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First-Order Logic Investigation of Relativity Theory with an Emphasis on Accelerated Observers

SUMMARY

Applying mathematical logic in foundations of relativity theories is not a new idea at all, it goes back to such leading mathematicians and philosophers as Hilbert, Reichenbach, Carnap, Gödel, Tarski, Suppes and Friedman among others. The work of our school of Logic and Relativity led by Andréka and Németi is continuation to their research.

This thesis is mainly about extensions of the first-order logic axiomatization of special relativity introduced by Andréka, Madarász and Németi. These extensions include extension to accelerated observers, relativistic dynamics and general relativity; however, its main subject is the extension to accelerated observers (AccRel). One surprising result is that natural extension to accelerated observers is not enough if we want our theory to imply certain experimental facts, such as the twin paradox. Even if we add the whole first-order theory of real numbers to this natural extension, it is still not enough to imply the twin paradox. Nevertheless, that does not mean that this task cannot be carried out within first-order logic since by approximating a second-order logic axiom of real numbers, we introduce a first-order axiom schema that solves the problem. Our theory AccRel nicely fills the gap between special and general relativity theories, and only one natural generalization step is needed to achieve a first-order logic axiomatization of general relativity from it. We also show that AccRel is strong enough to make predictions about the gravitational effect slowing down time.

Our general aims are to axiomatize relativity theories within pure first-order logic using simple, comprehensible and transparent basic assumptions (axioms); to prove the surprising predictions (theorems) of relativity theories from a few convincing axioms; to eliminate tacit assumptions from relativity by replacing them with explicit axioms formulated in first-order logic (in the spirit of the first-order logic foundation of mathematics and Tarski's axiomatization of geometry); and to investigate the relationship between the axioms and the theorems.