graph $G$ is defined as $Dg(G;X) = \sum_{v \in V(G)} X^{d_G(v)}$. A graph $G$ is $Dg$-unique, also called in the literature a *unigraph*, if it is determined by its degree sequence. An updated discussion on how to recognize unigraphs can be found in [A. Borri, T. Calamoneri and R. Petreschi, 2011].

A simple counting argument gives:

**Theorem 1:** Almost all graphs $G$ have an $Dg$-mate.

For a graph $G = (V(G), E(G))$ and $A \in V(G)$ we denote by $G[A]$ the subgraph of $G$ induced by $A$. Let $\mathcal{E}$ be the class of edgeless graphs and $\mathcal{C}$ be the class of complete graphs. The independence polynomial of a graph is defined as $Ind(G;X) = \sum_{A \in V(G): G[A] \in \mathcal{E}} X^{|A|}$. The clique polynomial of a graph is defined as $Cl(G;X) = \sum_{A \in V(G): G[A] \in \mathcal{C}} X^{|A|}$. Both were first studied in [C. Hoede and X. Li, 1994]. For a more recent survey on the independence polynomial see [V.E. Levit and E. Mandrescu, 2005].

**Theorem 2:** Almost all graphs $G$ have an $Ind$-mate and an $Cl$-mate.

The proofs use estimates for the independence number $\alpha(G)$ and the clique number $\omega(G)$ for random graphs, see [B. Bollobás and P. Erdős, 1976 and A.M. Frieze, 1990].

Theorem 2 can be generalized to the following:

Let $\mathcal{C}$ be a graph property of bounded tree-width and let $\hat{\mathcal{C}}$ be the class of complement graphs $\bar{G}$ of grphs $G \in \mathcal{C}$. Let $P_{\mathcal{C}}(G;X) = \sum_{A \in V(G): G[A] \in \mathcal{C}} X^{|A|}$. and $P_{\hat{\mathcal{C}}}(G;X) = \sum_{A \in V(G): G[A] \in \hat{\mathcal{C}}} X^{|A|}$.

**Theorem 3:** Almost all graphs $G$ have an $P_{\mathcal{C}}(G;X)$-mate and an $P_{\hat{\mathcal{C}}}(G;X)$-mate.

A vertex coloring of $G$ with at most $k$ colors is a proper coloring of $G$ such that every pair of colors occurs at most once along an edge. Let $\chi_{harm}(G;k)$ count the number of harmonious colorings of $G$. It was observed in [J.A. Makowsky and B. Zilber, 2006] that $\chi_{harm}(G;k)$ is a polynomial in $k$.

**Theorem 4:** Almost all graphs $G$ have an $\chi_{harm}$-mate.

The proof uses results from [E. Drgas-Burchardt and K. Gibek, 2017]. The same holds for an infinite set of less prominent graph polynomials, but the status of $P$-uniqueness remains open for $T(G;X,Y), \chi(G;X), m(G;X)$ and $char(G;X)$.

### The step Sidorenko property and non-norming edge-transitive graphs

**Taísa Martins**
University of Warwick, UK
joint with Daniel Král', Péter Pál Pach, Marcin Wrochna

Sidorenko's Conjecture asserts that every bipartite graph $H$ has the Sidorenko property, i.e., a quasirandom graph minimizes the density of $H$ among all graphs with the same edge density. We study a stronger property, which requires that a quasirandom multipartite graph minimizes the density of $H$ among all graphs with the same edge densities between its parts; this property is called the step Sidorenko property. We show that many bipartite graphs fail to have the step Sidorenko property and use our results to show the existence of a bipartite edge-transitive graph that is not weakly norming; this answers a question of Hatami [Israel J. Math. 175 (2010), 125–150].

### A New Probabilistic Proof of Selkows Bound on the Independence Number of Graphs

**Samuel Mohr**
Ilmenau, Germany
joint with Jochen Harant

For a graph $G$ with vertex set $V(G)$ and independence number $\alpha(G)$, S. M. Selkow (Discrete Mathematics, 132(1994)363–365) established the famous lower bound $\sum\limits_{v\in V(G)} \frac{1}{d(v)+1}\big(1 + \max\Big\{\frac{d(v)}{d(v)+1} - \sum\limits_{u\in N(v)} \frac{1}{d(u)+1}, 0\Big\}\big)$ on $\alpha(G)$, where $N(v)$ and $d(v) = |N(v)|$ denote the neighborhood and the degree of a vertex $v \in V(G)$, respectively. However, Selkow's original proof of this result is incorrect. We give a new probabilistic proof of Selkow's bound here.

### The CPE Network: Scientific Impact of the Combinatorial Problems and Exercises

**Roland Molontay**
Budapest, Hungary
joint with Béla Barabás, Ottilia Fülöp

László Lovász has a lot of influential and exceptionally highly cited papers. We consider here his Combinatorial Problems and Exercises book that has an essential role in education of many thousands of students, giving them the great opportunity to learn from a book enjoyed by its readers. This book has a great role in teaching of advanced mathematics via hundreds of useful progressive problem-solving strategies and techniques. It offers first a single idea (hint), then the full solution for those who intend to start research in combinatorics or related branches of mathematics. According to Google Scholar, 1848 scientific documents have referred to this book. We have access only to the most cited 1000 articles or books of the above-mentioned documents. We intend to illustrate the great influence of this book that has inspired hundreds of scientists.

Here we construct and analyze the co-authorship network determined by the collaborating authors of the citing documents.

### THE COMPLEXITY OF $t$-RESTRICTED OPTIMAL PEBBLING NUMBER OF GRAPHS

**László Papp F.**
Budapest, Hungary

Let $G$ be a graph with a distribution of pebbles on its vertices. A pebbling move consists of removing two pebbles from one vertex and placing one pebble on an adjacent vertex. The optimal pebbling number $\pi^*(G)$ is the smallest number of pebbles which can placed on the vertices of $G$ such that, for any vertex $v$ of $G$, there is a sequence of pebbling moves resulting in at least one pebble on $v$.

The optimal pebbling number is a well studied graph parameter. Milans and Clark proved that deciding whether $\pi^*(G) \leq k$ is NP-complete.

In the restricted version of pebbling a positive integer $t$ is given and at the initial stage we only consider pebble distributions where each vertex has at most $t$ pebbles. Therefore the $t$-restricted optimal pebbling number $\pi^*_t(G)$ is the smallest number of pebbles which can placed on the vertices of $G$ in such a way that no vertex of has more than $t$ pebbles and for any vertex $v$ of $G$, there is a sequence of pebbling moves resulting in at least one pebble on $v$.

Hedetniemi *et al.* showed several connections between the 2-restricted optimal pebbling number, the domination number and the Roman domination number.

We show that deciding wheter $\pi^*_t(G) \leq k$ is also NP-complete when $t \geq 2$.

### DECOMPOSING GRAPHS INTO EDGES AND TRIANGLES

**Yani Pehova**
University of Warwick, United Kingdom
joint with Dan Král', Taísa Martins and Bernard Lidický

We prove the following 30-year old conjecture of Győri and Tuza: the edges of every $n$-vertex graph $G$ can be decomposed into complete graphs $C_1, ..., C_l$ of orders two and three such that $|C_1| + ... + |C_l| \leq (1/2 + o(1))n^2$.
In a paper from 1966 Erdős, Goodman and Pósa show that for every graph $G$ of order $n$ there exists a decomposition of $E(G)$ into at most $n^2/4$ complete graphs. In fact, it is enough to consider only copies of $K_2$ and $K_3$ for this decomposition. This is best possible, as witnessed by the complete balanced bipartite graph. Later it was shown by Győri and Kostochka in 1980, and independently by Chung in 1981, that if one seeks to minimise not the number, but the sum of the sizes of cliques in a decomposition, the corresponding minimum is $n^2/2$. That is, every graph $G$ of order $n$ can be decomposed into cliques $C_1, ..., C_l$ such that $\sum |C_i| \leq n^2/2$. It was later conjectured by Győri and Tuza that using only $K_2$ and $K_3$ is enough for this problem as well.
We prove an asymptotic version of this conjecture, namely that every graph $G$ of order $n$ can be decomposed into cliques $C_1, ..., C_l$ of orders two and three, so that $|C_1| + ... + |C_l| \leq (1/2 + o(1))n^2$. Our proof has two main steps: first we obtain the corresponding result for a fractional decomposition, and then

translate that into an asymptotic statement about integer decompositions using well-known results in the area.
Many open problems around this topic remain. It is conjectured that in fact any graph has a triangle-edge decomposition with $|C_1| + ... + |C_l| \leq n^2/2 + 2$ (an exact version of our result). A similar open problem due to Erdős states that every graph has a decomposition into complete graphs $C_1, ..., C_l$ such that $\sum |C_i| - 1 \leq n^2/4$.
This is joint work with Dan Král', Taísa Martins and Bernard Lidický.

### Approximating Sparse Graphs: The Random Overlapping Communities Model

**Samantha Petti**
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joint with Santosh Vempala

What is the limit of the sequence of hypercube graphs? More generally, how can we approximate sparse graphs and sequences of sparse graphs (with average degree unbounded and $o(n)$)? At a qualitative level, Szemerédi's regularity lemma and graphons form an essentially complete theory for the approximation of *dense* graphs. As a consequence of the regularity lemma, one can approximate the homomorphism density of any fixed size graph (from the left or right) and the size of any cut; the partition itself can be constructed algorithmically and is easy to sample. Every sequence of dense graphs has a subsequence that converges to a graphon, a probability distribution over the unit square which captures the limit of homomorphism densities and normalized cuts of the graphs. Such a theory is missing for sparse graphs (with $o(n^2)$ edges).

We focus on approximating the first $k$ moments of the graph spectrum (equivalent to the numbers of closed $k$-walks) appropriately normalized. We introduce a simple, easy to sample, random graph model that captures the limiting spectra of many sequences of interest, including the sequence of hypercube graphs. The Random Overlapping Communities (ROC) model is specified by a distribution on pairs $(s, q)$, $s \in \mathbb{Z}_+$, $q \in (0, 1]$. A graph on $n$ vertices with average degree $d$ is generated by repeatedly picking pairs $(s, q)$ from the distribution, adding an Erdős-Rényi random graph of edge density $q$ on a subset of vertices chosen by including each vertex with probability $s/n$, and repeating this process so that the expected degree is $d$. We give a characterization based on the Stieltjes moment condition for when a vector of normalized closed walk counts can be realized by a family of ROC parameters.

We also show that the model is an effective approximation for individual graphs. For almost all possible triangle-to-edge and four-cycle-to-edge ratios, there exists a pair $(s, q)$ such that the ROC model with this single community type produces graphs with both desired ratios, a property that cannot be achieved by stochastic block models of bounded description size. Moreover, ROC graphs exhibit an inverse relationship between degree and clustering coefficient, a characteristic of many real-world networks.

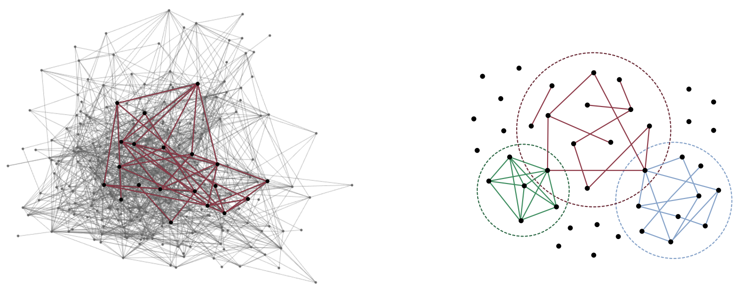

Figure 1: Left: In each step of the construction of a ROC$(n, d, s, q)$ graph, an instance of $G_{|S|,q}$ is added on a set of $S$ of randomly selected vertices. Right: Three communities in a ROC graph.

### A BRIDGE BETWEEN NUMERICAL SEMIGROUPS AND SETS OF SZEMERÉDI TYPE

**M. Pastora Revuelta**
Sevilla, Spain
joint with S.D. Adhikari, L. Boza, S. Eliahou and M.I. Sanz

Given any length $k \geq 3$ and density $0 < \delta \leq 1$, we introduce and study the set $\mathrm{Sz}(k, \delta)$ consisting of all positive integers $n$ such that every subset of $\{1, 2, \ldots, n\}$ of density at least $\delta$ contains an arithmetic progression of length $k$. A famous theorem of Szemerédi guarantees that this set is not empty. We show that $\mathrm{Sz}(k, \delta) \cup \{0\}$ is a numerical semigroup. In addition, given any real number $0 < d \leq 1$, we consider the set $\mathrm{SZ}(k, d)$ consisting of all $\mathrm{Sz}(k, \delta)$ where $d < \delta \leq 1$. We prove that $\mathrm{SZ}(k, d)$ is finite and explicitly determine $\mathrm{SZ}(3, 8/33)$ and $\mathrm{SZ}(4, 22/47)$.

### ON THE TRACTABILITY OF UN/SATISFIABILITY

**Latif Salum**
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X3SAT leads to the proof of $P = NP$, because it is easy to verify incompatibility of any literal $r_j$. A truth assignment $r_j = \mathbf{T}$ over any X3SAT formula $\phi$ is said to be incompatible if $\phi(r_j)$ is unsatisfiable, denoted by $\not\models \phi(r_j)$, in which $\phi(r_j) := r_j \wedge \phi$. Then, it is *tractable* to verify unsatisfiability of $\phi(r_j)$, because the exactly-1 disjunction transforms $\phi(r_j) := r_j \wedge \phi$ into $\phi(r_j) = \psi(r_j) \wedge \phi'(r_j)$ such that $\psi(r_j)$ and $\phi'(r_j)$ are disjoint, and $\psi(r_j)$ is a *minterm* (a conjunction of literals). Hence, it is *easy* to verify if $\not\models \psi(r_j)$, sufficient for $\not\models \phi(r_j)$. Furthermore, $\not\models \psi(r_j)$ is necessary, because $\phi'(r_j)$ the X3SAT becomes satisfiable iff $\psi(r_i)$ the minterm is satisfiable for every $r_i$ in $\phi$. Therefore, it is *tractable* to verify satisfiability of $\phi$ by removing any $r_j$ if $\not\models \psi(r_j)$. That is, $\phi$ is satisfiable iff $\psi(r_i)$ is satisfiable for every $r_i$ in $\phi$. The following introduces the basic idea.

Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be some X3SAT formula over clauses $C_k$, in which any clause $C_k = (r_i \odot r_j \odot r_u)$ denotes an exactly-1 disjunction $\odot$ of literals. Then, $C_k$ is satisfiable iff *exactly one* of $\{r_i, r_j, r_u\}$ is true.

Consider any assignment $r_j = \mathbf{T}$, hence $\phi(r_j) := r_j \wedge \phi$. This assignment leads to a *deterministic* chain of *reductions* of clauses $C_k$ over $\phi(r_j)$. This chain, initiated by $r_j \Rightarrow \neg\overline{r}_j$, is constructed as follows: $r_j \Rightarrow r_j \wedge \neg\overline{r}_i \wedge \neg r_u$ to satisfy any clause $(r_j \odot \overline{r}_i \odot r_u)$, which is said to *collapse* to the minterm $(r_j \wedge r_i \wedge \overline{r}_u)$. Hence, $r_j \Rightarrow r_j \wedge r_i \wedge \overline{r}_u$. Also, any $(\overline{r}_j \odot \overline{r}_u \odot r_v)$ *declines* by $\neg\overline{r}_j$ to $(\overline{r}_u \oplus r_v)$. Hence, $\overline{r}_u \Rightarrow \overline{r}_u \wedge \overline{r}_v$. Thus, $\phi(r_j) = r_j \wedge r_i \wedge \overline{r}_u \wedge \overline{r}_v \wedge \phi^*$. Then, $r_i \wedge \overline{r}_u \wedge \overline{r}_v$ lead to further reductions in $\phi^*$. If the reductions over $\phi(r_j)$ terminate, $r_j \wedge \phi$ is transformed into $\psi(r_j) \wedge \phi'(r_j)$ such that $\psi(r_j)$ and $\phi'(r_j)$ are disjoint. Otherwise, $r_j$ is incompatible. If the reductions over $\phi$ terminate, $\phi$ is transformed into $\phi_s = \psi \wedge \phi'$ such that $\psi$ and $\phi'$ are disjoint. Otherwise, $\phi$ is unsatisfiable.

**Assumption**: $\not\models \phi_s(r_j)$ is verified *solely* by $\not\models \psi_s(r_j)$ during the reductions over $\phi$. As a result, if $\psi_{\hat{s}}(r_j)$ is satisfiable, then whether $\not\models \phi'_{\hat{s}}(r_j)$ is *disregarded*, which is of X3SAT, while $\not\models \psi_s(r_i)$ is checked for every $r_i$ in $\phi$ and $s \geq \hat{s}$. Then, $r_j$ is removed from $\phi_s$ if $\not\models \psi_s(r_j)$, which results in $\neg r_j \Rightarrow \overline{r}_j$ and $\phi_{s+1} = \overline{r}_j \wedge \phi_s$.

**Claim**: $\phi$ is satisfiable iff $\psi(r_i)$ is satisfiable for every $r_i$ in $\phi$. Also, it is *redundant* to verify whether $\not\models \phi'_{\hat{s}}(r_j)$ for any $\hat{s}$, because it is *indirectly* verified by $\not\models \psi_s(r_i)$ for any $r_i$ in $\phi$ and $s \geq \hat{s}$. Hence, $\not\models \phi_s(r_j) \iff \not\models \psi_s(r_j)$ for any $s$.

**Justification**: $\psi(r_i)$ is constructed over $\phi$, and $\psi(r_i|r_j)$ over $\phi'(r_j)$. Because $\phi$ covers $\phi'(r_j)$, $\psi(r_i)$ covers $\psi(r_i|r_j)$. Therefore, if $\psi(r_i)$ is satisfiable, then $\psi(r_i|r_j)$ is satisfiable, hence $\psi(r_i) \models \psi(r_i|r_j)$. Furthermore, the minterms $\psi(r_{i_0})$, $\psi(r_{i_1}|r_{i_0})$, $\psi(r_{i_2}|r_{i_0}, r_{i_1})$, and $\psi(r_{i_3}|r_{i_0}, r_{i_1}, r_{i_2})$ form a *partition* of $\phi(r_{i_0})$ such that any minterm $\psi(r_{i_0})$, $\psi(r_{i_1}|r_{i_0})$, $\psi(r_{i_2}|r_{i_0}, r_{i_1})$, and $\psi(r_{i_3}|r_{i_0}, r_{i_1}, r_{i_2})$ is satisfiable, because $\psi(r_i)$ is satisfiable for any $r_i$ in $\phi$ when the reductions over $\phi$ terminate, and since $\psi(r_i) \models \psi(r_i|r_j)$ holds. As a result, both *independent* and *satisfiable* minterms compose $\phi(r_{i_0})$ the X3SAT. Therefore, $\phi(r_{i_0})$ is satisfiable. Consequently, any satisfiable assignment for $\phi_s = \psi \wedge \phi'$ can be constructed *arbitrarily*, e.g., $\psi \wedge \psi(r_{i_0}) \wedge \psi(r_{i_1}|r_{i_0}) \wedge \psi(r_{i_2}|r_{i_0}, r_{i_1}) \wedge \psi(r_{i_3}|r_{i_0}, r_{i_1}, r_{i_2})$.

## On the number of vertices of the polytope of integer partitions

**Vladimir A. Shlyk**
Minsk, Belarus

We consider every integer partition of a natural $n$ as a nonnegative integer point $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, a solution to the equation $x_1 + 2x_2 + \ldots + nx_n = n$. The polytope $P_n$ of integer partitions of $n$ is the convex hull of such points $x$. Of special interest are vertices of $P_n$ since every partition is a convex combination of some vertices. A difficulty in their study is that no criterion for vertices is known, though recognizing vertices is proved to be a polynomially solvable problem. In this work we concentrate on the function $v(n)$, the number of vertices of $P_n$.

Computation of vertices of $P_n, n \leq 105$, shows that their number does not grow up monotonously: it drops down at every even $n$ and its peaks at prime

$n$'s are higher than others. We show that the saw-toothed shape of $v(n)$ may be caused by a class of partitions of even $n$ that result by joining two partitions of $\frac{n}{2}$. They were counted by N. Metropolis. We demonstrate that most of partitions that are not vertices are convex combinations of two other partitions and the larger part of such partitions of even $n$ are of the Metropolis type.

We reveal that divisibility of $n$ by 3 also reduces $v(n)$ and explain this by large number of partitions that are convex combinations of three but not two others. We characterize the coefficients in convex representations of these partitions (actually, we do this for analogous integer points in arbitrary integral polytope).

To approach the prime $n$ phenomenon, we classify integer numbers by their proximity, in a certain sense, to primes. We consider the classes of integers $N_k = \{n|n = pk,\ p \text{ prime}\}$, $k = 2, 3, 4, 5, ...$, and the restriction functions $v_k(n) = v(n),\ n \in N_k$. Using Mathematica for approximating the known values of functions $v_k(n)$ allows us to demonstrate visually that the graph of $v(n)$ has the structure of a layered cake. It is stratified into layers, the graphs of $v_k(n)$. The topmost line is $v_1(n)$, which corresponds to the class of primes. The graphs of $v_7(n), v_5(n), v_3(n)$, and $v_2(n)$ are consecutively disposed below it. Our main conjecture claims that the number of vertices of $P_n$ depends on the divisors of $n$. The major influence on $v(n)$ is rendered by the smallest divisor of $n$ and every successive divisor makes its additional contribution to lowering it.

We also present an embryonic argument for the number of vertices of the corner polyhedron, the atom of integer programming, to have similar features. In conclusion, we outline some problems which the discovered dependence of $v(n)$ on factorization of $n$ gives rise to.

## The Multiset Dimension of Graphs

**Rinovia Simanjuntak**
Bandung, Indonesia
joint with Presli Bintang Mulia, Suhadi Wido Saputro, and Tomáš Vetrík

We introduce a variation of the metric dimension, called the multiset dimension.

The representation multiset of a vertex $v$ with respect to $W$ (which is a subset of the vertex set of a graph $G$), $r_m(v|W)$, is defined as a multiset of distances between $v$ and the vertices in $W$ together with their multiplicities. If $r_m(u|W) \neq r_m(v|W)$ for every pair of distinct vertices $u$ and $v$, then $W$ is called a resolving set of $G$. If $G$ has a resolving set, then the cardinality of a smallest resolving set is called the multiset dimension of $G$, denoted by $md(G)$. If $G$ does not contain a resolving set, we write $md(G) = \infty$.

We present some early results on the multiset dimension.

## Reconstructing Colourings of Finite Groups

**Jan Simon**
Aachen, Germany

In combinatorics, graph theory and related fields the term *reconstruction problem* typically refers to a problem of the following kind: One is given the so-called *deck* of an object, consisting of partial structures given only up to some notion of equivalence, and is then asked to retrieve the original object from this information – again only up to equivalence. Famous examples of this very general principle include the *(Edge) Reconstruction Conjecture* in graph theory and geometric reconstruction problems in the Euclidean plane.
Here we consider colouring reconstruction problems for a finite group $G$ acting on a finite set $X$. We denote the set of colours by $F$. The $G$-action on $X$ induces a $G$-action on the set $F^X = \{c : X \longrightarrow F\}$ of colourings of $X$ as well as on the set $\bigcup_{K \subseteq X} F^K$ of partial colourings of $X$. For $k \leq |X|$ the *combinatorial $k$-deck of a colouring* $c \in F^X$ consists of the multiset of $G$-classes $[c_{|K}]_G$ of the restrictions of $c$ to all $k$-element subsets $K \subseteq X$. In other words it counts, for each partial colouring of $X$ with domain of size $k$, how many equivalent copies of it can be found in $c$. This new notion of deck generalizes the well-known $k$-deck for subsets in finite groups and is a natural alternative to another (more analytical) notion of deck for real- or integer-valued functions considered by various authors. We compare these kinds of decks and show that the combinatorial deck is to arbitrary high degree stronger than the traditional analytical one. However, we also show that even for the combinatorial deck *there is no global colouring reconstruction number*, i.e. no $k_0 \in \mathbb{N}$, such that for all finite groups $G$, all finite $G$-sets $X$ and all (finite) colour sets $F$ each colouring $c \in F^X$ is reconstructible from its combinatorial $\min\{k_0, |X|\}$-deck.
The question whether there is a global *subset* reconstruction number is still open. By looking at certain subsets in the finite simple Mathieu groups we show that it would have to be at least 6, though.
In addition, we generalize a surprising reconstruction technique for the 3-deck – which has so far only been successfully applied in the case of subsets of cyclic groups – to colourings of arbitrary finite groups. As an application we show that if $G$ is a finite $p$-group (acting regularly on itself) then any colouring $c \in F^G$ possessing a colour class with size not divisible by $p$ is reconstructible from the 3-deck.

## On the properties of minimally tough graphs

**Kitti Varga**
Budapest, Hungary
joint with Gyula Y. Katona

Let $t$ be a positive real number. A graph is called $t$-tough, if the removal of any cutset $S$ leaves at most $|S|/t$ components. The toughness of a graph is the largest $t$ for which the graph is $t$-tough. A graph is minimally $t$-tough, if the toughness of the graph is $t$ and the deletion of any edge from the graph decreases the toughness. Kriesell conjectured that for every minimally 1-tough graph the minimum degree $\delta(G) = 2$. We show that in every minimally 1-tough graph $\delta(G) \leq \frac{n}{3} + 1$ (joint work with Dániel Soltész).

The complexity class DP is the set of all languages that can be expressed as the intersection of a language in NP and a language in coNP. We prove that recognizing minimally $t$-tough graphs is DP-complete for any positive integer $t$ and for any positive rational number $t \leq 1$ (joint work with István Kovács).

We also investigate the minimum degree and the recognizability of minimally $t$-tough graphs in the class of chordal graphs, split graphs, claw-free graphs and $2K_2$-free graphs.

### On degree thresholds of cycles in oriented graphs

**Jan Volec**
Montrel, Canada
joint with Roman Glebov, Andrzej Grzesik

A well-known conjecture of Caccetta and Häggkvist from 1978 states that if the minimum out-degree $\delta^+(G)$ of an oriented graph $G$ is at least $k$, then $G$ contains a cyclically oriented cycle of length at most $v(G)/k$. The conjecture was a generalization of an earlier conjecture of Behzad, Chartnard and Wall [Fund. Math. 69 (1970), 227–231], where the assumption on the out-degree of $G$ is replaced by a one on the so-called semi-degree of $G$. The semi-degree of an oriented graph $G$, which we denote by $\delta^\pm(G)$, is the minimum of $\delta^+(G)$ and $\delta^-(G)$. Despite a lot of effort given to these conjectures over the last 40 years, both of them remain widely open.

Inspired by these two conjectures, Kelly, Kühn and Osthus [J. Combinatorial Theory Ser. B 100 (2010), 251–264] conjectured a precise value of the minimum semi-degree that guarantees an oriented graph to contain a cyclically oriented cycle of a given length $\ell$. More precisely, for an integer $\ell \geq 4$, let $k(\ell)$ be the smallest integer greater than 2 such that $k$ does not divide $\ell$, and let $\vec{C}_\ell$ be the cyclically oriented cycle of length $\ell$.

**Conjecture**[Kelly, Kühn, Osthus] For every integer $\ell \geq 4$ there exists $n_0 = n_0(\ell)$ such that the following holds. If $G$ is an oriented graph with $v(G) \geq n_0$ and $\delta^\pm(G) \geq \left\lfloor \frac{v(G)}{k(\ell)} \right\rfloor + 1$, then $G$ contains $\vec{C}_\ell$ as a subgraph.

In their work, Kelly, Kühn and Osthus proved the conjecture for all the cycle lengths $\ell$ not divisible by 3.

In our work, we first confirm the variant of this conjecture for closed walks instead of cycles, which immediately yields an asymptotic solution of the conjecture.

**Theorem** For every integer $\ell \geq 4$ the following holds. If $G$ is an oriented graph with $\delta^\pm(G) \geq \left\lfloor \frac{v(G)}{k(\ell)} \right\rfloor + 1$, then $G$ contains a closed walk of length $\ell$.

Next, we observe that the conjecture as stated is actually false, and for all $\ell \geq 4$ such that $k(\ell) \neq 4$ or $k(\ell) = 4$ and $\ell \not\equiv 3 \mod 4$, we construct arbitrary large oriented graphs $G_\ell$ with $\delta^\pm(G) = \left\lfloor \frac{v(G_\ell)}{k(\ell)} \right\rfloor + 1$ that are $\vec{C}_\ell$-free.

The last result of our work provides an exact bound for oriented graphs that is within a constant factor from the originally conjectured bound.

**Theorem** For every $\ell \geq 4$ there exists a constant $f(\ell)$ such that the following holds. If $G$ is an oriented graph with $\delta^{\pm}(G) \geq \left\lfloor \frac{v(G)}{k(\ell)} \right\rfloor + f(\ell)$, then $G$ contains $\vec{C}_\ell$ as a subgraph.

## Sparse Kneser graphs are Hamiltonian


**Bartosz Walczak**
Kraków, Poland
joint with Torsten Mütze, Jerri Nummenpalo


For integers $k \geq 1$ and $n \geq 2k+1$, the *Kneser graph* $K(n,k)$ is the graph whose vertices are the $k$-element subsets of $\{1,\ldots,n\}$ and whose edges connect pairs of subsets that are disjoint. The Kneser graphs of the form $K(2k+1,k)$ are also known as the *odd graphs.* We settle an old problem due to Meredith, Lloyd, and Biggs from the 1970s, proving that for every $k \geq 3$, the odd graph $K(2k+1,k)$ has a Hamilton cycle. This and a known conditional result due to Johnson imply that all Kneser graphs of the form $K(2k+2^a,k)$ with $k \geq 3$ and $a \geq 0$ have a Hamilton cycle. We also prove that $K(2k+1,k)$ has at least $2^{2^{k-6}}$ distinct Hamilton cycles for $k \geq 6$. Our proofs are based on a reduction of the Hamiltonicity problem in the odd graph to the problem of finding a spanning tree in a suitably defined hypergraph on Dyck words.

## Bypassing Intractability via Errorless Heuristics


**Rupei Xu**
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joint with András Faragó


Heuristic algorithms have been used for a long time to tackle problems that are known or conjectured intractable. A heuristic algorithm is one that provides a correct decision for most inputs, but may fail on some. We focus on the case when failure exclusively means that the algorithm does not return any answer, rather than returning a wrong result. These algorithms are called *errorless heuristics,* because they never return an incorrect answer.

A reasonable quality measure for heuristics is the failure rate over the set of $n$-bit instances. When no efficient exact algorithm is available for a problem, then, ideally, we would like one with vanishing failure rate. We show, however, that this is hard to achieve: unless a complexity theoretic hypothesis fails, some *NP*-complete problems cannot have a polynomial-time errorless heuristic algorithm with any vanishing failure rate, i.e., with any failure rate that converges to 0.

On the other hand, we prove that vanishing, even exponentially small, failure rate is achievable, if we use a somewhat different accounting scheme to count the failures. This is based on special sets, that we call *$\alpha$-spheres.* These are the images of the $n$-bit strings under a bijective, polynomial-time computable and polynomial-time invertible encoding function $\alpha$.

Our main result is that a large class of decision problems has polynomial-time errorless heuristic algorithms, with exponentially vanishing failure rates on the $\alpha$-spheres. The class contains the set of paddable languages, which, in turn, includes *all known intuitively natural NP*-complete problems. Furthermore, the proof actually supplies a general method to construct the desired encoding and the errorless heuristic, thus providing a novel algorithm design technique. This, in a certain sense, "bypasses" the intractability of all known natural *NP*-complete problems.

Beyond the proof, we also demonstrate the method experimentally, showing its performance on some natural *NP*-complete problems.

## Legal Assignments, the EADAM Algorithm, and Latin Marriages

**Xuan Zhang**
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joint with Yuri Faenza

We investigate the many-to-one assignment problem arising in matching markets and generalize the well-known stable matching problem. Consider the classical *school-student (or hospital-residence) assignment problem*, where each student has a strict preference order over a (possibly incomplete) set of schools, and each school in turn has a strict preference order over a (possibly incomplete) set of students. Each school also has a maximum number of students it can be assigned to. An assignment is stable if there does not exist a student and a school that prefer each other to their current assignments. It is well-known that given an instance, stable assignments always exist, and the set of stable assignments forms a distributive lattice, where one can move from the supremum of the lattice to the infimum via *rotation elimination* operations. This structural property enables many fast algorithms on a wide range of optimization / enumeration problems.

A recent paper [3] introduced the concept of *Legal assignments*, which is a superset of stable assignments. This concept corresponds to *von Neumann-Morgenstern (vNM) stable set* for cooperative games. In [3], it is shown that the set of legal assignments also has a lattice structure. In addition, the student-optimal legal assignment is Pareto-optimal for the students and it coincides with the output of Kesten's *Efficiency Adjusted Deferred Acceptance Mechanism* (*EADAM*) [1].

The goal of our work is to provide an improved structural and algorithmic understanding of those concepts. Our main results are as follows. We first show that legal assignments are *structurally* equivalent to stable assignments, and *algorithmically* not harder than stable *marriages* (i.e., one-to-one assignments). We show that, for any instance, the set of *legal* assignments coincides with the set of *stable* assignments in a sub-instance, which is obtained by removing a set $\overline{E}$ from the edges $E$ of the original instance. This implies that legal assignments inherit all structural properties of stable assignments. We also show that $\overline{E}$ can be found in time $O(|E|)$. This enables us to extend the concept of rotations to legal assignment settings, which implies that optimization of linear functions over and enumeration of the set of legal assignments can be solved efficiently and output-efficiently, respectively.

Lastly, we explore the relationship between legal matchings and *Latin marriages*, in the spirit of a classical, long-standing open question of Knuth [2] on the maximum number of stable matchings an instance with $n$ men and $n$ women may have. We first show that all Latin instances, which are known for having *many* stable matchings, are legal, i.e. $\overline{E} = \emptyset$. We then construct instances admitting one stable matching, but *legalization* of which unveils the underlying Latin instance. This implies that legalization can increase the number of stable matchings by an exponential factor.


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[2] Knuth, D. E. (1997). Mariages stables. Les Presses de L'Université de Montréal, 1976. English translation in Stable marriage and its relation to other combinatorial problems, volume 10 of CRM Proceedings and Lecture Notes. American Mathematical Society.

[3] Morrill, T. (2016). Which School Assignments Are Legal?. Working paper, North Carolina State University.

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