SCHLICHT GAP SERIES WHOSE CONVERGENCE ON THE UNIT CIRCLE IS UNIFORM BUT NOT ABSOLUTE

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There exist Taylor series (1) $\sum a_k z^{n_k}$ which converge uniformly, but not absolutely, on the unit circle C. On the other hand, if the function represented by a series (1) is bounded and schlicht in the unit disc, then $\sum n_k |a_k|^2 < \infty$; therefore the condition of schlichtness and uniform, nonabsolute convergence on C imposes restrictions on the gaps which may occur in (1)—for example, it prohibits the convergence of the series $\sum n_k^{-1}$. As a contribution toward the determination of the extent of these restrictions, the authors exhibit a series (1) which is schlicht in the unit disc, converges uniformly but not absolutely on C, and has the property $n_{k+1} - n_k \to \infty$. The series maps the unit disc into a disc whose periphery bears many short, highly branched needles; the needles and their branches point away from the center of the disc.

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