PROBLEMS

1. Is it true that there is an absolute constant c so that every graph of m edges contains a subgraph of $cm^{3/4}$ edges which does not contain a rectangle (i.e. a circuit of four edges)? The complete graph of $\binom{n}{2} = m$ edges shows that this conjecture if true is certainly best possible.

Let us now assume that our graph has n vertices and m edges and the valence of each vertex is less than $c_1 \frac{m}{n}$. Then our graph contains perhaps a subgraph of $c_2 (mn)^{1/2}$ edges which does not contain a rectangle. The order of magnitude $(mn)^{1/2}$ if true is certainly best possible.

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2. Trivially every $3k$ chromatic graph contains k_1 odd vertex independent circuits. Perhaps every $3k - 1$ chromatic critical graph having more than $n_0(k)$ vertices contains k odd vertex independent circuits. In particular, is it true that every 5-chromatic critical graph having sufficiently many vertices contains two odd vertex independent circuits? GALLAI showed that this is false for 4-chromatic graphs.

LOVÁSZ in trying to prove this made the following conjecture: Let G be a k-chromatic graph which does not contain a complete k-gon and let $a > 1$ and $b > 1$ be arbitrary positive integers satisfying $a + b = k + 1$. Then we can split the vertices of G into two classes so that the graph spanned by the vertices of the first class has chromatic number $\geq a$ and the graph spanned by the vertices of the second class has chromatic number $\geq b$.

By taking $a = 3$ we obtain from LOVÁSZ's conjecture that every graph of chromatic number $3k - 1$ which does not contain a complete $(3k - 1)$-gon contains k vertex-independent odd circuits.

LOVÁSZ further remarks that even the following special case $(a = 2)$ does not seem to be easy to prove. Every k-chromatic graph G which does not contain a complete k-gon contains two vertices, x_1 and x_2, which are joined by an edge so that $G - x_1 - x_2$ has chromatic number $\geq k - 1$.

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3. Let G be a graph of n vertices. Assume that for every $m \leq n$ every subgraph spanned by m vertices of G contains an independent set having at least $\frac{m-k}{2}$ vertices. Is it then true that G has chromatic number $\leq k+2$? The complete $(k+2)$-gon shows that this conjecture if true is certainly best possible.

The conjecture is trivial for $k = 0$ but seems to present difficulties already for $k = 1$.

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4. Is it true that in any finite connected graph there exists a vertex such that every longest simple path of the graph contains this vertex?

Added in prof: H. WALTHER proved that the answer to the above question is negative. His construction will apper in the *J. Combinatorial Theory.*

5. *A distancial generalization of Menger's theorem.* Let G be a finite connected graph with the vertex set $V(G)$. x, y, z will denote vertices of G. If G' is a subgraph of G, then $x \in G'$ means $x \in V(G')$. For any $x, y \in G$ the distance of x and y (in G) will be denoted by $d(x, y)$. If P_1 and P_2 are simple paths of G and $z \in G$, then let

$$d(P_1, P_2) = \min_{x \in P_1, y \in P_2} d(x, y) \text{ and } d(z, P_1) = \min_{x \in P_1} d(z, x).$$

Let A and B be two fixed disjoint subsets of $V(G)$. A path which has one endpoint in A and the other in B and has no other vertex in common with $A \cup B$ will be called an AB-path. S will denote the set of all simple AB-paths of G.

The simplest case of MENGER's theorem states that if $d(P_1, P_2) = 0$ for every $P_1, P_2 \in S$, then there exists an $x \in G$ with $d(x, P) = 0$ for every $P \in S$.

Suppose now that $d(P_1, P_2) \leq 1$ for every $P_1, P_2 \in S$. Then in the special case in which $A \cup B = V(G)$ and G contains only AB-edges (i.e. G is a bipartite graph) there exists an $x \in G$ with $d(x, P) \leq 1$ for every $P \in S$. However, in the general case there does not exist such an x. Is it true that in any case there exists an $x \in G$ for which

$$d(x, P) \leq 2 \quad \text{for every} \quad P \in S?$$

6. B. BOLLOBÁS and L. PÓSA proved that in any finite undirected graph G in which any two circuits have a vertex in common there are three vertices so that every circuit of G pathes through one of these vertices. Let C be the class of all finite directed graphs in which any two directed circuits have a vertex in common. Does there exist a fixed natural number k so that in any $G \in C$ there are at most k vertices for which any directed circuit of G pathes through one of these vertices?

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