CONVERGENCE OF APPROXIMATING RATIONAL FUNCTIONS OF PRESCRIBED TYPE

J.L.Walsh (Cambridge, USA)

Let E be a closed bounded set whose complement is connected, and regular in the sense that it possesses a Green's function $G(z)$ with pole at infinity. Let E_σ denote generically the locus $G(z) = \log\sigma,\ (\sigma > 0)$. Let the function $f(z)$ be analytic on E, meromorphic with precisely poles interior to E_ρ, $1 < \rho \leq \infty$. Let the rational functions $r_{n\nu}(z)$ of respective types (n, ν), namely of form

$$r_{n\nu}(z) = \frac{a_0 z^n + a_1 z^{n-1} + \dots + a_n}{b_0 z^\nu + b_1 z^{\nu-1} + \dots + b_\nu}, \quad \sum |b_k| \neq 0,$$

satisfy

$$\text{(1)} \qquad \lim_{n \to \infty} \sup \| f(z) - r_{n\nu}(z) \|^{1/n} \leq \rho$$

with the Tchebycheff (uniform) norm. Then for n sufficiently large the function $r_{n\nu}(z)$ has precisely ν finite poles, which approach $(n \to \infty)$ respectively the ν poles of $f(z)$ interior to E_ρ. If D denotes the interior of E_ρ with the ν poles of $f(z)$ deleted, the sequence $r_{n\nu}(z)$ converges to $f(z)$ throughout D, uniformly on compact sets. If the $r_{n\nu}(z)$ are rational functions of the given types of best approximation to $f(z)$ on E, and if ρ is the largest number such that $f(z)$ is meromorphic with precisely ν poles interior to E_ρ, then (1) holds with the equality sign.

ON SOME APPLICATIONS OF PROBABILITY METHODS TO FUNCTION THEORY

P. Erdös (Budapest, Hungary)

A sequence of integers $m_1 < m_2 < \cdots$ is said to satisfy gap condition $\mathcal{A}$ if there is a sequence $k_i - l_i \to \infty$ so that

/I/ $$(m_{k_i} - m_{l_i})^{1/(k_i - l_i)} \to 1.$$

Every sequence having Hadamard gaps clearly satisfies /I/ but /I/ does not imply $m_k^{1/k} \to 1$.

In this note we will point out the common probabilistic source of several theorem involving condition $\mathcal{A}$ e.g.

I. Let $m_1 < m_2 < \cdots$ satisfy condition $\mathcal{A}$, then there is a power series $\sum_1^\infty a_k z^{m_k}$ converging uniformly but not absolutely in $|z| \leq 1$.

II. Let $m_1 < m_2 < \cdots$ satisfy condition $\mathcal{A}$, then there is a power series $\sum_1^\infty a_k z^{m_k}$, $|a_k| \to 0$, which diverges for every z satisfying $|z| = 1$.

Several other examples will be cited. It seems possible that our gap condition $\mathcal{A}$ is best possible in all these cases, but this has never been proved.

TRUNCATION ERROR ESTIMATES FOR STIELTJES FRACTIONS

By Peter Henrici and Pia Pfluger. (Schweiz)

Let C be a Stieltjes continued fraction (not necessarily convergent)

$$C(z) = \frac{a_1|}{|z} + \frac{a_2|}{|1} + \frac{a_3|}{|z} + \frac{a_4|}{|1} + \cdots$$

($a_n > 0$, $n = 1, 2, \ldots$; $|\arg z| < \pi$) with the approximants

$$w_0 = 0,\ w_1 = \frac{a_1}{z},\ w_2 = \frac{a_1}{z + a_2}, \ldots .$$

For each $n = 1, 2, \ldots$, we denote by γ_n the circular arc from w_{n-1} to w_n, passing through w_{n+1}, and by Ω_n the compact set bounded by γ_n and by that portion of γ_{n-1} which lies between w_{n-1} and w_n. Theorem 1 : For each $n = 1, 2, \ldots$ (a) Ω_n is convex, (b) $\Omega_{n+1} \subset \Omega_n$, (c) if $0 < |\arg z| < \pi$ he interior of Ω_n is not empty, and every of its points value of a terminating Stieltjes fraction whose first n proximants are $w_1, w_2, \ldots, w_n$. (d) If $C(z)$ is convergent, its value is contained in each Ω_n, and the diameter